Chapter 5   Fuel - Coolant Heat Transfer

5.1   Introduction

The interface between the fuel and the coolant is centrally important to reactor design since it is here that the limit to power output occurs. Nuclear fission can provide a virtually unlimited heat generation rate, far more than can be transported away by the coolant. Herein we investigate the heat transfer at the fuel site so that this limitation can be factored into the reactor design.

5.2   General Heat Conduction Equation

For a solid, the general energy thermal energy balance equation of an arbitrary volume, $\forall$, is:

$$\iiint_{\forall} \frac{\partial(\rho e)}{\partial t} \, d\forall \; = \; \iiint_{\forall} q'''(\overline{r},t) \, d\forall \; - \; \iint_{S} \overline{q}''(\overline{r},t) \cdot \hat{n} ds \tag{1}$$

where ρ is the material density, e is the internal energy, $\forall$ is the volume, S is the surface area, q''' is the volumetric heat generation, q'' is the heat flux and $\hat{n}$ is the unit vector on the surface. We replace the internal energy with temperature, T, times the heat capacity, c. Using Gauss' Law to convert the surface integral to a volume integral and dropping the volume integral everywhere:

$$\frac{\partial(\rho c T)}{\partial t} \; = \; q'''(\overline{r},t) \; - \; \nabla \cdot q''(\overline{r},t) \tag{2}$$

We further need a relation to specify the heat flux in terms of temperature. In a solid, Fourier's law of thermal conduction applies:

$$q''(\overline{r},t) \; = \; -k \, \nabla T(\overline{r},t) \tag{3}$$

where k is the thermal conductivity. This gives the usable form:

$$\frac{\partial(\rho c T)}{\partial t} \; = \; q'''(\overline{r},t) \; - \; \nabla \cdot k \nabla T(\overline{r},t) \tag{4}$$

The parameters have the following units:
ρ kg/m^3
c   J/(kg $^\circ$K)
k   J/(m$^\circ$K-sec)
q''   J/(m^2-sec) = W/m^2
q'''   J/(m^3-sec) = W/m^3
T $^\circ$K
α defined as k/ρc = m^2/sec.

5.3   Radial Heat Transfer

Consider a typical cylindrical fuel pin composed of fuel meat surrounded by a metal clad, as shown in figure 5.1. There is usually a small gap between the fuel and the clad which offers substantial resistance to heat transfer. The flowing coolant surrounds the pin. We will look at the fuel, gap, clad and coolant separately to develop the temperature profile in each material. Then, we will combine the equations to give the full fuel to coolant temperature profile. It is sufficient for our purposes to focus on the steady state.

5.3.1   Fuel Meat

Equation 4 in the steady state is:
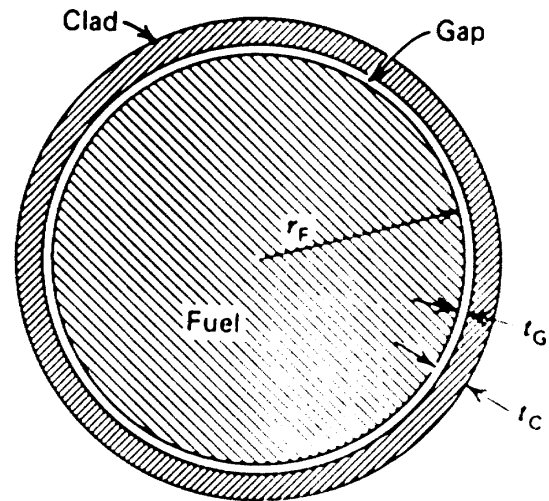
$$- \nabla \cdot \, k_f \nabla T(\overline{r},t) \; = \; q^{\,///}(\overline{r},t) \tag{5}$$

The pins are much longer than their diameter, hence, axial heat conduction can be ignored. In radial coordinates:

$$- \frac{1}{r} \frac{d}{dr}\left(k_f \; r \; \frac{dT}{dr} \right) \; = \; q^{\,///}(r) \tag{6}$$

This can be directly integrated to give:

$$k_f \; r \; \frac{dT}{dr} \; = \; - \; \frac{r^2}{2} \; q^{\,///}(r) \tag{7}$$

Figure 5.1 Radial fuel pin geometry [Source: DUD76, figure 12-3]

