# Chapter 4 - Probability Tools and Techniques

## Introduction

### Chapter Content

This chapter presents basic probability tools and techniques, drawing heavily from McCormick[1] for the basic probability theory. Alan Monier guided the bulk of the remainder. Paul Santamaura contributed to improving the chapter.

The objective of this chapter is to provide the basic probability tools and techniques needed to explore reactor safety analysis. This will allow the quantification of the concepts and designs developed in the rest of the course.

### Chapter Layout

First, the general rules of probability (AND and OR rules) and Bayes Equation are introduced but, for the most part in this course, we will rely on the approximations of rare and independent events. Time dependent systems are addressed, covering failure rates, repair, continuous operation, and demand systems.

We encounter a simple shutdown system, illustrating the concept of testing to increase system availability. We also consider the basic '2 out of 3' system so prevalent in reactor safety systems. By way of contrast to the shutdown system, which is a demand type system, the emergency core cooling system is also examined as an example of a demand system with a mission time.

## Definitions and Rules

First, you may want to refresh your memory with the basic rules of Boolean algebra in Appendix 3.

If event A occurs *x* times out of *n* repeated experiments then:

$$\begin{aligned} P(A) &= \text{probability of event A} \\ &= \lim_{n \to \infty} \left(\frac{x}{n}\right) \end{aligned} \tag{1}$$

$$\textit{(Axiom \#1)} \qquad 0 \le P(A) \le 1 \tag{2}$$

$$\textit{(Axiom \#2)}\text{:} \qquad P(A) + P(\bar{A}) = 1 \quad \text{where } \bar{A} \text{ means "not A".} \tag{3}$$

In other words, an event must either occur or not occur - there is no third possibility..

The *intersection* of 2 events, $A_1$ and $A_2$, is denoted:

$$\begin{gathered} A_1 \cap A_2 \quad \text{or } A_1 A_2 \quad \text{or } A_1 \text{ AND } A_2 \\ \text{(This is } \underline{\text{not}} \text{ } A_1 \text{ times } A_2\text{)} \end{gathered} \tag{4}$$

$A_1A_2$ means that *both* events occur, so $P(A_1A_2)$ is the probability that both events occur.

The *conditional probability* $P\ (A_1 \mid A_2)$ means the probability of $A_1$ given that $A_2$ has occurred.

The product rule for probabilities states:

$$\textit{(Axiom\#3)} \qquad \begin{aligned} P(A_1\ A_2) &= P(A_1|A_2)\ P(A_2) \\ &= P(A_2|A_1)\ P(A_1) \end{aligned} \tag{5}$$

For example, if $A_1$ is the probability that part 1 fails and $A_2$ is the probability that part 2 fails then
$P(A_1\ A_2)$ = probability that both part 1 fails and part 2 fails
= probability that part 2 fails **and** (probability that part 1 fails given that part 2 fails)

The attached Figure 4-1 shows this graphically; yellow represent all events; green those events with outcome A1; blue with outcome A2; red, with outcome *both* A1 and A2.

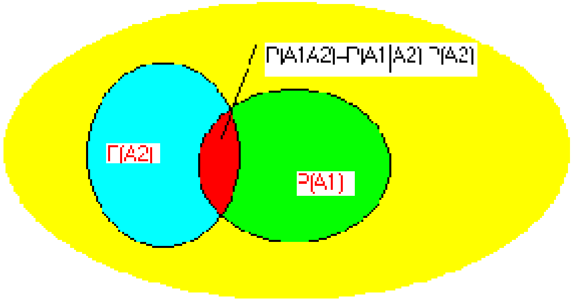


Figure 4-1 - Probability of Both of two Events

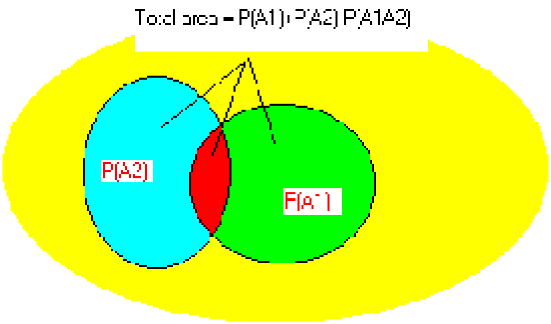


Figure 4-2 - Probability of Either of two Events

See Appendix 1 for an example.

If the failures are independent,

$$P(A_2 \mid A_1) = P(A_2).$$

This can be extended to give:

$$P(A_1A_2....A_N) = P(A_1)P(A_2|A_1)....P(A_N|A_1A_2....A_{N-1}) \qquad (6)$$

If events are independent:

$$P(A_1A_2....A_N) = P(A_1)P(A_2)....P(A_N) \qquad (7)$$

The *union* of two events is denoted:

$$A_1 \cup A_2 \quad \text{or} \quad A_1+A_2 \quad \text{or} \quad A_1 \text{ OR } A_2. \qquad (8)$$

This means the cases where *either* event occurs, including the cases where *both* events occur.

We have:

$$P(A_1+A_2) = P(A_1) + P(A_2) - P(A_1A_2) \qquad (9)$$

as shown in Figure 4-2. The reason for subtracting $P(A_1A_2)$ is because what you want is the total area encompassed by the combination of the blue and green ovals in the diagram. If you just add $P(A_1)$ and $P(A_2)$, you count the intersection where both events occur (in red) twice. So you have to subtract one of them away.

In general:

$$\begin{aligned} P(A_1+A_2+\ldots+A_N) &= \sum_{n=1}^{N} P(A_N) - \sum_{n=1}^{N-1}\sum_{m=n+1}^{N} P(A_nA_m) \\ &\pm\ldots+(-1)^{N-1}P(A_1A_2\ldots A_N) \end{aligned} \qquad (10)$$

If events are independent:

$$1 - P(A_1+A_2+\ldots.+A_N) = \prod_{n=1}^{N} [1-P(A_N)] \qquad (11)$$

See Appendix 2 for an example.

*Rare events approximation* means $P(A_n)<<1$, and assuming they are independent:

$$P(A_1+A_2+ \ldots A_N) = \sum_{n=1}^{N} P(A_N) \qquad (12)$$

and we previously had (equation 7):

$$P(A_1A_2\ldots.A_N) = P(A_1)P(A_2)\ldots..P(A_N) \qquad (13)$$

## The Bayes Equation

Given an event or hypothesis, B, and $A_n$ mutually exclusive events or hypotheses (n=1, 2....N):

$$P(A_nB) = P(A_n)\ P(B|A_n) = P(B)P(A_n|B) \qquad (14)$$

$$\therefore\ P(A_n|B) = P(A_n)\left[\frac{P(B|A_n)}{P(B)}\right] \tag{15}$$

Now, since the events, $A_n$ are mutually exclusive:

$$\sum_{n=1}^{N} P(A_n|B) = 1 \tag{16}$$

Multiplying by P(B):

$$\begin{aligned} P(B) &= \sum_{n=1}^{N} P(B)\ P(A_n|B) \\ &= \sum_{n=1}^{N} P(A_n B) \\ &= \sum_{n=1}^{N} P(A_n)\ P(B|A_n) \end{aligned} \tag{17}$$

Substituting (17) into (15):

$$P(A_n|B) = \frac{P(A_n)\ P(B|A_n)}{\sum_{m=1}^{N} P(A_m)\ P(B|A_m)} \tag{18}$$

So if we know $P(B|A_n)$ then we can calculate $P(A_n\ |B)$. This is an important result because it enables you to "reverse" the order of information. This is especially useful for analysing rare events.

## Example - Pipe Inspection

Suppose you are radiographing a Class I pipe for a defect. You know from past experience that the likelihood of a defect is one per 100,000 radiographs. You also know that the likelihood of the instrument indicating a defect when there is *no* defect (false positive) is 1%, and the likelihood of indicating a defect when there *is* a defect is 99%. Your test indicates a defect. What is the probability that the pipe actually has a defect?

Solution:

Apply Bayes theorem to two events:
A: pipe has a defect, so P(A) = 0.00001

B: instrument says that pipe has a defect, so P(B)=0.01[a]
B|A: instrument says pipe has a defect when it has a defect, so P(B|A) = 0.99

What we want is P(A|B), the probability that the pipe actually has a defect when the instrument says it has one.

Using Bayes theorem:

P(A|B) = [P(B|A)][P(A)]/P(B)

= 0.99 x 0.00001 / 0.01

= 0.00099

Comment:

This seems counterintuitive and suggests the test is not very good in detecting defects, despite the instrument's good accuracy rate. However the fact that the defect is so rare (we need about a hundred thousand samples before we have chance at seeing a real positive) magnifies the small false positive rate so that most positive tests are false positives.

This is quite important in medical tests - even a very accurate test for a rare cancer will often give far more false positives than real ones.

[a] This is a bit of a simplification using the fact that P(A) is small.. Actually
P(B) = P(B|A) P(A) + P(B|notA) P(notA)
= 0.99 x .00001 + 0.01 x 0.99999
= 0.0100098
or approx = 0.01 as stated

In English, the first term is the 99% chance of detecting the defect in the one pipe in 100,000 that has the defect; plus the second term, which is the 1% chance of indicating a false positive in the remaining 99,999 pipes out of 100,000.

## Example - Core Monitoring System

A Core Monitoring System (CMS) is composed of the 3 sensors as shown. All sensors are required to work for the core monitoring system to work.

