UNENE Graduate Course
Reactor Thermal-Hydraulics Design and Analysis

McMaster University
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Nodalization

Dr. Nik Popov

Outline

- Macro conservation equations
- Node-link diagrams
- Nodal diffusion
- Matrix notation

Macro Conservation Equations

Mass

$$\frac{\partial M_i}{\partial t} = \sum_j \rho_j \cdot v_j \cdot A_j = \sum W_j$$

Momentum

$$\frac{\partial W}{\partial t} = \frac{A}{L}\left[\left(P_{IN} - P_{OUT}\right) - \left(\frac{f \cdot L}{D} + k\right) \cdot \frac{W^2}{2g_c \cdot \rho \cdot A^2}\right] - A \cdot \rho \cdot g / g_c \cdot \sin(\theta)$$

Energy

$$\frac{\partial H}{\partial t} = \sum W_{IN} \cdot h_{IN} - \sum W_{OUT} \cdot h_{OUT} + Q$$

Macro Conservation Equations

- The mass and energy equations are averaged; do not capture details within the volume
- Knowing mass and energy, and using the equation of state, gives the pressure
- Flow is driven by the pressure difference between two points
- Flow generates pressure drop
- Momentum equation is applied between points of known pressure; i.e., between volumes
- In the node-link diagram (staggered grid method) volumes are represented by nodes, flow paths are represented by links

Nodal Diffusion

- In the node - link representation of flow in a pipe, no flow detail is considered as the fluid moves along the pipe
  - Therefore, no diffusion, dispersion, advection, flow profiles or flow regimes are explicitly permitted
  - Not too crude an approximation for the calculation of pressure drops and flows
  - Perfect mixing at the nodes is assumed
- For plug flow in a pipe, flow is transmitted along the pipe relatively undisturbed. If no diffusion or turbulent dispersion existed, a sharp discontinuity in a property would propagate undisturbed.
- A very large number of nodes are needed to transmit a disturbance without appreciable distortion.
- Thus, nodalization creates a form of diffusion in much the same manner as finite difference schemes create numerical diffusion. Attaining convergence in nodalization is, in essence, converging the model to plug flow behaviour.