The constant of integration is zero since the temperature gradient at r=0 is zero. The thermal conductivity, k, is a strong function of T in fuel. Hence, the subsequent integration of equation 7 is:

$$\int_{T_0}^{T_F} k_f(T) \; dT \; = \; - \; \frac{r_F^2}{4} q^{\,///} \equiv \; \overline{k}_f \; (T_F \, - \, T_0) \tag{8}$$

where the subscript 0 indicates the centre point and the subscript F indicates the fuel meat radius. Since $T=T_0$ at r=0, the constant of integration is again zero. Finally we have:

$$\Delta T_{fuel} \; \equiv \; T_0 \, - \, T_F \; = \; \frac{r_F^2}{4 \overline{k}_f} \; q^{\,///}$$

$$= \; \frac{q^{\,/}}{4 \pi \overline{k}_f} \tag{9}$$

where $q^{\,/} \equiv \pi r_F^2 \; q^{\,///}$ = linear power density

Note that we get the same ΔT for a given q' no matter what the fuel radius. For UO_2 ceramic, k_F is typically 0.02 - 0.03 W/cm°K. At a q' of 500 W/cm, the ΔT is about 1400°C.

5.3.2 Gap

Equation 4 in the steady state for the gap region is:

$$\frac{1}{r}\frac{d}{dr}\left(k_G \, r \, \frac{dT}{dr}\right) = 0 \tag{10}$$

This can be directly integrated to give:

$$k_G \, r \, \frac{dT}{dr} = \text{constant} \tag{11}$$

The constant of integration is determined by considering the heat flux, q'' at the fuel - gap interface:

$$-k_G\frac{dT}{dr}\bigg|_{r=r_F} = q'' = \frac{q'}{2\pi r_F} \tag{12}$$

Substituting equation 12 into 11, we get:

$$k_G \, r \, \frac{dT}{dr} = -\frac{q'}{2\pi} \tag{13}$$

$$\therefore k_G \, \frac{dT}{dr} = -\frac{q'}{2\pi r} \tag{14}$$

Integrating again we have:

$$k_G \, \Delta T_{GAP} = k_G \, (T_F - T_C) = \frac{q'}{2\pi}\ln\left(\frac{r_F + t_G}{r_F}\right) \tag{15}$$

where the subscript C indicates the gap-clad interface. The boundary conditions $T=T_C$ at $r=r_F+t_G$ is incorporated into the above solution. Finally we have:

$$\therefore \Delta T_{GAP} = \frac{q'}{2\pi k_G}\ln\left(\frac{r_F + t_G}{r_F}\right)$$
$$\approx \frac{q'}{2\pi r_F}\left(\frac{t_G}{k_G}\right) \quad \text{since} \quad \ln(1+x)\approx x \tag{16}$$

The gap conductivity k_G is ~ 0.002 W/cm°K but it varies considerably with the amount of fission product gases. For a gap thickness of 0.005 cm, we get a ΔT_{GAP} of about 300°C for a q' of 500 W/cm. Since the

fuel will swell to touch the clad (although not perfectly since the surfaces have a finite roughness), an effective heat transfer coefficient, h_G is used:

$$h_G(\Delta T_{GAP}) = q''$$

(17)

Thus:

$$\Delta T_{GAP} = \frac{q'}{2\pi r_F h_G}$$

(18)

A heat transfer coefficient of 0.5 - 1.1 W/cm^2°K give a ΔT_{GAP} less than 300°C.

5.3.3 Clad

As per the gap region, the steady state equation for the clad region is:

$$\frac{1}{r}\frac{d}{dr}\left(k_C \, r \, \frac{dT}{dr}\right) = 0$$

(19)

This is solved in the same manner as for the gap to give:

$$k_C \, \Delta T_{CLAD} = k_C \, (T_C - T_S) = \frac{q'}{2\pi}\ln\left(\frac{r_F+t_G+t_C}{r_F+t_G}\right)$$

(20)

where the subscript S indicates the clad-coolant surface interface. The boundary conditions $T=T_S$ at $r=r_F+t_G+t_C$ is incorporated into the above solution. Finally we have:

$$\therefore \Delta T_{CLAD} = \frac{q'}{2\pi k_C}\ln\left(\frac{r_F+t_G+t_C}{r_F+t_G}\right)$$
$$\approx \frac{q'}{2\pi(r_F+t_G)}\left(\frac{t_G+t_C}{k_C}\right) \quad \text{since} \quad \ln(1+x)\approx x$$

(21)

The clad conductivity k_C is ~ 0.11 W/cm°K giving a ΔT_{CLAD} of about 80°C for a q' of 500 W/cm.