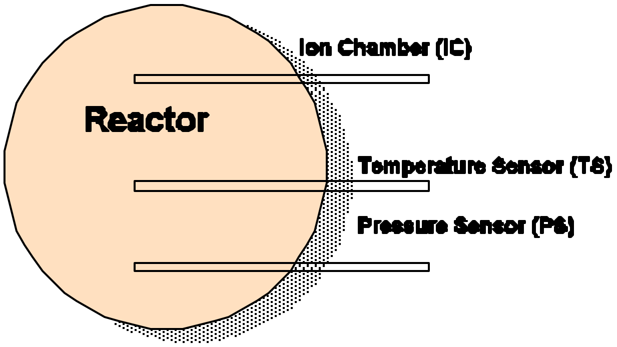


Figure 4-3 - Components of Core Monitoring System

We know from the manufacturer the failure probabilities over the period of time under consideration (this is the axiomatic data):

P(IC) = 0.02
P(TS) = 0.04
P(PS) = 0.01

Testing of the installed system shows that P(CMS|IC) = 0.10 (i.e., when IC fails, the CMS fails 10% of the time.

Also P(CMS|TS) = 0.15
P(CMS|PS) = 0.10

What is the chance that when CMS fails, TS has also failed?

Solution:

$$\begin{aligned} P(TS|CMS) &= \frac{P(TS)\ P(CMS|TS)}{P(IC)\ P(CMS|IC)\ + P(TS)\ P(CMS|TS)\ +\ P(PS)\ P(CMS|PS)} \\ &= \frac{0.04 \times 0.15}{0.02 \times 0.10 + 0.04 \times 0.15 + 0.01 \times 0.10} \\ &= 0.667 \end{aligned} \qquad (19)$$

Comment:

Based on the axiomatic data P(IC), P(TS) & P(PS) one would expect the TS to be a problem in proportion to its failure rate relative to the other devices i.e.,

$$\frac{0.04}{0.02+0.04+0.01} = \frac{4}{7} \qquad (20)$$

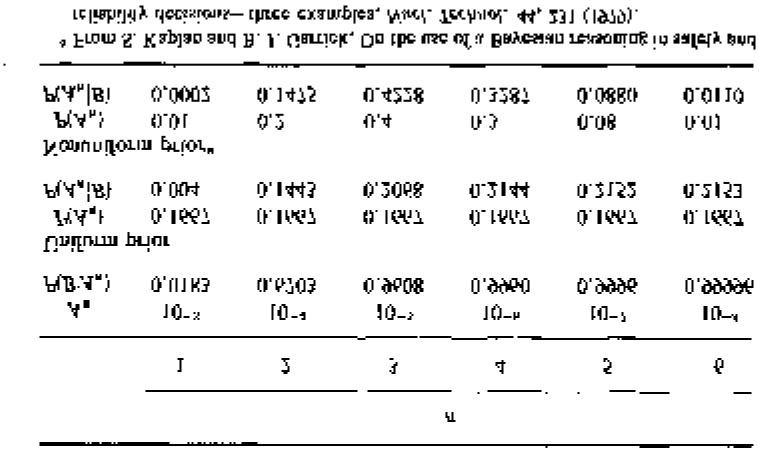
So, in the above example, the testing data, P(B)|$A_n$) is used to modify the axiomatic data to yield a revised relative frequency of sensor failure, given a system failure, by $P(A_n|B)$. This is called *a posteriori* probability.

## Failure rate estimation when no failures have occurred

We can use Bayes Equation to glean information from non-events as well.

Consider the case where 4000 fuel shipments have been made with no radioactive release. Can we determine the probability of release per shipment?

Let B = 4000 shipments with no release



| | n | | | | | |
|---|---|---|---|---|---|---|
| | 1 | 2 | 3 | 4 | 5 | 6 |
| $A_n$ | $10^{-3}$ | $10^{-4}$ | $10^{-5}$ | $10^{-6}$ | $10^{-7}$ | $10^{-8}$ |
| $P(B\|A_n)$ | 0.0183 | 0.6703 | 0.9608 | 0.9960 | 0.9996 | 0.99996 |
| Uniform prior | | | | | | |
| $P(A_n)$ | 0.1667 | 0.1667 | 0.1667 | 0.1667 | 0.1667 | 0.1667 |
| $P(A_n\|B)$ | 0.004 | 0.1443 | 0.2068 | 0.2144 | 0.2152 | 0.2153 |
| Nonuniform prior[a] | | | | | | |
| $P(A_n)$ | 0.01 | 0.2 | 0.4 | 0.3 | 0.08 | 0.01 |
| $P(A_n\|B)$ | 0.0002 | 0.1475 | 0.4228 | 0.3287 | 0.0880 | 0.0110 |

[a] From S. Kaplan and B. J. Garrick, On the use of a Bayesian reasoning in safety and reliability decisions—three examples, Nucl. Technol. 44, 231 (1979).

Figure 4-4 - Bayesian calculations for the example [Source: MCC81, page 19]

What we do now is take six cases, in each of which we hypothesize the value of the release probability. We then use Bayes theorem to test how good our hypotheses are (i.e. calculate the probability that each hypotheses is correct). We label our hypotheses $A_1$ to $A_6$.

$A_1$ = release prob. = $10^{-3}$
$A_2$ = release prob. = $10^{-4}$
.
.
.
$A_6$ = release prob. = $10^{-8}$

If $A_1$ were true, then:
$P(B|A_1) = (1\text{-}10^{-3})^{4000} = 0.0183$
since we can assume shipments are independent, the probability of a single success is $1\text{-}10^{-3}$, and $P(B|A_1)$ is just the intersection of 4000 events.

Likewise we find (as shown in Figure 4-4):

$P(B|A_2) = 0.6703$

$P(B|A_3) = 0.9608$

If we know $P(A_1)$,...$P(A_6)$ we could calculate $P(A_n|B)$ or the probability of our statement $A_n$ being actually true. If we <u>assume</u> $P(A_n) = 1/N = 1/6$, we find that $P(A_1|B) = 0.04$, ie, it is not too likely. If we use a more likely $P(A_n)$ we see that $P(A_n|B)$ is adjusted downwards and we conclude that the failure rate is significantly less than $10^{-3}$. The practical application of this is in assigning a frequency - e.g., large pipe break - in a Probabilistic Safety analysis, when none have actually occurred and all we have is the number of reactor-years of experience.

Note that one of the criticisms of Bayes theorem when used this way is that the answer depends on the appropriateness of the initial hypotheses. If there is little data and you put in strange hypotheses, you get back strange answers.

## Probability Distributions

Let p(x)dx be the probability that an event occurs in an interval x to x+dx - the probability density function. Let P(X) be the cumulative probability that the event occurs somewhere between $x_{min}$ and X. Then

$$\begin{aligned} P(X) &= \int_{x_{min}}^{x} p(x)dx \\ &= \text{cumulative probability} \\ &= P(x < X) \\ \text{where } p(x) &= \text{probability density function} \end{aligned} \tag{21}$$

If p(x) is a constant, $p_o$, then $P(X) = p_o(X-x_{min})$ as expected.

There are two types of systems:

1) Those that operate on demand (i.e., safety systems)
2) Those that operate continuously (i.e., process systems)

### Demand Systems

We define:

$D_n$ = $n^{th}$ demand

$P(D_n)$ = probability of success on demand n

$P(\bar{D}_n)$ = probability of failure on demand n

$W_n$ = case where system works for each demand up to and including demand n.

What is the probability that it works for *n-1* demands and fails on demand *n*?

$$\therefore \quad P(W_{n-1}) = P(D_1 \; D_2 \; D_3 \; \ldots \; D_{n-1}) \tag{22}$$

$$P(\bar{D}_n \; W_{n-1}) = P(\bar{D}_n | W_{n-1}) \;\; P(W_{n-1}) \tag{23}$$

So

$$\begin{aligned} P(D_1 D_2 D_3 \ldots D_{n-1} \; \bar{D}_n) &= P(\bar{D}_n | W_{n-1}) \; P(W_{n-1}) \\ &= P \; (\bar{D}_n | D_1 D_2 \ldots D_{n-1}) \; . \; P \; (D_{n-1} | D_1 D_2 \ldots D_{n-2}) \ldots . P(D_2 | D_1) \; P(D_1) \end{aligned} \tag{24}$$

If all demands are alike and independent, this reduces to:

$$P(D_1 D_2 \ldots D_{n-1} \bar{D}_n) = P(\bar{D}) \; [1 - P(\bar{D})]^{n-1} \tag{25}$$

Data for demand failure is often published using the symbol $Q_d$.

<u>Example</u>:

$P(\bar{D})$ for a switch is $10^{-4}$. What is the probability that the switch fails at the end of 3 years when the switch is used 20 times per week?

<u>Solution</u>:

Number of demands = 20x52x3 = 3120.

$$\begin{aligned} \therefore \; P(\bar{D}_{3120} | W_{3119}) &= 10^{-4} \; (1 - 10^{-4})^{3119} \\ &= 0.732 \times 10^{-4}. \end{aligned} \tag{26}$$

This is the same as the probability of any single specified failure, say on demand 25 or 87, out of 3120 demands (i.e., it doesn’t matter when the failure occurs).

If the switch were repaired immediately upon any failure, then the probability that it would fail *once* at anytime within the 3 years is just 3120 times the probability that it would fail at any specified demand, i.e., 3120 x 0.732 x $10^{-4}$ = 0.228.