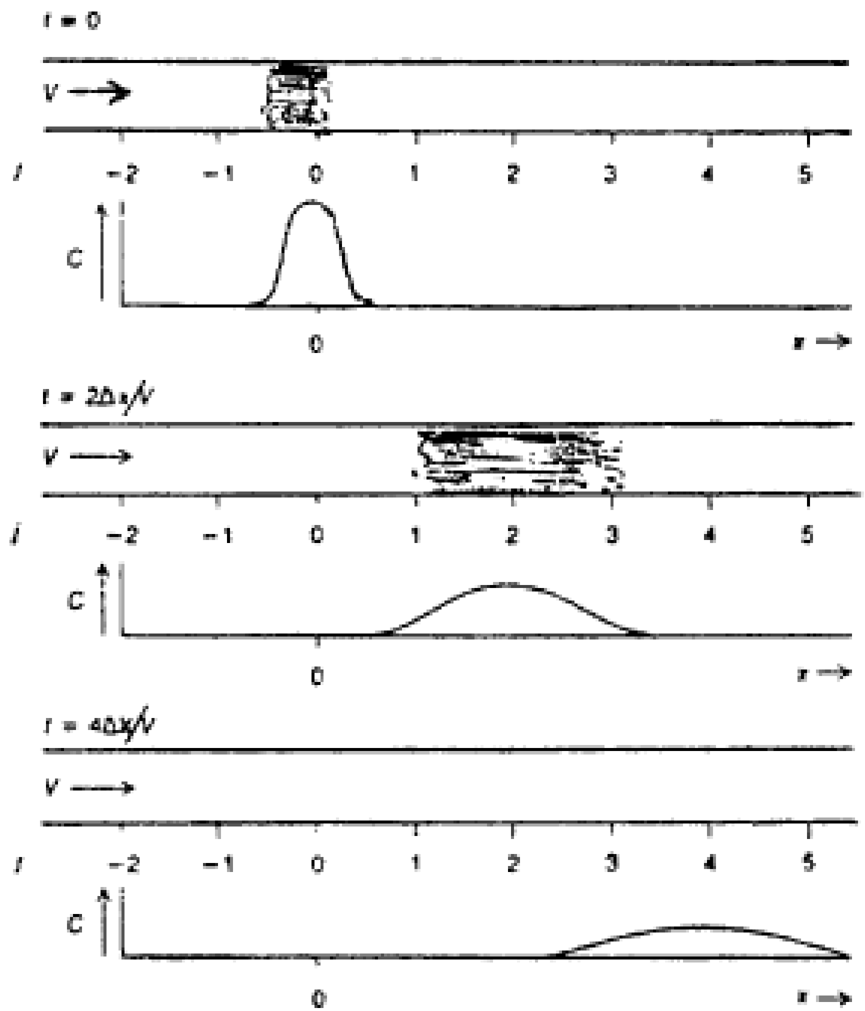
Nodal Diffusion

$$C_{OUT} = C_{IN} \cdot e^{-\frac{t}{\tau}}$$

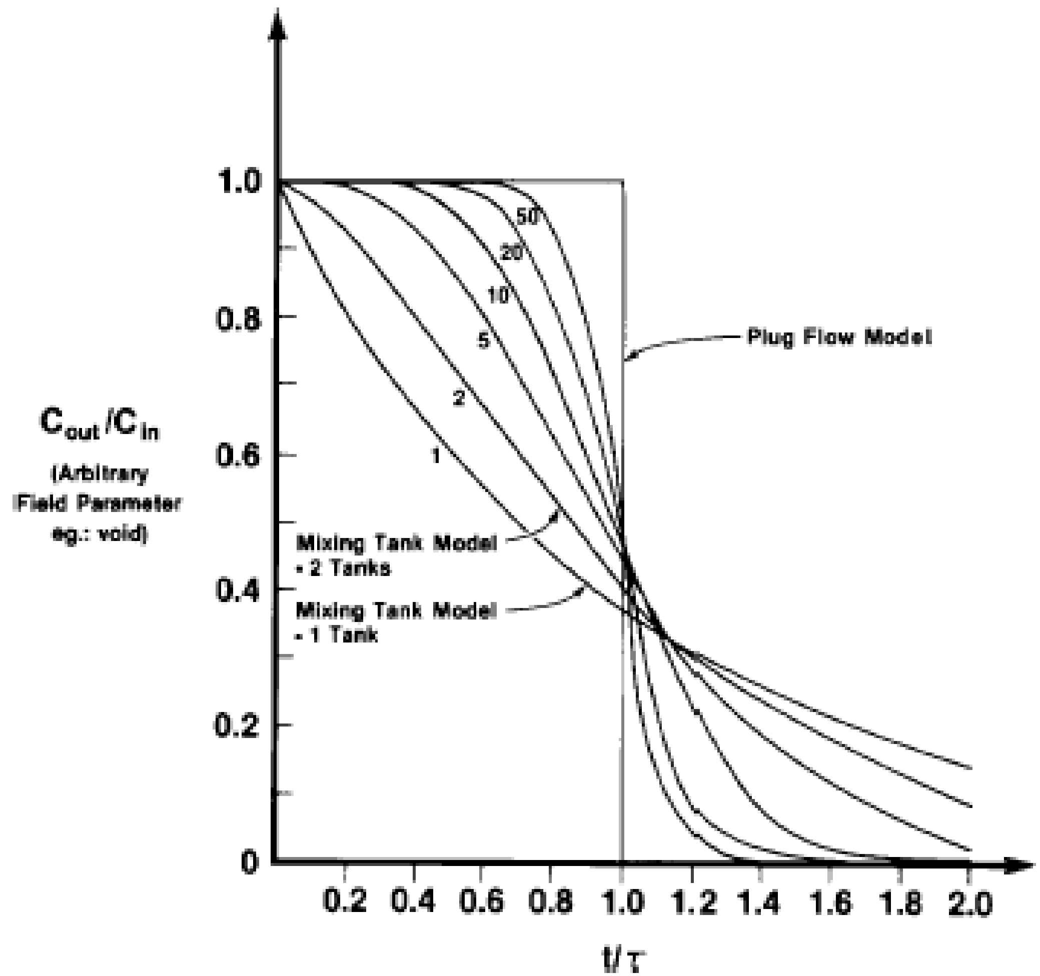
$$C_{OUTNODE2} = C_{INNODE1} \cdot \left(1 + \frac{2t}{\tau}\right) e^{-\frac{2t}{\tau}}$$

