The Infinite Lattice and the Infinite-Lattice Multiplication Constant
and POWDERPUFS-V
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1. Introduction

The first stage of practical reactor-physics calculations is the study of the “infinite lattice”. This is a hypothetical configuration where a basic lattice cell is assumed repeated identically and infinitely in all directions. In this view, the complexities of considering cells of different composition or configuration are ignored. Also, leakage is ignored, since the configuration is infinite and since all cells are identical.

Despite the simplification, one can learn a lot from the study of the infinite lattice.

The figure below (not to scale) shows the CANDU basic lattice cell. Note that it consists of the fuel, coolant, pressure and calandria tubes, and moderator, but that reactivity devices (which are not present everywhere in the core) are not included.

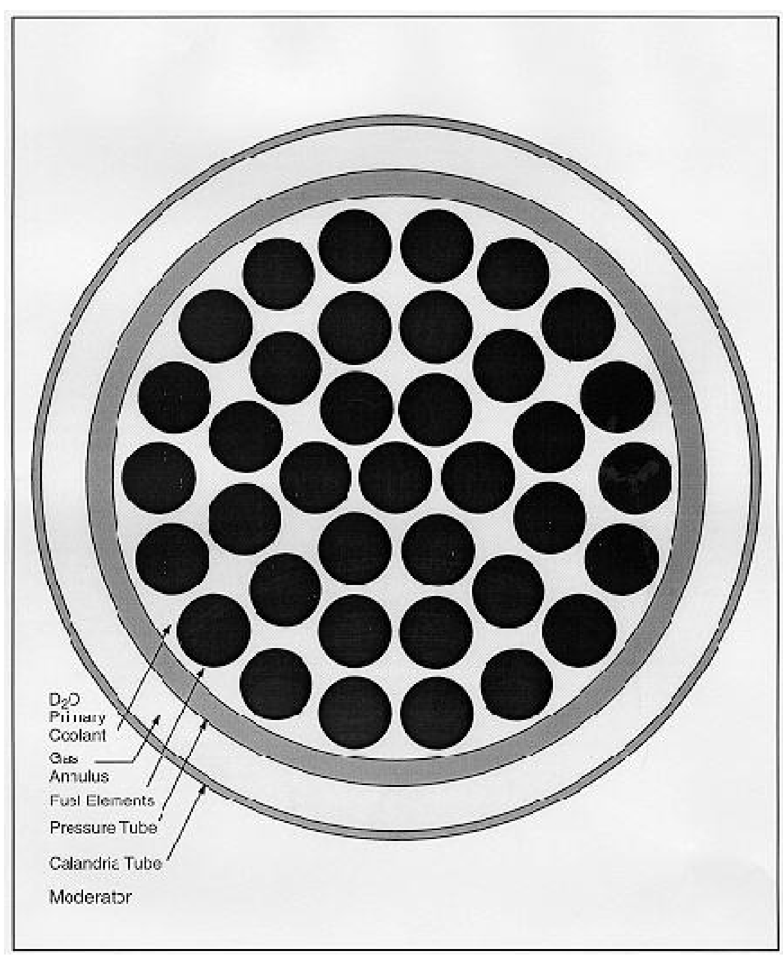


Figure 12.1 CANDU Basic-Lattice Cell (not to scale)

In the real reactor, different cells may have different local parameters or conditions, such as densities, temperatures, even changes in dimensions, and of course the most important difference, which is the fuel burnup. The infinite lattice, however, considers the average conditions in the core, although with different calculations differences in local conditions can be studied. However, the lattice code must be able to provide the properties of the cell throughout the burnup history of the fuel – this is called a “burnup” or “depletion” calculation.

2. Lattice or Cell Codes

Two computer codes, mainly, are used for lattice calculations for CANDU. One is WIMS-IST, which has a strong foundation in transport theory. The other is POWDERPUFS-V, a semi-empirical code developed at CRL in the 19670s and 70s, and based on the results of a large number of measurements on D_2O-moderated lattices in research reactors.

WIMS-IST is an Industry Standard Tool and is intended to replace POWDERPUFS-V for all official calculations. In particular, safety analyses and licensing submissions to the CNSC are to be based on WIMS-IST. POWDERPUFS-V is still used in many core-tracking applications at CANDU generating stations.

WIMS-IST is too complicated for us to use it here. POWDERPUFS-V is a code which executes very quickly, and we will use it here to do “real-life”.