## Failure Dynamics

Failures are not static events. Let's look at failure dynamics.

$$\begin{aligned} f(t)dt &= \text{probability of failure in the interval dt at time t} \\ F(t) &= \text{accumulated failure probability} \\ &= \int_0^t f(t')dt' \end{aligned} \tag{27}$$

Assuming that the device eventually fails, the reliability, R(t) is defined as

$$\begin{aligned} R(t) &= 1 - F(t) \\ &= \int_0^\infty f(t')dt' - \int_0^t f(t')dt' \\ &= \int_t^\infty f(t')dt' \end{aligned} \tag{28}$$

So,

$$f(t) = -\frac{dR(t)}{dt} = \frac{dF(t)}{dt} \tag{29}$$

If λ(t) dt = probability of failure at time t given successful operation up to time t (defined as the conditional failure rate), then:

$$\begin{aligned} f(t)dt &= \lambda(t)\ dt\ R(t) \\ \text{or}\quad f(t) &= \lambda(t)\ R(t) \\ &= -\frac{dR}{dt} \end{aligned} \tag{30}$$

$$\therefore \frac{dR}{dt} = -\lambda(t)\ R(t) \tag{31}$$

$$\therefore \frac{dR}{R} = -\lambda(t)\ dt \tag{32}$$

$$\therefore \int_{R(0)}^{R(t)} \frac{dR}{R} = -\int \lambda(t)dt = \ln R(t) - \ln R(0) \tag{33}$$

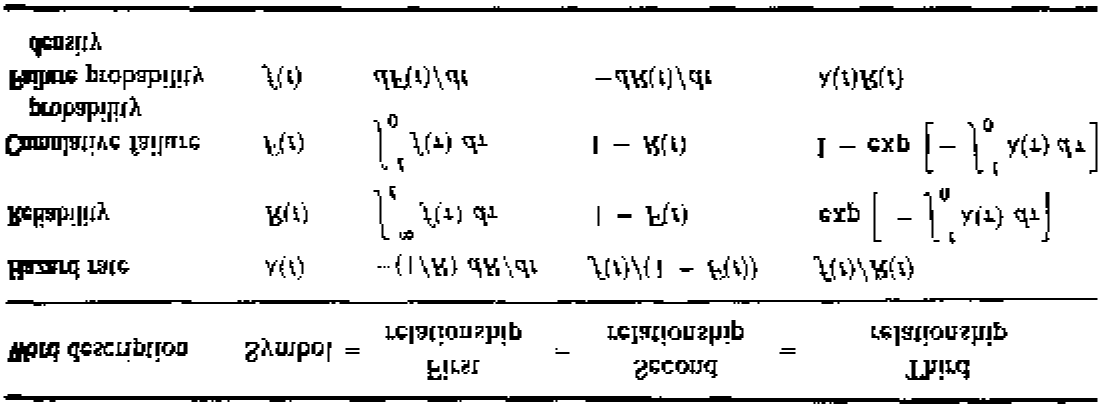
Since R(0) = 1,

$$R(t) = \exp\left[-\int_0^t \lambda(t)dt\right] \tag{34}$$

If λ is constant, (i.e., random failures):

$$R(t) = e^{-\lambda t}. \tag{35}$$

Given λ(t), we can determine everything else. See Figure 4-5 for a summary of commonly used terms and relationships. See Figure 4-6 for typical λ vs t.



| Word description | Symbol = | First relationship | = | Second relationship | = | Third relationship |
|---|---|---|---|---|---|---|
| Hazard rate | $\lambda(t)$ | $-(1/R)\,dR/dt$ | | $f(t)/(1-F(t))$ | | $f(t)/R(t)$ |
| Reliability | $R(t)$ | $\int_t^\infty f(\tau)\,d\tau$ | | $1-F(t)$ | | $\exp\left[-\int_0^t \lambda(\tau)\,d\tau\right]$ |
| Cumulative failure probability | $F(t)$ | $\int_0^t f(\tau)\,d\tau$ | | $1-R(t)$ | | $1-\exp\left[-\int_0^t \lambda(\tau)\,d\tau\right]$ |
| Failure probability density | $f(t)$ | $dF(t)/dt$ | | $-dR(t)/dt$ | | $\lambda(t)R(t)$ |

Figure 4-5 - A summary of equations relating λ(t), R(t), F(t), and f(t)

## Mean time to failure (MTTF)

$$\begin{aligned} MTTF &= \frac{\int_0^\infty t\ f(t)dt}{\int_0^\infty f(t)dt} = \int_0^\infty t\ f(t)dt \\ &= \int_0^\infty t\ \lambda\ e^{-\lambda t}\ dt \quad \text{(assuming } \lambda = \text{random)} \\ &= \frac{1}{\lambda} \end{aligned} \tag{36}$$

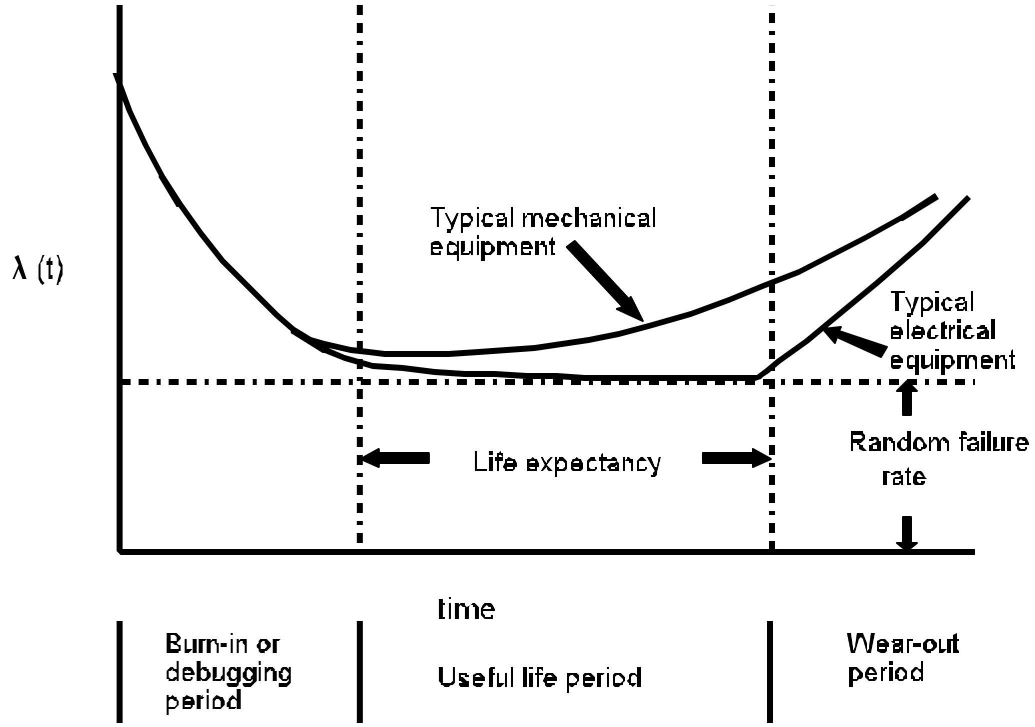


Figure 4-6 - Time dependence of conditional failure (hazard) rate [Source: Ref. 1, page 26]

## Availability, A(t)

If a device undergoes repair then R(t) → A(t)

$$R(t) \leq A(t) \leq 1. \tag{37}$$

A(t) = R(t) for devices that are not repaired.

## Continuous operation with Repair

Assume random failures. This implies

$\lambda$ = constant
R(t) = $e^{-\lambda t}$ = reliability, illustrated in Figure 4-7.

Failure probability = F(t) $\equiv$ 1 - R(t) $\equiv$ 1 - $e^{-\lambda t}$
illustrated in Figure 4-8.

Let repair occur at time interval, $\tau$. Then F(t) is a sawtooth as illustrated in Figure 4-9.

If $\tau << \lambda$ then

$$\begin{aligned} F(t) &= 1 - \left(1 - \lambda t + \frac{\lambda^2 t^2}{2} \ldots\right) \\ &= \lambda t \quad \text{for } \ t < \tau \text{ in any interval} \\ &\qquad \text{and } t \text{ is measured the time of last repair.} \end{aligned} \tag{38}$$

$$\therefore \langle F \rangle = \frac{\lambda\tau}{2} \ . \tag{39}$$

This is a useful rule of thumb but you can always calculate accurately from:

$$\begin{aligned} \langle F \rangle &= \frac{\int_0^\tau F(t)dt}{\int_0^\tau dt} = \frac{t \Big|_0^\tau + \frac{e^{-\lambda t}\Big|_0^1}{\lambda}}{\tau} \\ &= \frac{\lambda\tau + e^{-\lambda\tau} - 1}{\lambda\tau}. \end{aligned} \tag{40}$$

A common design task is to design a system (composed of components that have a known failure rate) to meet some target unavailability $\bar{A}$ $(\bar{A} = F)$ . Given a design, the repair interval is the remaining variable. A frequent repair cycle (low $\tau$) gives a low $\bar{A}$ , but such frequent repair may be untenable due to excessive cost on downtime or even hazard to repair personnel. In such a situation, alternative designs would have to be considered.

Often, repair may not be required in order to return F to 0. It may be sufficient to simply test the components to ensure that they are available. This is usually the case for “demand” systems.