5.3.4 Coolant

From the clad to the coolant, the heat flux is determined by:

$$q'' = h_S \, (T_S - T_{FL})$$

(22)

where T_{FL} is the bulk temperature of the coolant fluid. Thus the temperature drop from the clad surface to the bulk fluid temperature is:

$$\Delta T_{COOL} = \frac{q'}{2\pi h_S(r_F+t_C+t_G)} \tag{23}$$

A heat transfer coefficient of ~4.5 W/cm^2°K give a ΔT_{COOL} of about 10 - 20°C.

5.3.5   Overall Temperature Difference

Adding up all the temperature differences we find:

$$T_0 - T_{FL} = \frac{q'}{2\pi}\left(\frac{1}{2\bar{k}_f}+\frac{1}{h_G r_F}+\frac{t_G+t_C}{k_C(r_F+t_G)}+\frac{1}{h_S(r_F+t_G+t_C)}\right) \tag{24}$$

Thus, given a bulk coolant temperature, the centre line fuel temperature floats on top of the coolant temperature by an amount that depends on the heat being generated and the various resistances to heat flow. For a given fuel design, most of the parameters are fixed under normal operation. The one exception is h_S. As illustrated in figure 5.2, h_S (defined as the slope of the q" - ΔT curve) can vary considerably. If the surface temperature is too high, a vapour blanket forms at the surface and the heat cannot flow out of the fuel. In effect, h_S drops. This is the dreaded fuel cooling crisis that can occur if power regulation is lost, if a loss of coolant flow occurs or if a loss of coolant inventory occurs. The result of such a crisis is clad failure and release of fission products to the coolant system, and possibly to the turbine cycle and the atmosphere.

5.4   General Thermal Energy Equation

To determine the axial temperature distribution in the coolant, we need to consider the axial heat transport

$$\iiint_\forall \frac{\partial(\rho e)}{\partial t}\, d\forall = -\iint_S \rho e\bar{v}\cdot\hat{n}ds + \iiint_\forall q'''(\bar{r},t)\, d\forall - \iint_S \bar{q}''(\bar{r},t)\cdot\hat{n}d. \tag{25}$$

$$+ \iiint_\forall \tau:\nabla\bar{v}d\forall = \iiint_\forall P\nabla\cdot\bar{v}d\forall$$

Figure 5.2 Heat flux vs. ΔT for pool-boiling heat transfer [Source: DUD76, figure 12-9]

mechanisms. For this we need the general form of the thermal energy equation:
where the last two terms are the viscous heat dissipation (friction heating) and pressure work terms, respectively (more on this in chapter 9). The first term on the right hand side of equation 25 represents the flow of energy through the surfaces, i.e., energy transport. As we shall see, this can be rearranged in terms of enthalpy (h = e + P/ρ):

$$\iiint\limits_{\forall} \frac{\partial(\rho h - P)}{\partial t}\, d\forall \;=\; -\iint\limits_{S} \rho h \bar{v}\cdot\hat{n}\, ds \;+\; \iiint\limits_{\forall} q'''(\bar{r},t)\, d\forall \;-\; \iint\limits_{S} \bar{q}''(\bar{r},t)\cdot\, \hat{n}\, ds$$

$$+\; P\bar{v} \text{ and friction terms that are relatively small and tend to cancel}$$

(26)

5.5 Axial Temperature Distribution

In typical power reactors, $\partial P/\partial t \ll \partial \rho h/\partial t$ so that term can be ignored. For the steady state situation, the energy balance on a lump of fluid coolant of length dz surrounding the fuel pin (see figure 5.3) is, thus:

$$Adz\frac{\partial(\rho h)}{\partial t} = 0 = (A\rho hv)|_z - (A\rho hv)|_{z+dz} + q''(z)2\pi r_F dz \qquad (27)$$