3. Basic-Lattice-Cell Representation in POWDERPUFS-V

POWDERPUFS-V uses a simplified representation of the CANDU lattice cell. This is shown in the figure below.

The fuel elements (fuel and sheath) and most of the coolant have been homogenized into one region. In the program, the homogenized-region radius is the radius of a circle having an area equal to the cross sectional area enclosed by a rubber band touching the sheath of each outermost element and wrapped around the bundle at its midplane. There is an annulus of coolant between the homogenized region and the pressure tube.

Not shown in the figure are the bundle end regions, which contain end caps and end plates but no fuel. In POWDERPUFS-V, the total quantity of fuel and cladding material per bundle has been homogenized over the length of the bundle. The lattice cell is thus divided into three separate regions, viz. the homogenized fuel region, the annulus region, and the moderator region.

The detailed POWDERPUFS-V input file and options for running POWDREPUFS-V are covered in a separate document.

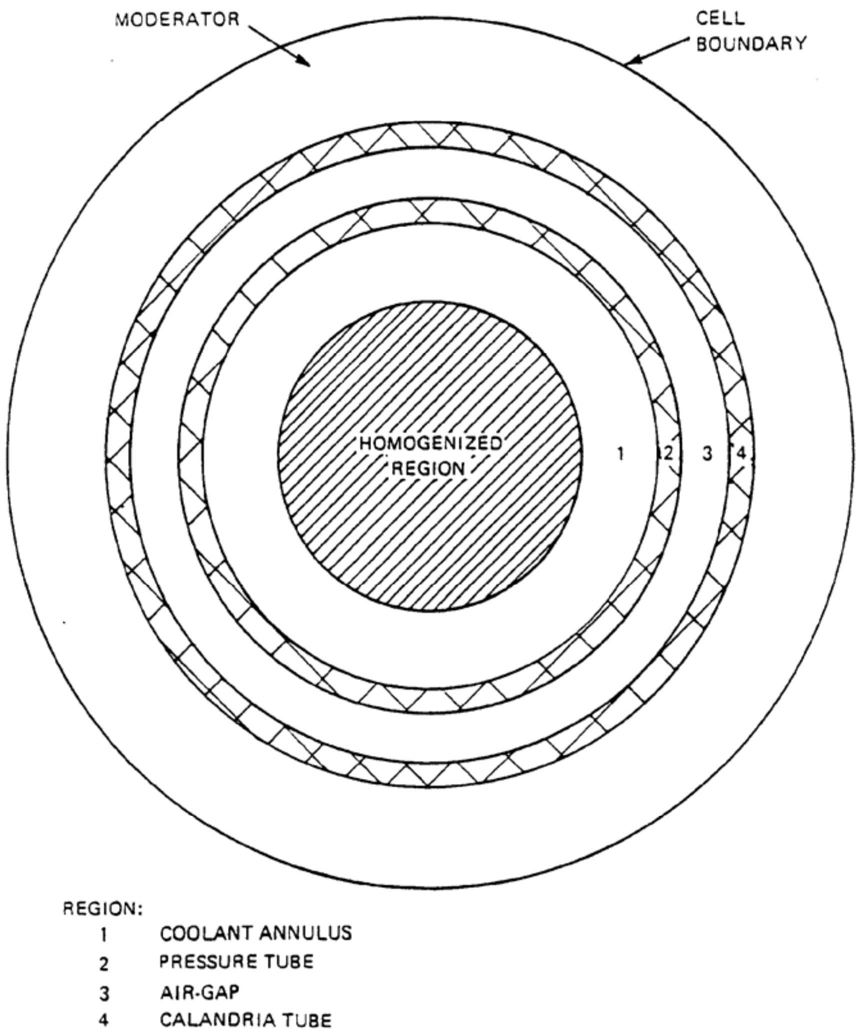


Figure – Lattice-Cell representation in POWDERPUFS-V

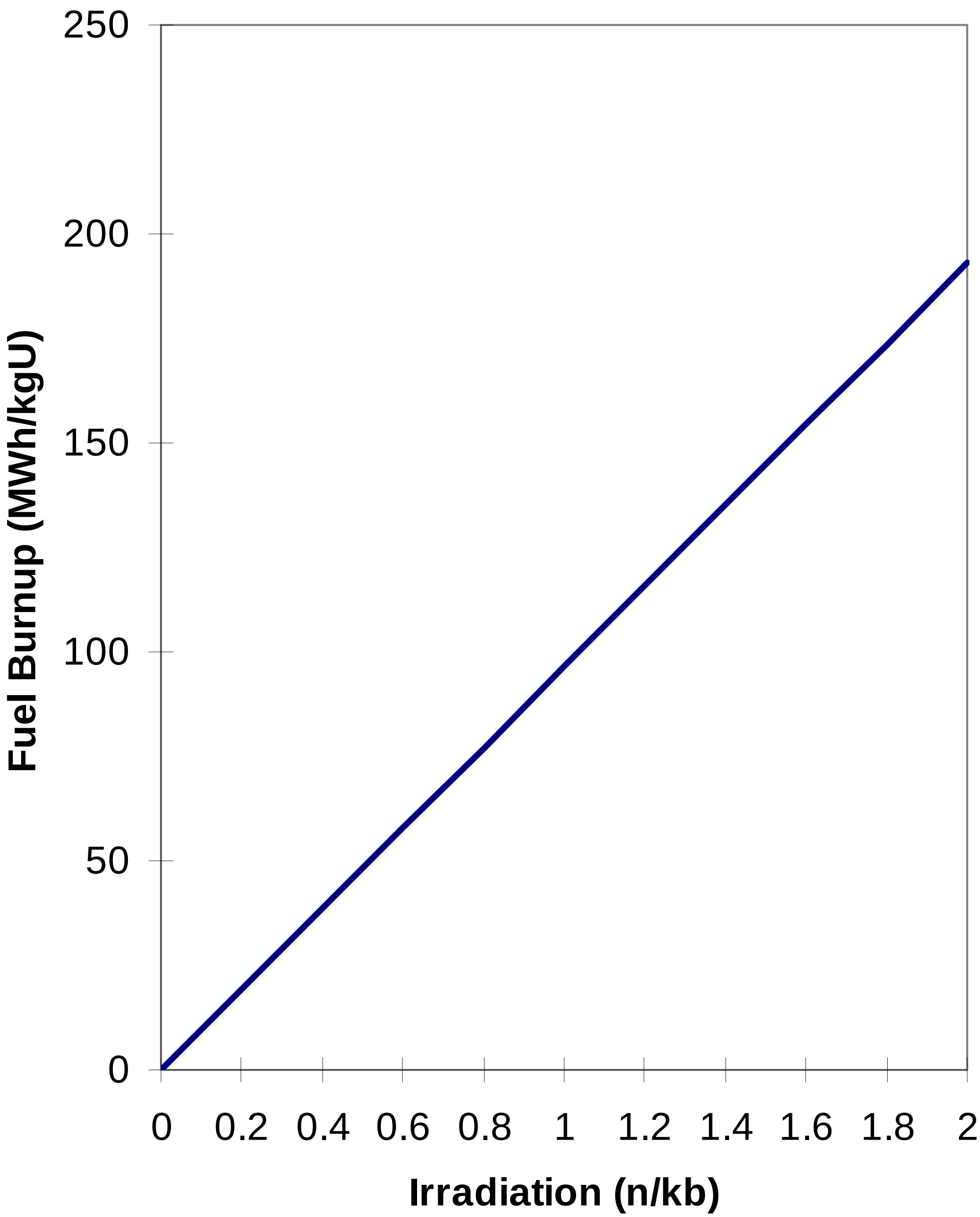
4. Burnup and Irradiation

Most lattice codes use burnup as the independent variable in the depletion calculation. POWDERPUFS-V uses fuel irradiation.

For completeness, the definitions are:

- "Burnup" is the amount of energy released in fission by a unit mass of fissionable material (here, uranium) in the fuel, from the time the fuel entered the reactor. Thus, burnup is an instantaneous quantity, and it increases with time as the fuel produces energy. Units of burnup are MW.d/Mg(U) or MW.h/kg(U).
- The "exit burnup" or "discharge burnup" of fuel is its burnup on exit from the reactor. Thus, it is the total amount of energy per unit mass of uranium that the fuel has released in fission during its residence time in the reactor. This is an economic quantity. Obviously, the higher the exit burnup of a given amount of fuel, the lower the contribution of fuel to the cost of electricity.
- The irradiation ω of a material (here we are interested in the fuel) is the product of neutron flux ϕ with time t: $\omega = \phi t$. That is, it measures the time the material has remained in a given neutron flux. [If the flux varies with time, the defining equation for ω becomes an integral of flux with time.] Irradiation is therefore, just like burnup, an instantaneous quantity, which increases with time. Since ϕ has units of n.cm^{-2}s^{-1}, the units of ω are n.cm^{-2}, or, more usually, n/kb (neutrons per kilobarn), where 1 barn = 10^{-24} cm^2.