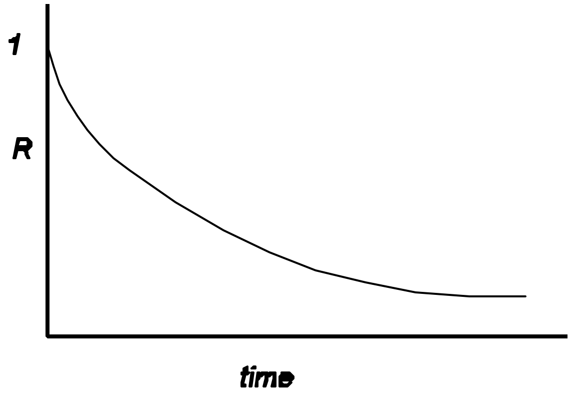


Figure 4-7 - Reliability vs. Time

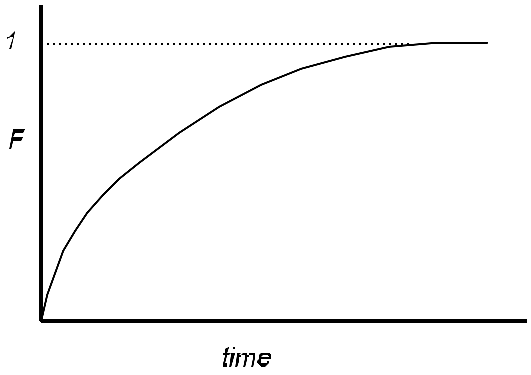


Figure 4-8 - Failure Probability vs. Time

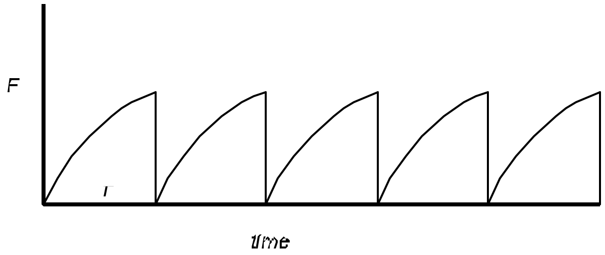


Figure 4-9 - Failure probability with repair

**Example - Shutdown System**

Consider the case of a single shutoff rod (SOR) for a reactor. Given a failure rate based on previous experience of $\lambda = 0.002$/year and a required unavailability of $\leq 10^{-3}$, what is the required test period, $\tau$?

$$\bar{A} \approx \frac{\lambda\tau}{2} = 0.001\ \tau \qquad (41)$$

To meet the $\bar{A}$ target of $10^{-3}$,

$$\tau \leq \frac{10^{-3}}{0.001/\text{year}} = 1\ \text{year} \qquad (42)$$

This is certainly a reasonable test period. But if the $\bar{A}$ target were $10^{-6}$ or if the failure rate were 2 / year, then the required test period would be $10^{-3}$ years or about 3 times per day! This would not be reasonable.

A more realistic shutdown system would have a bank of, say, 6 SORs, as illustrated in Figure 4-10.

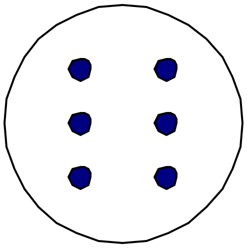


Figure 4-10 - Core with
Six SORs (from top)

When the shutdown system (SDS) is activated some, all or none of the rods drop into the core. The possible events are enumerated in Table 4-1.

Assuming that the rods fail independently and that the failure rate is λ, then the probability of a given rod failing on average is:

$$\langle F \rangle \approx \frac{\lambda T}{2} \quad (\equiv p \ \text{ for conciseness}) \qquad (43)$$

as before. And the success probability is 1-p. In general the probability for event $E_k$, k = 1, 2... N is

$$P(E_k) = \frac{N!}{(N-k)!k!} (1-p)^{N-k} p^{k} \qquad (44)$$

The factor $\frac{N!}{(N-k\ )!k!}$ gives the number of possible ways for that event to happen, the factor $(1-p)^{N-k}$ is the probability that N-k rods all successfully drop and the factor $p^k$ is the probability that k all fail to drop.

Table 4-1 - SDS event possibilities

| Event | # rods drop | # rods fail to drop |
|---|---|---|
| E0 | 6 | 0 |
| E1 | 5 | 1 |
| E2 | 4 | 2 |
| E3 | 3 | 3 |
| E4 | 2 | 4 |
| E5 | 1 | 5 |
| E6 | 0 | 6 |

Thus:

$P(E_o) = (1-p)^6$
$P(E_1) = 6(1-p)^5p$
$P(E_2) = 15\ (1-p)^4p^2$
$P(E_3) = 20\ (1-p)^3p^3$
$P(E_4) = 151-p)^2p^4$
$P(E_5) = 6(1-p)p^5$
$P(E_6) = p^6$

Since these are the only possibilities, they sum to unity, i.e:

$$\sum_{k=0}^{N} P(E_k) = 1 \qquad (46)$$

Normally, there are more SORs than necessary for reactor shutdown and it is sufficient to require that, say, 4 of the 6 rods must drop. If this were the design criteria, then events $E_o$, $E_1$ and $E_2$ represent the successful deployment of the SDS. Events $E_3 \rightarrow E_6$ represent system failures.

The system unavailability for a 4 out of 6 criterion is thus:

$$\begin{aligned}\bar{A} &= \sum_{k=3}^{6} P(E_k) = 1 - \sum_{k=0}^{2} P(E_k) \\ &= 1 - (1-p)^6 - 6(1-p)^5 p - 15(1-p)^4 p^2 \\ &\text{where } p = \frac{\lambda\tau}{2}\end{aligned} \tag{47}$$

Given a $\lambda$ and an assumed $\tau$, the $\bar{A}$ is calculated and compared to the required unavailability.

The $\tau$ is then adjusted until the $\bar{A}$ target (say $10^{-3}$) is met. For a $\lambda$ of, say 0.02/year, we find that

$\bar{A}$ is 2 x$10^{-5}$ for a $\tau$ of 1 year. Thus testing every year is more than enough for this design to meet the unavailability target.

The above assumes that, when testing occurs, any deficiencies are immediately and instantaneously repaired so that the "clock" is effectively reset and the failure probability is reset to zero. However, repairs cannot usually be made right away. The plant will have to operate with less than 6 SORs available and the unavailability target must still be met.

For instance, assume that the operator finds that one rod fails the test and has to be declared "out of service". The above calculation needs to be repeated based on a 4 out of 5 criterion rather than a 4 out of 6.

Thus:

$$\begin{aligned}\bar{A} &= 1 - \frac{5!}{5!0!}(1-p)^5 - \frac{5!}{4!1!}(1-p)^5 p \\ &= 1 - (1-p)^5 - 5(1-p)^4\, p \\ &\equiv \bar{A}_1 \text{ (to denote unavailability with 1 rod out of service)}\end{aligned} \tag{48}$$

A $\tau$ of 1 year gives $\bar{A}_1$ = 0.00098, which just meets the $\bar{A}$ target of $10^{-3}$.

We continue in this way by also considering the case where 2 rods fail their test and are taken out of service. Now the SDS must operate on a 4 out of 4 basis. All remaining rods must drop. In this case the unavailability is

$$\overline{A_2} = 1 - (1\text{-}p)^4$$

For $\tau$ = 1 year, we find $\bar{A}_2$ = 0.039 and the operator must step up the testing programme dramatically ($\tau$ = 0.02 years or once every week) to achieve $\bar{A}$ = $10^{-3}$ or better.

<u>To summarize</u>:

Table 4-2 - SDS summary

| Case | $\bar{A}_2$ | $\tau$<br>(per year) | Operator Action |
|---|---|---|---|
| 0 rods fail test | 2 x $10^{-5}$ | 1 | None |
| 1 rod fail test | 0.00098 | 1 | Repair rod |
| 2 rods fail test | .0008 | .02 | Repair rods<br>Test every week until rods are repaired |
| 3 or more rods fail test | 1 | | Shutdown since need at least 4 rods available |

## Fault Tree Example

A more systematic way to carry out the same analysis as per the previous section is to develop a fault tree. We start by identifying the end result (SDS1 fails to deploy) and itemize all the ways that this can happen. In this case, SDS1 can fail in any one of its 7 modes:

| | |
|---|---|
| Event $E_0$ | 0 rods out of service |
| Event $E_1$ | 1 rods out of service |
| Event $E_2$ | 2 rods out of service |
| Event $E_3$ | 3 rods out of service |
| Event $E_4$ | 4 rods out of service |

These modes are automatic failures since at least 4 rods are required. The reactor is not operated in these modes.

| | |
|---|---|
| Event $E_5$ | 5 rods out of service |
| Event $E_6$ | 6 rods out of service |

All these modes are mutually exclusive so we OR their probabilities of failures. The fault tree is shown in Figure 4-13.

We digress briefly to explain the symbols used in the fault tree. One starts at the top with the event of interest, usually the system failure. Then one determines each and every *immediate* cause of such an outcome. If either of several immediate causes is sufficient to cause the "top" event, then they are joined by an "OR" gate, which looks like Figure 4-11. It means: Event A OR event B must occur in order for event C to occur.

Conversely, if *all* of several immediate causes must occur in order to cause the "top" event, then they are joined by an "AND" gate, which looks like Figure 4-12. It means: Event A AND event B must occur in order for event C to occur. It can also be represented by the same symbol containing a · sign

In Figure 4-13 below, the "OR" gate is represented by a symbol as in Figure 4-11. It can also be represented by the same symbol containing a + sign, or just by a line junction (uncommon).