$$C_{OUTNODE2} = C_{INNODE1} \cdot e^{-\frac{n \cdot t}{\tau}} \sum_{k=1}^{N} \left(\frac{n \cdot t}{\tau}\right)^{k-1} \frac{1}{(n-1)!}$$

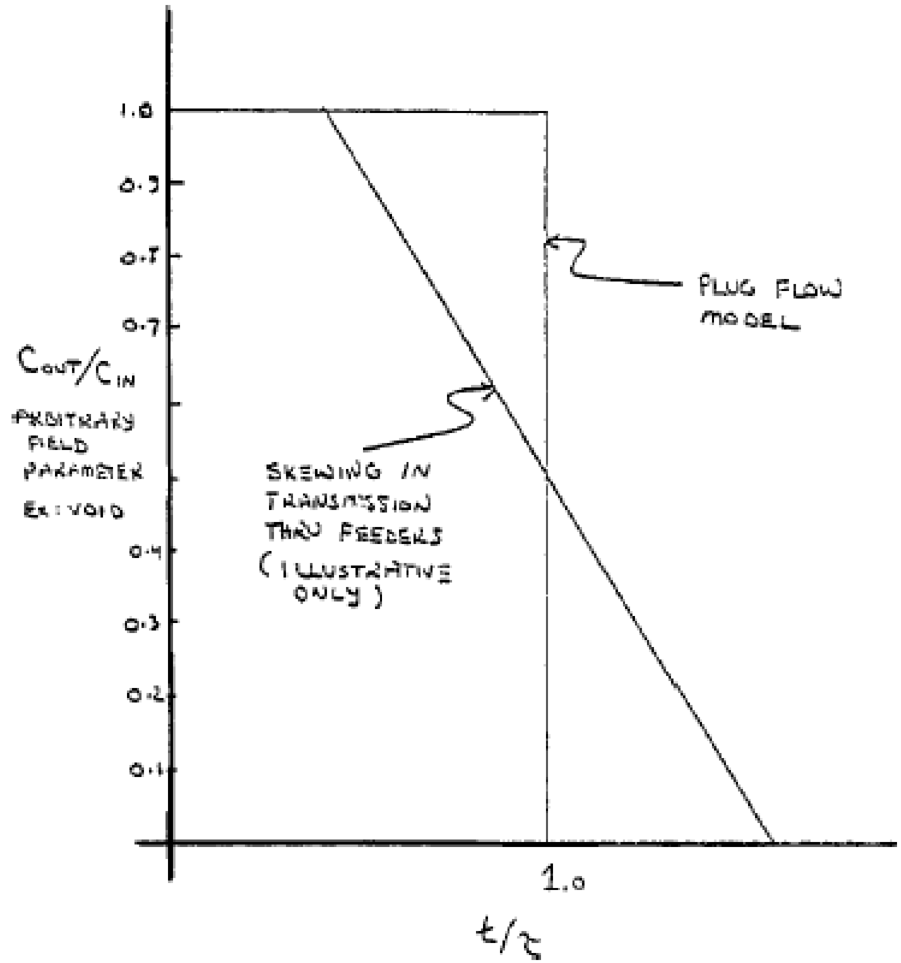
Nodal Diffusion



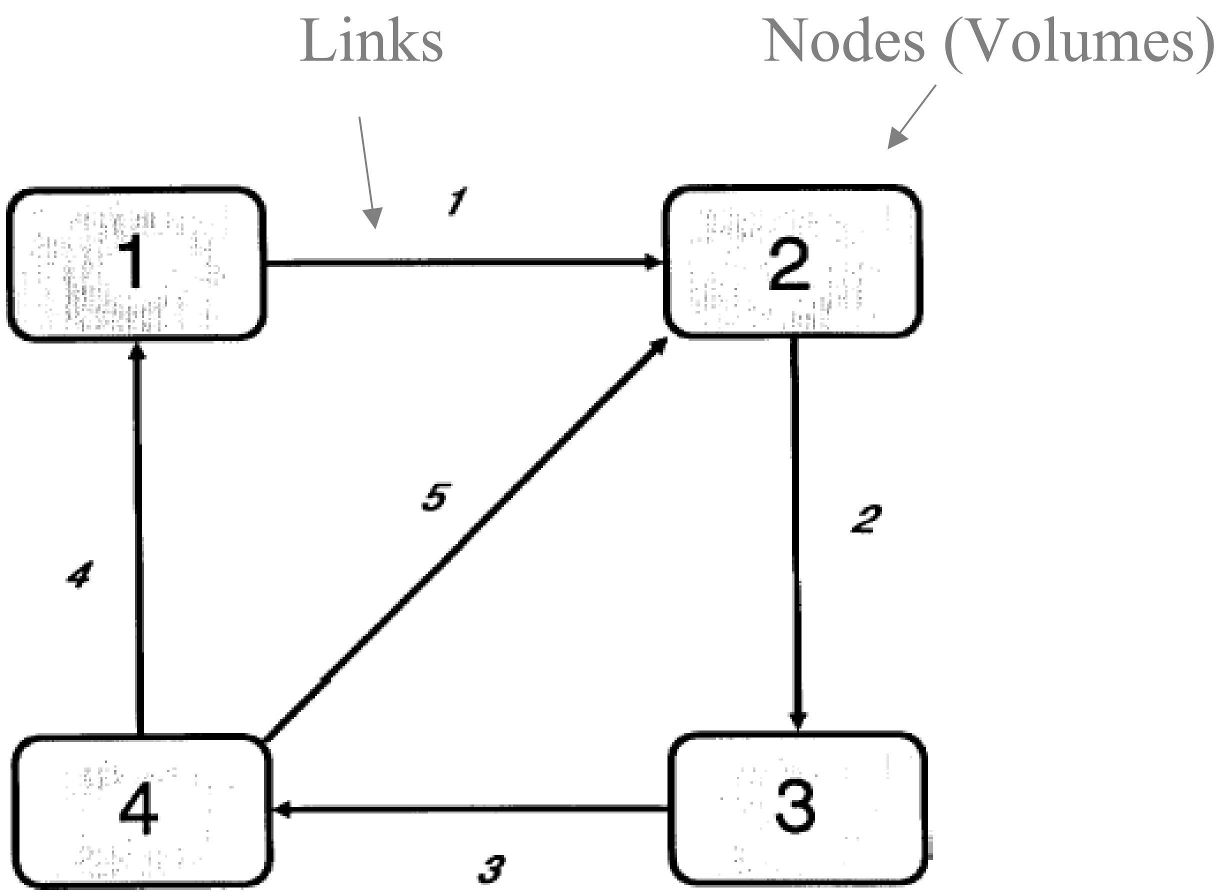
Nodal Diffusion



Nodal Diffusion



Node-Link Diagrams



Matrix Form of Equations

$$\frac{dM_1}{dt} = -W_1 + W_4$$

$$\frac{dM_2}{dt} = W_1 - W_2 + W_5$$

$$\frac{dM_3}{dt} = W_2 - W_3$$

$$\frac{dM_4}{dt} = W_3 - W_4 - W_5$$

$$\begin{pmatrix} \dot{M}_1 \\ \dot{M}_2 \\ \dot{M}_3 \\ \dot{M}_4 \end{pmatrix} = \begin{pmatrix} -1 & 0 & 0 & 1 & 0 \\ 1 & -1 & 0 & 0 & 1 \\ 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & -1 & -1 \end{pmatrix} \cdot \begin{pmatrix} W_1 \\ W_2 \\ W_3 \\ W_4 \\ W_5 \end{pmatrix} \equiv \dot{m} = A^{MW} \cdot W$$

Questions?