Because the fission cross section in natural-uranium fuel does not vary a lot with the "age" of the fuel, and because the energy release in plutonium fission is not very different from that in uranium fission, there is an almost linear relationship between irradiation and burnup. The figure below shows that the burnup in units of MW.h/kg(U) is approximately 100 times the irradiation from POWDERPUFS-V in units of n/kb.

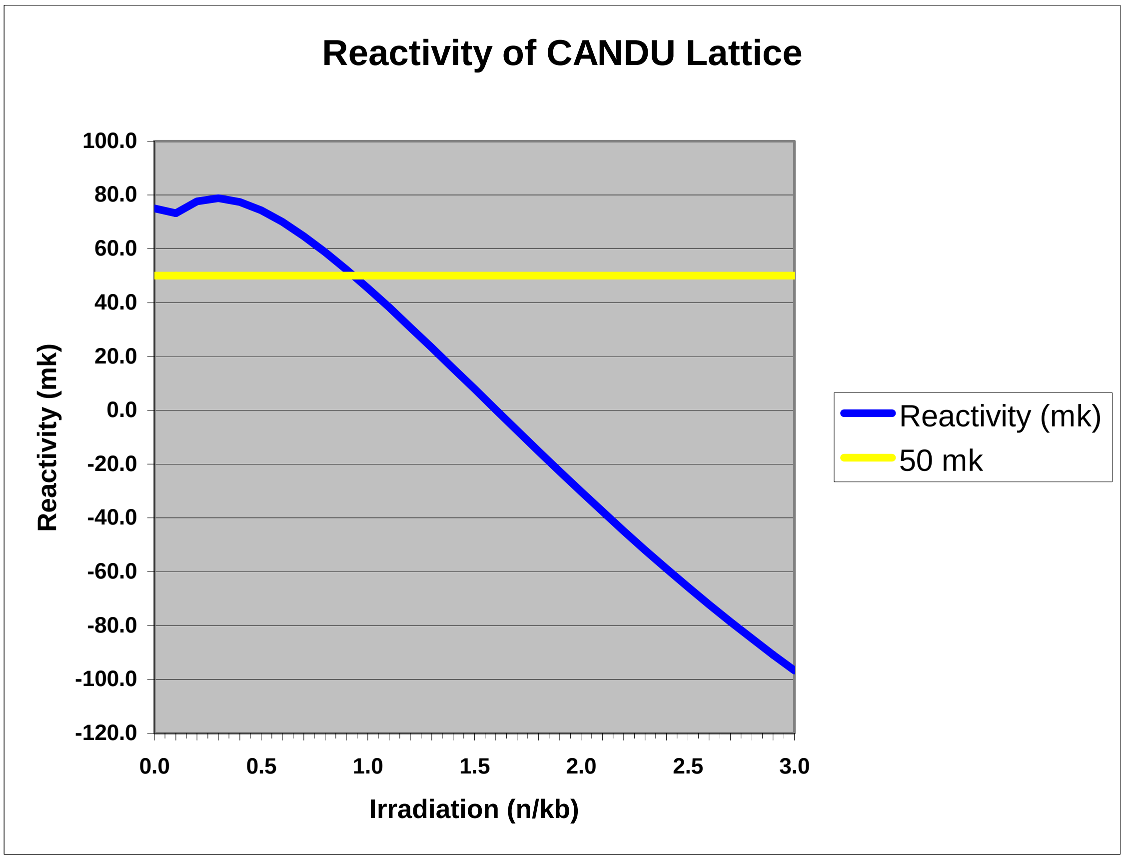


250
200
150
100
50
0
Fuel Burnup (MWh/kgU)
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2
Irradiation (n/kb)

5. Infinite-Lattice Multiplication Constant

In this part of the course, you will be asked to do a number of POWDERPUFS-V calculations, with the object of learning various aspects of the reactor physics of CANDU.

However, for the moment, I jump a bit ahead to show you below a figure of the typical infinite-lattice multiplication constant k_∞ for the standard CANDU-6 lattice fuelled with natural uranium.



The figure shows that the lattice is just under 80 milli-k supercritical for fresh fuel (zero irradiation) - at full power, i.e., ^{135}Xe and the other saturating fission products are assumed to have already built in.

The reactivity decreases initially for a short time. This is related to the start of depletion of ^{235}U, while plutonium production is delayed by the hold-up in neptunium: ^{238}U absorbs a neutron and beta-decays to ^{239}Np, and the latter will also beta-decay to ^{239}Pu, but ^{239}Np has a half-life of ~2 days.