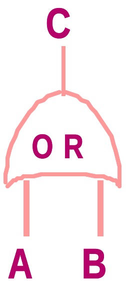


Figure 4-11 - OR Gate

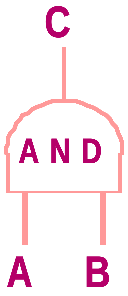


Figure 4-12 - AND Gate

We expand each option until we can no longer decompose the event or we arrive at a point where we know the probability of failure.

For the case of 0 rods out of service, the probability of being in that mode is $(1-p)^6$ as before.

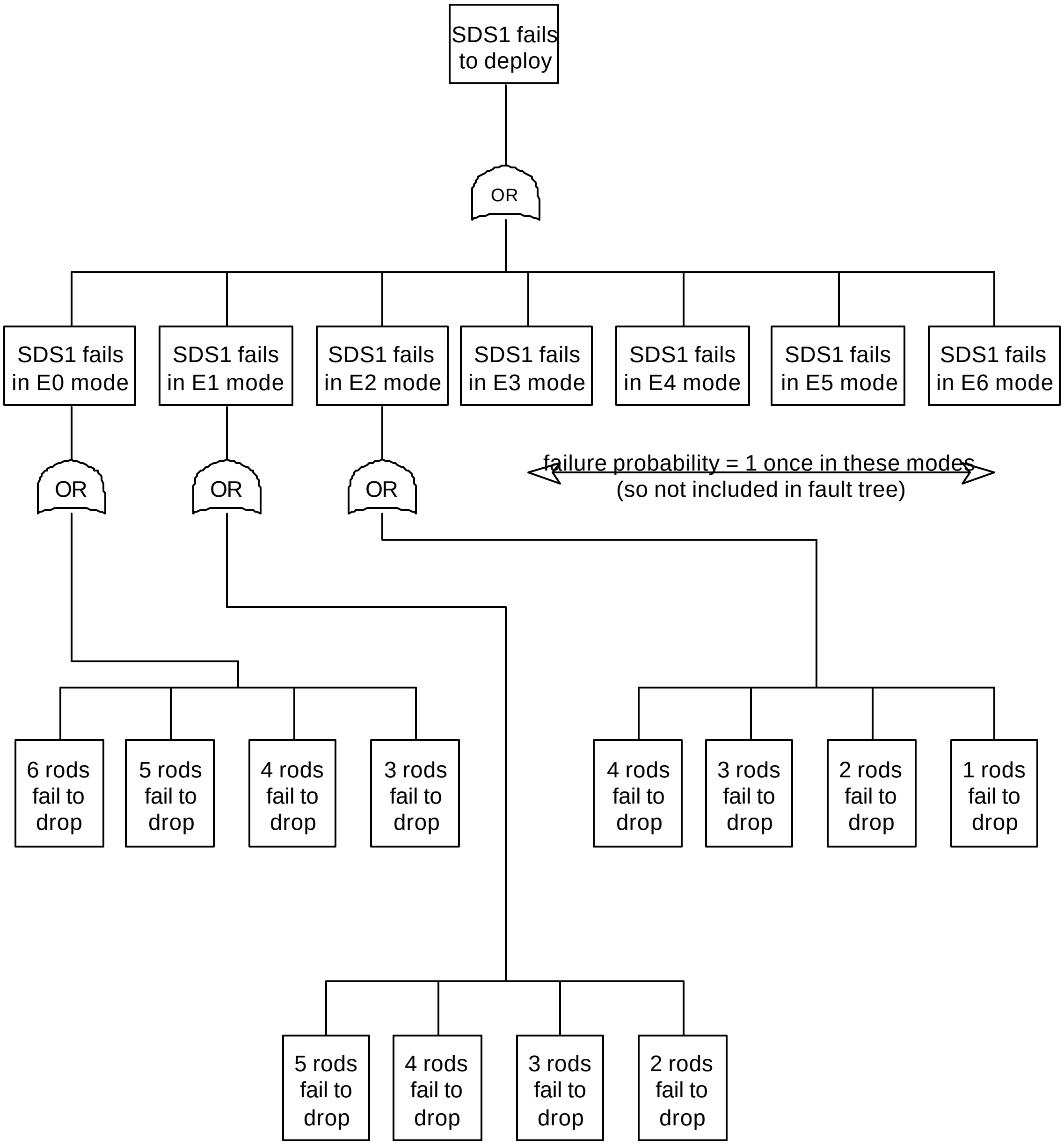


Figure 4-13 - Six ShutOff Rod System Fault Tree

Within that mode, failure occurs if either:

6 rods fail to drop [probability of this failure mode = $p^6$]
5 rods fail to drop [probability of this failure mode = $6\ (1\text{-}p)\ p^5$]
4 rods fail to drop [probability of this failure mode = $15\ (1\text{-}p)^2\ p^4$]
3 rods fail to drop.[probability of this failure mode = $20\ (1\text{-}p)^3\ p^3$]

These events are mutually exclusive. Thus that portion of the tree is expanded as shown. The unavailability of SDS1 while in the $E_0$ mode is simply:

$$\begin{aligned}\bar{A}_0 &= \sum \text{failure modes when 0 rods are out of service}\\ &= p^6 + 6(1-p)p^5 + 15(1-p)^2p^4 + 20(1-p)^3p^3\\ &\text{where } p = \frac{\lambda\tau}{2}\end{aligned} \tag{49}$$

The contribution to unavailability of the system for this segment of the fault tree is:

$$\bar{A}\ (\text{no rods out of service}) = (1-p)^6\ \bar{A}_0 \tag{50}$$

The other modes can be expanded in like fashion to give:

$$\begin{aligned}\bar{A}_1 &= \sum \text{failure modes when 1 rod is out of service}\\ &= p^5 + 5(1-p)p^4 + 10(1-p)^2p^3 + 10(1-p)^3p^2\end{aligned} \tag{51}$$

$$\begin{aligned}\bar{A}_2 &= \sum \text{failure modes when 2 rods are out of service}\\ &= p^4 + 4(1-p)p^3 + 2(1-p)^2p^2 + 4(1-p)^3p\end{aligned} \tag{52}$$

Finally, the total system unavailability is:

$$\bar{A} = (1-p)^6\ \bar{A}_0 + 6(1-p)^5p\ \bar{A}_1 + 15(1-p)^4p^2\ \bar{A}_2 \tag{53}$$

Note that the system unavailability does *not* include the unavailability for modes 3 through 6 since these are modes where the unavailability is *known*. In those cases, the plant would be shut down and put in a fail safe mode by other means. Thus, these modes do not contribute to operating unavailability.

Also note that, in contrast to the example developed in the previous section, the above is based on a common $\tau$. In the previous example $\tau$ was varied within each mode to meet the target unavailability so that:

$$\bar{A} = \bar{A}_0 = \bar{A}_1 = \bar{A}_2 = \bar{A}_{target} \tag{54}$$

## 2 / 3 Logic Example

Figure 4-14 illustrates a relay setup that operates on a 2 out of 3 logic, or 2/3 logic. There are 3 physical relays, D, E and F but each relay has two sets of terminal pairs, allowing them to be connected as shown. The relays are normally open but close when a signal (D, E or F) from their respective channels are received. If any two channels are activated, then the circuit is completed and current can flow between top and bottom. If the sub-circuit is in a safety system circuit, the safety system is activated when two or more of channels D, E and F are TRUE. If the probability of failure of any relay is p, what is the overall unavailability of the sub-circuit?

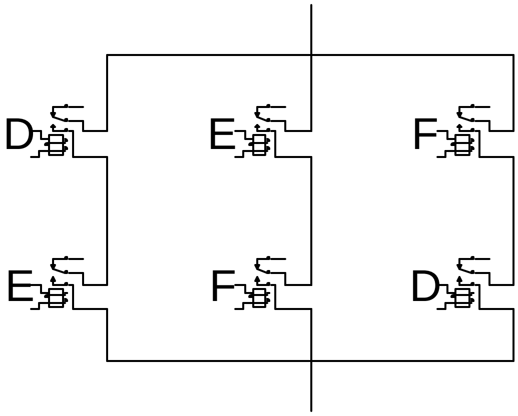


Figure 4-14 - '2 out of 3' Logic - Relay example

This situation is, in fact, completely similar to the SOR case previously examined. Here success is defined as 2 out of 3 events occurring. The unit fails if 3 relays fail or if 2 relays fail. All other states constitute a working sub-system. This is summarized in Table 4-3. All the states are mutually exclusive. The unavailability, then of the unit is simply the sum of the failure probabilities:

$$\begin{aligned}\bar{A} &= \frac{3!}{3!\ 0!}p^3 + \frac{3!}{2!\ 1!}\ p^2\ (1-p) \\ &= p^3 + 3\ p^2\ (1-p)\end{aligned} \qquad (55)$$

In general, for a M out of N system:

$$\begin{aligned}\bar{A} &= \sum_{k=M}^{k=N} \frac{N!}{(N-k)!k!}\ (1-p)^{N-k}p^k \\ &= 1 - \sum_{k=0}^{k=M-1} \frac{N!}{(N-k)!k!}\ (1-p)^{N-k}p^k\end{aligned} \qquad (56)$$

Table 4-3 - Possible sub-system states and probabilities

| Condition of relays DEF<br>(1 = OK,<br>0 = FAILED) | Condition of sub-system | Probability |
|---|---|---|
| 000 | 0 | $p^3$ |
| 001 | 0 | $p^2$ (1-p) |
| 010 | 0 | $p^2$ (1-p) |
| 011 | 1 | p $(1-p)^2$ |
| 100 | 0 | $p^2$ (1-p) |
| 101 | 1 | p $(1-p)^2$ |
| 110 | 1 | p $(1-p)^2$ |
| 111 | 1 | $(1-p)^3$ |

## Ladder Logic

Consider now the system shown in Figure 4-15 (a) where the relays D, E and F have two sets of terminals just like the previous example. In the standby or ready state, the relays are energized closed, providing a current path from top to bottom. When the system “fires”, i.e., when signals are received at the relays, the current path is broken if at least 2 relays change state (go from closed to open). Failure of a component (a relay in this case) occurs when it fails to change state as requested. The failure modes are the same as for the previous example and are given in Table 4-3. We conclude that the system depicted by Figure 4-15 is entirely equivalent to that of Figure 4-14.