Since there is no heat generation in the coolant itself (apart from some minor turbulence heating), $q''' = 0$. Defining the mass flow as $W = A\rho v$ (kg/sec) and converting q'' to q', we have:

$$W(h|_{z+dz} - h|_z) = q'(z)dz \qquad (28)$$

We note that W is constant along the channel length since mass is neither created nor destroyed. Also note that the heat flux is a function of axial position since the power generation axial distribution in a reactor is not uniform. To a first approximation it is a cosine shape. In single phase, then:

$$W\,c\,dT = q'(z)dz \equiv q_0\,\cos(\pi\frac{z}{H})\,dz \qquad (29)$$

where H is the channel length, $z = 0$ at the channel midpoint and c is the fluid heat capacity. Integrating from the channel entry ($z = -H/2$) to the channel outlet ($z = +H/2$) gives:
This is plotted in figure 5.4

$$T_{FL}(z) - T_{INLET} = \frac{q_0 H}{\pi c W}\left(\sin\left(\frac{\pi z}{H}\right)+1\right) \qquad \text{where } z \in (-H/2,+H/2) \qquad (30)$$
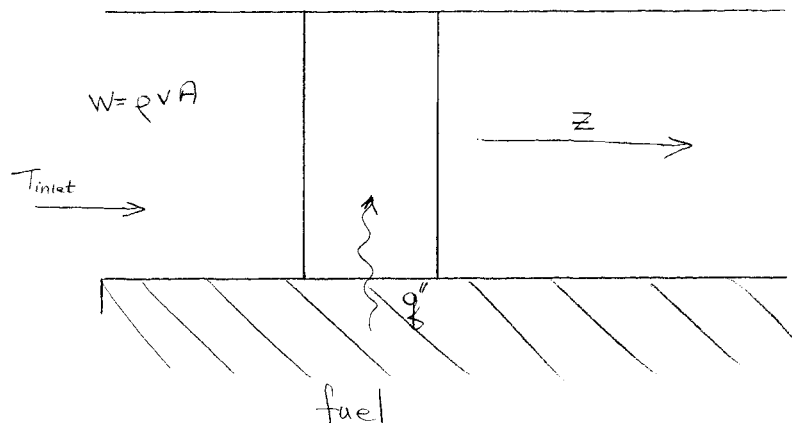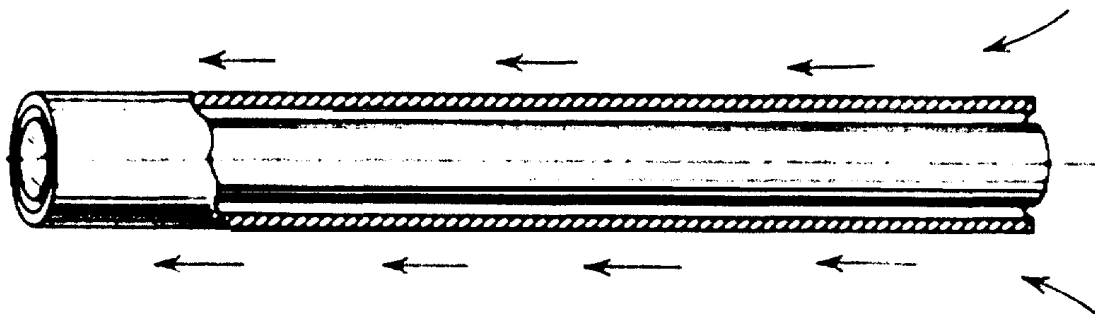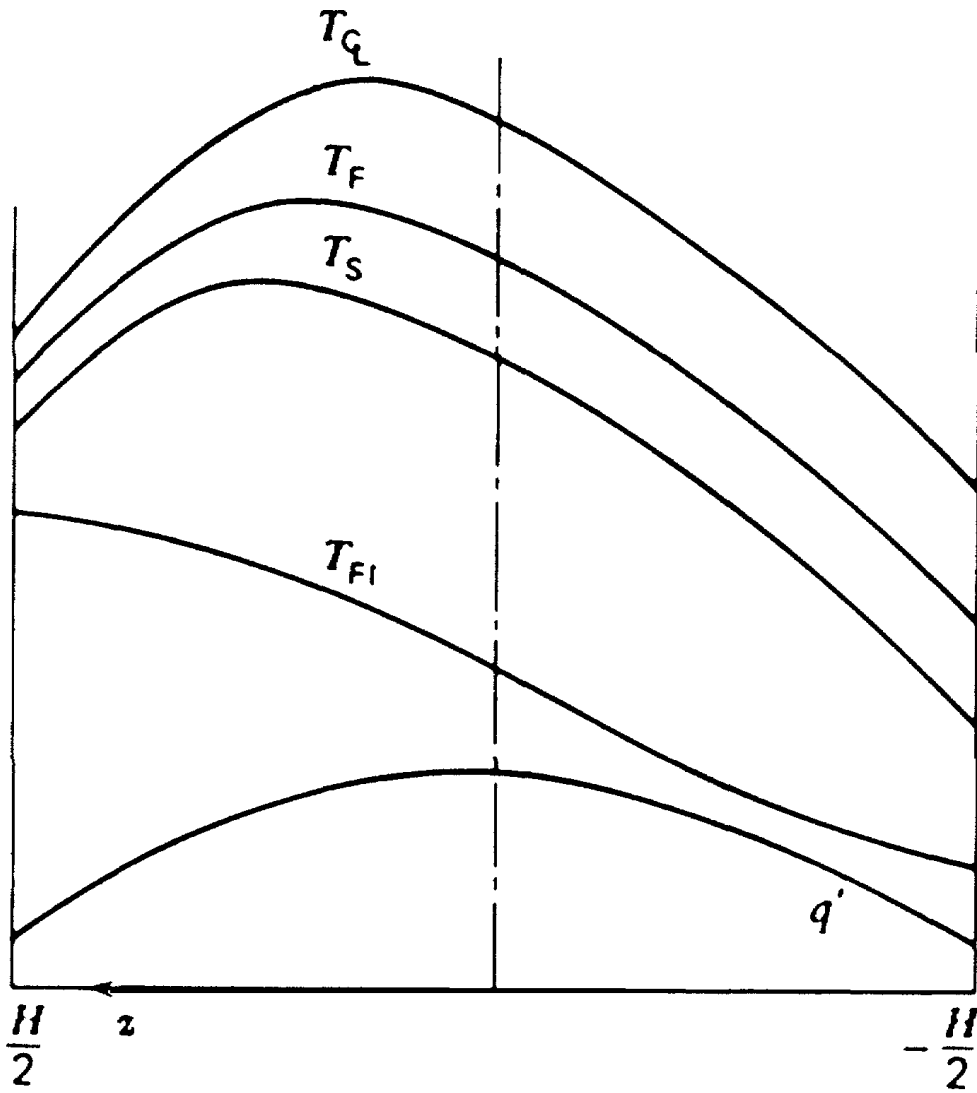