Then reactivity starts to **increase** with increasing irradiation, reaching a maximum at approximately 0.4-0.5 n/kb. This phenomenon is due to the start of the contribution of plutonium to the fission chain reaction. This reactivity maximum is consequently known as the plutonium peak. Note that it is **not** the plutonium concentration which is at a peak – it will continue to increase. It is the **reactivity** which is at a maximum.

Beyond the plutonium peak, the reactivity starts to decrease with increasing irradiation, on account of the continuing depletion of ^{235}U, the slowing down of the net increase in plutonium concentration, and the increasing fission-product load. The lattice reaches zero reactivity at an irradiation of about 1.6-1.7 n/kb.

6. Implications for Finite Core

The desired outcome of building a nuclear reactor is that it be able to run and produce electricity. Ideally, the reactor should keep purring along, producing electricity at a constant rate all the time. That is, the reactor must be kept exactly critical.

If reactivity is positive, "external" absorption can normally be easily added to compensate for the positive reactivity and bring the reactor critical. When, on the other hand, reactivity is negative, it is usually more difficult to make it go to 0. Practically speaking, then, the net reactivity must be kept 0 or positive for the reactor to run.

From the figure above, then, it appears that the infinite lattice can be made to produce electricity to a fuel irradiation of ~1.6 n/kb. That is approximately what the fuel exit irradiation would have to be.

However, the infinite lattice, by construction, does not account for neutron losses by leakage and by "parasitic" absorption in reactivity devices. The total such losses amount to about 50 mk. Therefore the finite reactor will have a reactivity about 50 mk <u>lower</u> than that of the infinite lattice: for the finite reactor, we need to lower the infinite-lattice reactivity curve uniformly by 50 mk. Or, alternatively, let us consider where the infinite-lattice curve crosses the +50-mk line, instead of the 0-mk line.

From the figure above, the reactivity is 50 mk at an irradiation of ~0.9 n/kb. Thus, it would appear that the exit irradiation in CANDU is limited to about 0.9 n/kb. **This would be true if CANDU were operated in batch-refuelling mode.**

However, it is not! It is operated in on-power-refuelling mode. Therefore, fuel that is "older" than 0.9 n/kb, and with net negative reactivity, can be left in the reactor to compensate for the positive net reactivity of fresh and "young" fuel in the reactor. **That is, the net excess reactivity is compensated not by "external" absorption, but by leaving the fuel in the reactor even after it has become a net absorber.** This allows the exit irradiation (or burnup) to be much higher than that limited by the +50-mk line, which would be the case with batch refuelling.

Exactly how much longer the fuel can remain in the reactor, i.e., to what exit burnup it can be taken, can be determined with POWDERPUFS-V, and we will do that. However, we can think of it in a simple manner as follows. The CANDU reactor is essentially refuelled every day. Therefore, there is fuel of all irradiations in the core, from 0 to some exit irradiation (which we would like to determine). Essentially, we are saying that the average fuel in the core has a lattice reactivity of +50 mk (which is taken away by leakage and absorption in devices). We can think of "integrating" the reactivity curve less 50 mk with irradiation. As we integrate from 0 irradiation, the integral starts in the positive direction and increasing, since reactivity is greater than 50 mk, but, when reactivity crosses the 50-mk line, the integral will start to decrease, and at some point it will reach zero. That point marks the exit irradiation. A graphical way of thinking of this is to look at the area of the curve above the 50-mk line, and estimate at what irradiation the area under the 50-mk line will exactly compensate: that is the exit irradiation. Just by visual examination, that point would appear to be approximately 1.7 n/kb, corresponding to an exit burnup of about 170 MW.h/kg(U).

7. POWDERPUFS-V Options

Depletion calculations in POWDERPUFS-V can be of two types:

- "Instantaneous" calculations. In this case the independent variable is the instantaneous irradiation ω =. These runs give results of the type in the figure above.
- "Reaction-Rate-Averaged" calculations. In this case the variable is the exit irradiation ω_{exit}, which is the exit irradiation to which the fuel is to be taken (in an on-power-refuelled CANDU).

From the discussion in the previous sections, it is clear that "instantaneous" calculations provide the properties of a single cell, whereas "reaction-rate-averaged" calculations provide approximate average properties of the reactor, i.e., the properties of the "equilibrium core"

We will be doing POWDERPUFS-V calculations of both kinds.