Since safety systems are generally wired so that a power failure will invoke the safety system, the ready state has the relays powered closed and the relays open when power is lost. The relays are designed to fail open, thereby tending to fire the safety system if the safety system logic or components fail. The McMaster Nuclear Reactor safety trip signals, for instance, are all wired in series and any one signal breaks the current to the magnetic clutches holding up the shutoff rods.

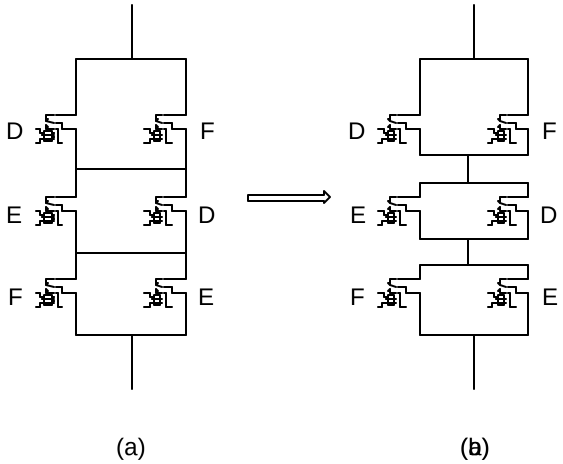


Figure 4-15 - '2 out of 3' Ladder Logic

In actual systems, the relays of the ladder shown in Figure 4-15 do not have dual terminals. Rather, separate relays are used, depicted as D1, D2, etc. in Figure 4-16.

Failure of the system due to relay failures now occurs when all 3 ladder steps fail, ie, when step 1 fails AND step 2 fails AND step 3 fails. The system will succeed if any step succeeds in breaking the circuit (assuming signals at all 3 channels D, E and F).

Step 1 fails if either D1 or F2 fails to switch state upon demand (from closed to open). The fault tree is shown in Figure 4-17. The system unavailability is thus:

$$\begin{aligned}\bar{A} &= (\bar{D}1+\bar{F}2).(\bar{E}1+\bar{D}2).(\bar{F}1+\bar{E}2) \\ &= (2p)^3 = 8p^3\end{aligned} \qquad (57)$$

if all relays fail with probability p. Since p<<1, the unavailability of this circuit with 6 relays is significantly lower than the previous example which uses 3 relays.

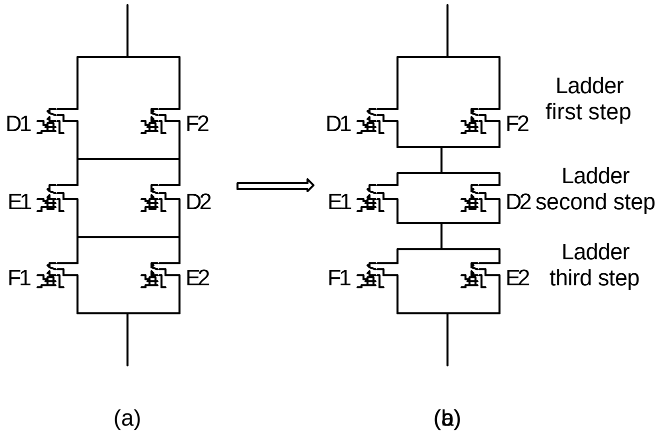


Figure 4-16 - '2 out of 3' Ladder Logic - Separate Relays

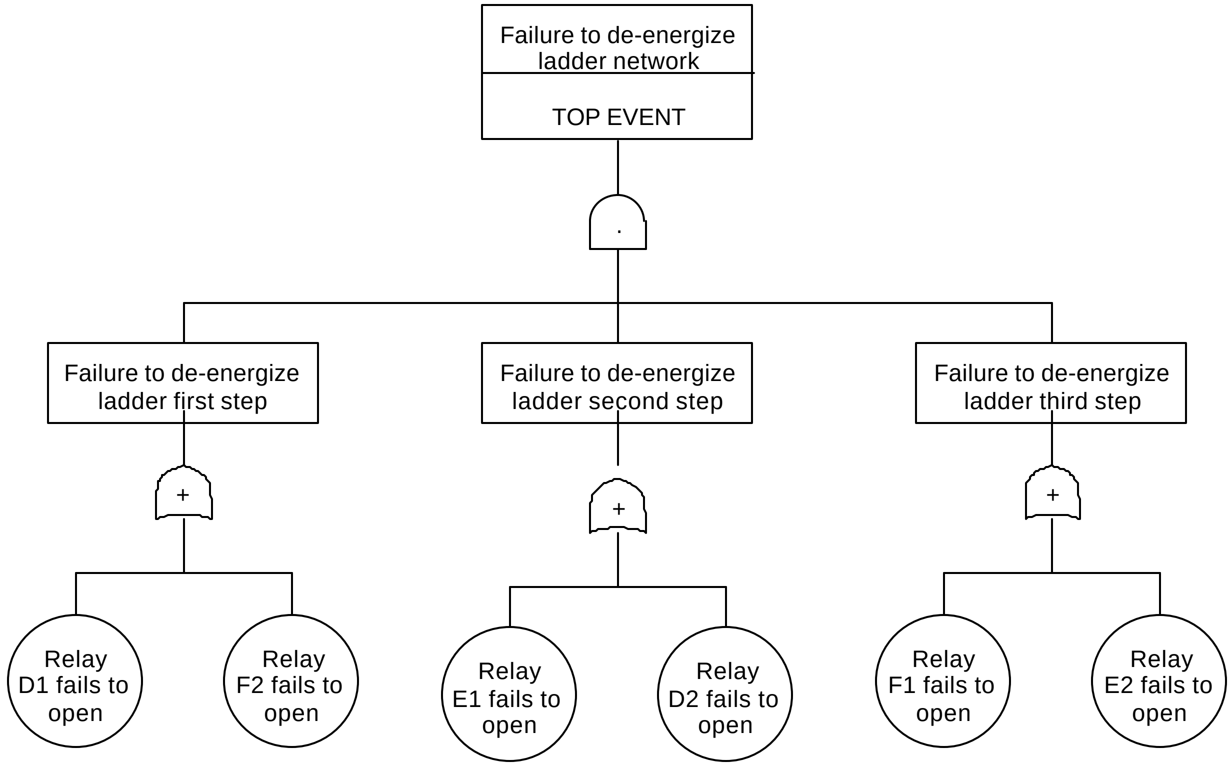

Failure to de-energize ladder network
TOP EVENT
·
Failure to de-energize ladder first step
Failure to de-energize ladder second step
Failure to de-energize ladder third step
+
+
+
Relay D1 fails to open
Relay F2 fails to open
Relay E1 fails to open
Relay D2 fails to open
Relay F1 fails to open
Relay E2 fails to open

Figure 4-17 - Fault Tree for the Ladder Logic Relays

## Unavailability Targets

The unavailability of a system at any given time is, in general, a function of the system configuration. Valves, switches, etc., fail from time to time. System configuration is a function of time. Hence, unavailability is a function of time, as illustrated in Figure 4-18. Safety targets can be defined in terms of some average unavailability or in terms of an instantaneous unavailability. In the later case, the operating station would need to continuously monitor the plant status in order to continuously calculate the station "risk" level. This is likened to having a "risk meter" for the station. Station personnel would respond to equipment failures that lead to a rise in station risk by fixing equipment, maintaining equipment or invoking standby or alternate systems. Working to an average unavailability, on the other hand, does not require such vigilance; instantaneous risk can be permitted to rise in the short term as long as the averages are achieved. This is more workable but less precise in maintaining control of station risk.

Having said that, many stations are using what is effectively an (analytical) 'core melt' meter. Core melt being the only event that can lead to significant public health effects, it is important to know whether changes in station configuration - such as equipment unavailability - lead to a significant increase in the likelihood of core melt during that period. This is particularly true during maintenance outages - what degradation in heat sink redundancy is acceptable, for example?