Figure 5.3 Axial energy balance

Figure 5.4 Axial temperature profile [Source: DUD76, figure 12-8]

5.6   Axial Quality Distribution

Equation 28 can be used to calculate the axial quality distribution by noting that:

$$h \ = \ h_{f_{SAT}} \ + \ x h_{fg} \tag{31}$$

where x is the weight fraction of steam in a two-phase mixture, h_{fSAT} is the saturated liquid enthalpy and h_{fg} is the latent heat of vaporization. Thus:

$$W \left(h(z) - h_{INLET} \right) \ = \ \int_{-H/2}^{+H/2} q^{\,\prime}(z) dz \tag{32}$$

If the axial position of the start of bulk boiling (the point where $h(z) = h_{fSAT}$) is defined as z_{BB}:

$$W \left(h(z) - h_{f_{SAT}} \right) \ = \ \int_{z_{BB}}^{z} q^{\,\prime}(z) dz \tag{33}$$
$$= \ x(z) \ W \ h_{fg}$$

The quality as a function of axial position is, finally:

$$x(z) \ = \ \frac{1}{W h_{fg}} \int_{z_{BB}}^{z} q^{\,\prime}(z) dz \tag{34}$$

5.7   Critical Heat Flux

The local quality is of central importance to the margin to dryout in a reactor channel since x is the one parameter that was experimentally found to relate to centre line melting and sheath dryout, two phenomena that serve as indicators of the onset of a heat transfer crisis. Figure 5.5 shows a typical plot of heat flux and quality as a function of axial position. We shall discuss the explicit experimental correlations used in the industry in chapter 9 but, in the meantime, figure 5.5 also shows a sketch of the Critical Heat Flux (CHF) as a function of local quality. Re-plotting actual heat flux vs. quality on the same graph allows an estimation of the margin to dryout or centre line melting.   If the channel power were to increase, this curve will move up and to the right, approaching the CHF curve. The channel power that causes the two curves to touch is the limiting or critical channel power. The Critical Power Ratio, or CPR, is defined as the ratio of this critical power or heat flux and the nominal power or heat flux.
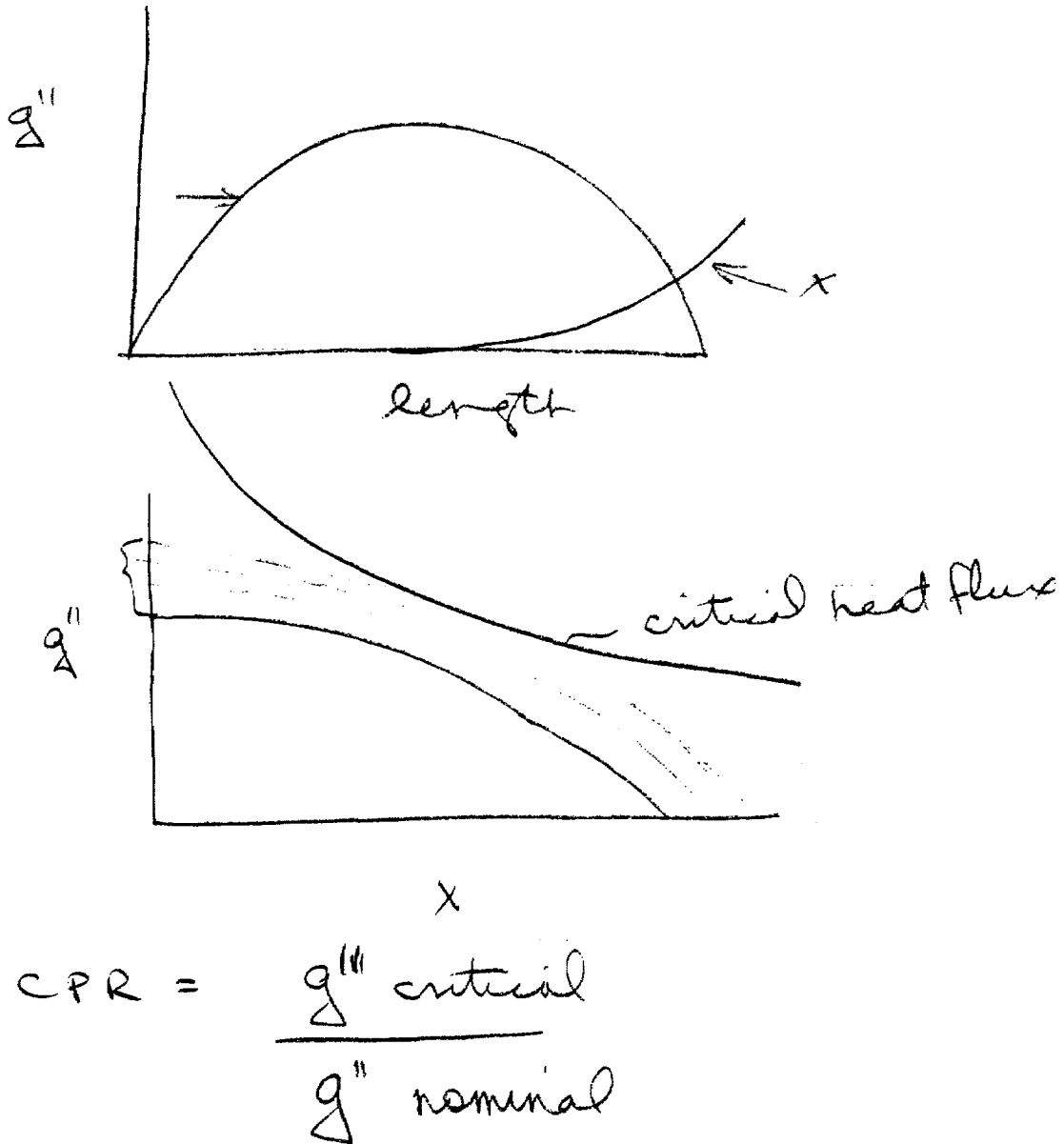
$$CPR = \frac{g''\ critical}{g''\ nominal}$$

Figure 5.5 CHF and CPR

5.8   Summary

This chapter has dealt with the heat transfer situation in the fuel channel. Heat flux limitations here set the limit for plant power output. In previous chapters we have covered the basic notions of overall plant thermalhydraulic and thermodynamic phenomena and the simplified governing equations for those phenomena. We are now in a position to appreciate and investigate the following topics:

- overall plant control
- overall plant design optimization
- thermalhydraulic design evolution
- detailed systems equations and modelling
- design verification.

This topics are covered in the subsequent chapters.