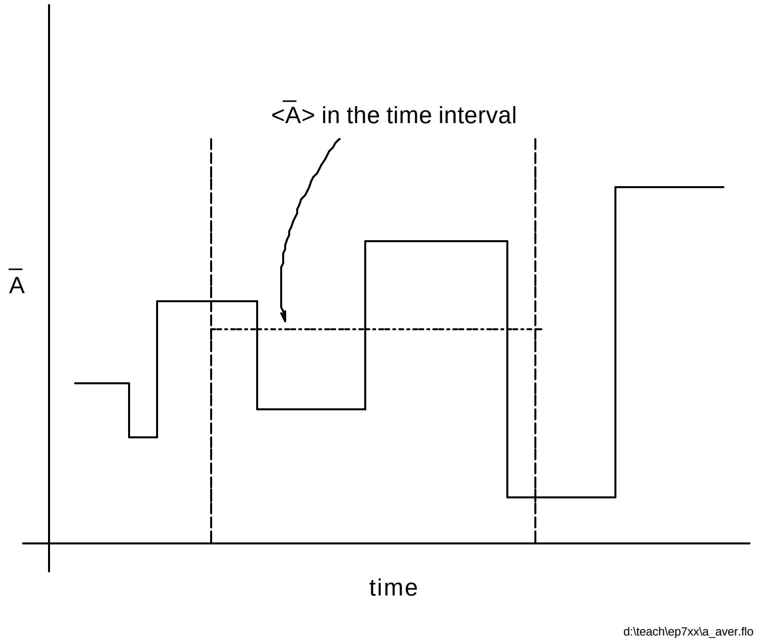


Figure 4-18 - Time dependent unavailability

## Dormant vs active systems

So far we have focussed on systems that are normally dormant and are required to operate on demand. Safety systems generally fall into this category. However, some systems, like the Emergency Core Cooling System (ECCS), are required to activate on demand *and* to continue to function for some defined mission time. The normal response of the ECC to a Heat Transport System (HTS) break (Loss of Coolant Accident or LOCA) is for the ECC to detect the event and initiate the injection of high pressure (HP) cooling water (strictly speaking the water injection function of ECC is called ECI, or Emergency Coolant Injection, since it has other functions such as steam generator cooldown and loop isolation[b]). Then, after the HTS has depressurized, medium pressure and finally low pressure water is injected. The HP water is supplied, for example, from a water tank (accumulator) pressurized by huge gas cylinders. Medium pressure cooling water can be  supplied from a water tank via ECC pumps; and low pressure water is retrieved from the sumps, cooled and pumped back into the HTS. For CANDU reactors a mission time of 1 to 3 months has been set[c]. The ECCS is consequently divided into two separate fault trees for the purposes of analysis: Dormant ECC and Long Term ECC (designated DECC and LTECC respectively). The DECC fault tree focusses on failure to detect the LOCA event, failure to initiate high pressure (HP) cooling water, failure to distribute the flow, and failure to provide medium and low pressure water. The LTECC fault tree focusses on the failure to provide long term low pressure cooling due to pump failure, valve failure, flow blockage, loss of electrical power and loss of coolant supply.

[b] Such a distinction is made in Canada but most other places just use ECC

[c] The mission time is calculated as the time beyond which the decay heat can be removed from the fuel to the moderator *without any water in the fuel channel*, so as to prevent any further fuel failures due to overheating.

## Exercises

1. For the example fault tree of shutoff rods, calculate $\bar{A}_0$ from the success modes. Which way is better
   a. in the 4/6 case
   b. in the 26/28 case?

2. From Ref. 1: A horn on a car operates on demand 99.96% of the time. Consider each event independent from all others. How many times would you expect to be able to honk the horn with a 50% probability of not having a single failure?

3. From Ref. 1: A light bulb has a $\lambda(t) = 5\text{x}10^{-7}$ t, where t is the time in days. What is the MTTF for the bulb?

4. What can you say quantitatively about the probability of a large LOCA in a CANDU?

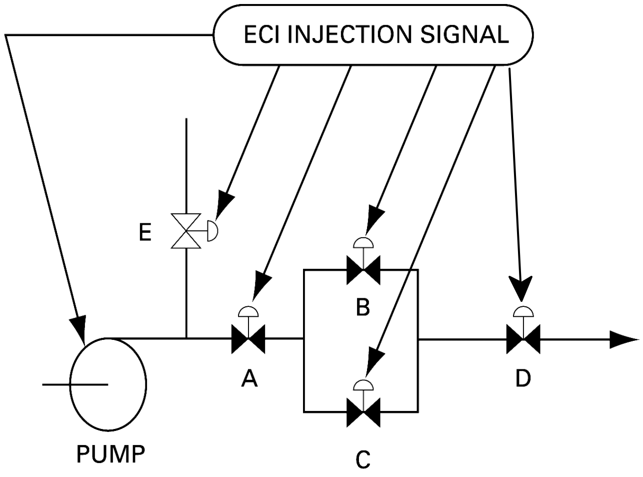
5. In a fuel plant, two assembly lines, L1 and L2 produce 40% and 60% respectively of the fuel pellets. From past experience, it is known that 0.1% of L1 and 2% of L2 of the fuel pellets are defective. If a fuel pellet was chosen randomly and found defective, what is the probability that it was produced by L1?

6. Describe the advantages and disadvantages of Bayesian techniques for risk evaluations. What applications could you use Bayesian techniques in an operating nuclear power plant?

7. Two x 100% electrical motor driven auxiliary feedwater pumps are located in the turbine building and depend on recirculated cooling water and two independent power supplies. Identify common mode and cause failures. The utility wants to extend the life of the plant and is planning refurbishment. What mitigation features or modifications would you do to reduce or eliminate these failures?

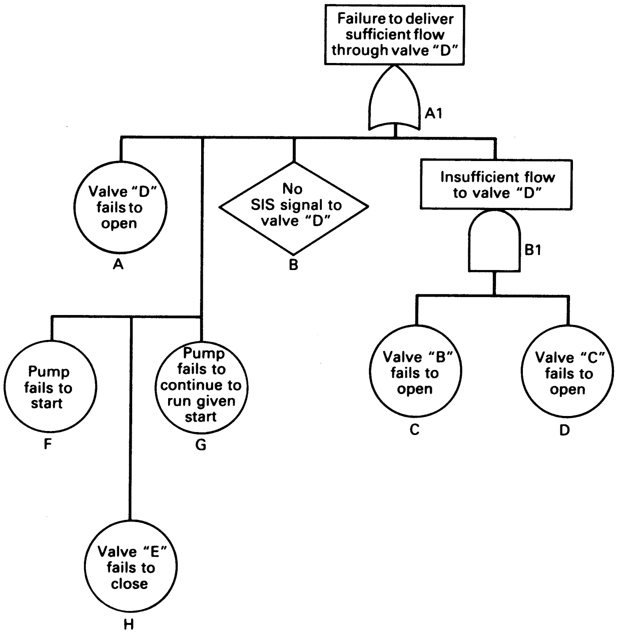
8. Based on the slide below showing a simple emergency coolant injection system and the results of the fault tree analysis below, identify any weaknesses of the design. Draw a schematic of the system to improve the system reliability. Calculate the unavailability if the pump and valves

have a failure rate of 0.01/y and 0.002/y respectively. How would you meet an unavailability of 1E-3? List any assumptions.

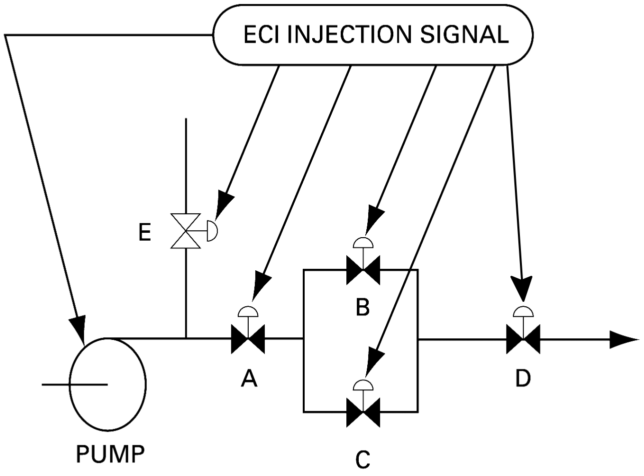


Actions for necessary for success:

1 Start pump
2 Close valve ‘E’
3 Open valves ‘A’ and ‘D’
4 Open either or both valves ‘B’, ‘C’


Failure to deliver sufficient flow through valve "D"
A1
Valve "D" fails to open
A
No SIS signal to valve "D"
B
Insufficient flow to valve "D"
B1
Pump fails to start
F
Pump fails to continue to run given start
G
Valve "B" fails to open
C
Valve "C" fails to open
D
Valve "E" fails to close
H

9. Consider the simplified ECC system shown below.



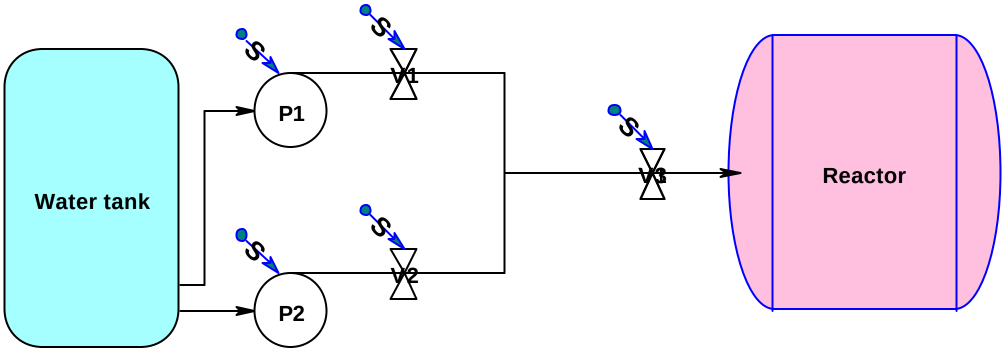
Assuming the signal has a demand failure probability of 0.005, the pump has a demand failure probability of 0.02 and each valve has a demand failure probability of 0.01, work out the demand failure probability of the system.

Note: Actions for necessary for success:

1 Start pump
2 Close valve 'E'
3 Open valves 'A' and 'D'
4 Open either or both valves 'B', 'C' (i.e. valves are 2 x 100%)

10. Consider the simplified ECC system shown below. Calculate the demand failure probability to deliver sufficient flow to the reactor:
(a) using Boolean logic, and then
(b) by drawing the fault tree and calculating it out
Note that there are two 100% trains - i.e. each pump can deliver 100% of the required flow. Note also that the valves are normally closed and the pumps are normally stopped, and require the signal to open them and start them respectively.

Define your system boundaries specifically and very carefully.



**Demand failure probabilities for each component:**

**Pump (P): 0.01**
**Valve (V): 0.01**
**Signal (S): 0.001**

## Appendix 1 - Example of Common Cause Failure

Consider two shutoff rods, each of which as a probability of failure of 0.001 per demand. What is the probability that they both fail when required?

If they are independent, then P(A1A2) = P(A1)P(A2) = $(0.001)^2 = 10^{-6}$ per demand.

Suppose there is a common cause failure 10% of the time. That is:

$$P(A1) = P(A2) = 0.0009 \text{ (random)} + 0.0001 \text{ (CC)}$$

so the probability of one rod failing *given that the other has failed* is 90% random and 10% common cause (with probability 1):

$$P(A1|A2) = 0.9 * 0.001 + 0.1 * 1 = 0.1009$$

or

$$P(A1A2) = 0.1009 * 0.001 = 0.0001009 \sim 10^{-4}$$

Thus a 10% common cause probability has increased the combined failure by a factor of 100!

## Appendix 2 - Example of Probabilities for “OR”ed Events

Recall for independent events:

$$1 - P(A_1+A_2+....+A_N) = \prod_{n=1}^{N} [1-P(A_N)] \tag{58}$$

Let’s take an example to see how this works. Take two dice. What is the probability that die 1 shows a six OR die 2 shows a six (i.e., that there is *at least* one six). Recall:

$$P(A_1+A_2) = P(A_1) + P(A_2) - P(A_1A_2) \tag{59}$$

Since $P(A_1) = P(A_2) =1/6$, and $P(A_1A_2) = 1/36$, then clearly

$$P(A_1+A_2) = 1/6 + 1/6 - 1/36 = 11/36.$$

Confirm by counting:

| **Die 1** | **Die 2** | **Number of Cases showing ‘six’** |
|---|---|---|
| 1 | 123456 | 1 |
| 2 | 123456 | 1 |
| 3 | 123456 | 1 |
| 4 | 123456 | 1 |
| 5 | 123456 | 1 |
| 6 | 123456 | 6 |
| **Total Combinations Showing ‘six’** | | 11 |

[Aside: Note that if the question had been “What is the probability that there is *only* one six showing (i.e., that die 1 shows a six OR die 2 shows a six but not both), then you have to subtract off the intersection again, i.e.,

$$P(A_1+A_2) = P(A_1) + P(A_2) - 2\ P(A_1A_2) \tag{60}$$

or

$$P(A_1+A_2) = 1/6 + 1/6 - 1/36 - 1/36 = 10/36.]$$

Here is another way of looking at it. The probability of getting one or more sixes is [1 - the probability of getting no sixes], since the events are mutually exclusive and complete:

P(at least one six) = 1 - P(no sixes)

The probability of getting no sixes for each die is [1 - the probability of getting a six]. The probability of getting no sixes for both dies is the product of the probability of getting no six for each die, since the events are independent (see equation 7):

P(no six for die 1) = 1 - P(six for die 1)
P(no six for die 2) = 1 - P(six for die 2)

So:

P(no six for die 1 AND no six for die 2) = [1 - P(six for die 1)][1 - P(six for die 2)]

Hence

P(at least one six) = 1 - P(no sixes) = 1 - [1 - P(six for die 1)][1 - P(six for die 2)]

= 1 - [1- 1/6][1 - 1/6] = 1 - 25/36 = 11/36

as we had before.

There are two lessons from this example:

- if you’re having trouble understanding some of the arcane equations of probability theory, work through some examples. Mathematicians and purists may cringe - but real science goes from example to theory, not the other way around.

- sometimes it is easier to work from the probability of the complementary event - P(not A) or $P(\overline{A})$ - rather than the probability of the event - P(A), remembering that

$$P(\overline{A}) = 1 - P(A)$$

In this example it didn't make much difference. However suppose you had a thousand dice and asked the same questions - which approach (equation 10 or equation 11) would be easier to use?

## Appendix 3 - Boolean Logic

This is a quick refresher in the basics of Boolean logic.

Boolean Algebra is a set of laws formulated by the British mathematician George Boole. It deals with statements (here represented by A, B, C, etc.) which can be either true or false - often denoted by the numbers 1 and 0 respectively. So for example A=0 means the statement A is false.

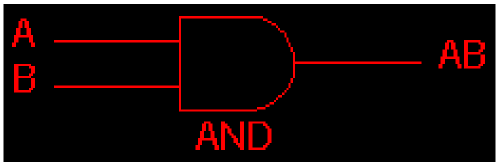
E.g. Let A represent “the earth is flat”. Then A=0.

### Operators

There are several operators which act on these statements:

**AND** is an operator that gives the answer 1 only if *both* of its inputs are 1; and 0 otherwise. It is represented as:

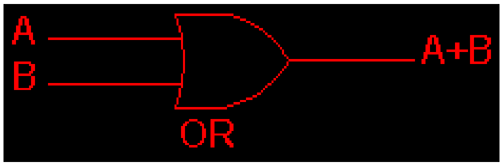
A.B or AB or A∩B or A.AND.B, or graphically as



It can be thought of as the intersection of sets A and B.
Examples:
If A is true and B is true, then A∩B is true.
If A=1 and B=0, then A.B=0

**OR** is an operator that gives the answer 1 only if *either or both* of its inputs are 1; and 0 otherwise. It is represented as:

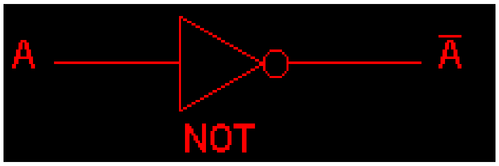
A+B or A∪B or A.OR.B, or graphically as



It can be thought of as the union of sets A and B.
Examples:
If A is true and B is false, then A∪B is true.
If A=1 and B=0, then A+B=1.

NOT is an operator which inverts the input.
It is represented at NOT.

The output of .NOT.A is denoted as A’ or Ā.
It is represented graphically as:



It can be thought of as “the opposite of” or “the complement of”set A.
Examples:
If A is true then .NOT.A is false.

There are four lesser used operators (.NAND., .NOR., .XOR. and .XNOR. which you can look up.

### Basic principles

A=0 or A=1
(i.e. something can only be either true or false, not both)

If A=0, A∩A=0
(i.e. if A is false, and since A∩A=A, then A∩A is false. Sometimes this is written as 0.0=0)

If A=1, A∪A=1
(i.e. if A is true, and since A∪A=A, then A∪A is true. Sometimes this is written as 1+1=1 - remember, you aren’t doing addition!)

If $A=0$, then $A\cup A=0$.
(This is obvious if you think of set theory as we did above. Sometimes it is written $0+0=0$).

If $A=1$, then $A\cap A=1$
Again, obvious from set theory - also written $1.1=1$.

If $A=1$ and $B=0$, then $A\cap B = B\cap A = 0$
(i.e. if A is true and B is false, then the intersection of A and B can never be true, and vice versa. Sometimes this is written as $1.0 = 0.1 = 0$.)

If $A=1$ and $B=0$, then $A\cup B = B\cup A = 1$.
(i.e. if either one of A or B is true, then the union of A and B (A.OR.B) is always true. This can be written $1+0 = 0+1 = 1$.)

### Theorems

These sound abstract but are obvious once you draw Venn diagrams. So I've used set theory symbols. You can substitute the mathematical symbols + and . For ∪ and ∩ if that is more intuitive for you.

**Commutative Law**
$A\cup B = B\cup A$
$A\cap B = B\cap A$

**Associative Law**
$(A\cup B)\cup C = A\cup(B\cup C)$
$(A\cap B)\cap C = A\cap(B\cap C)$

**Distributive Law**
$A\cap(B\cup C) = (A\cap B) \cup (A\cap C)$
$A\cup(B\cap C) = (A\cup B) \cap (A\cup C)$

**Identity Law**
$A\cap A = A$
$A\cup A = A$

**Completeness**

$(A \cap B) \cup (A \cap B') = A$

$(A \cup B) \cap (A \cup B') = A$

**Redundancy**

$A \cup (A \cap B) = A$

$A \cap (A \cup B) = A$

**Mathematical**

$1 + A = 1$

$1.A = A$

$0 + A = A$

$0.A = 0$

$A + \bar{A} = 1$

$A.\bar{A} = 0$

$A + \bar{A}B = A + B$

$A.(\bar{A} + B) = A.B$

DeMorgan's Theorem

$(A+B)' = A'.B'$

$(AB)' = A' + B'$

These will be useful when you work out fault trees mathematically.

## References

1. Norman J. McCormick, *Reliability and Risk Analysis*, Academic Press, 1981, ISBN 0-12-482360-2.