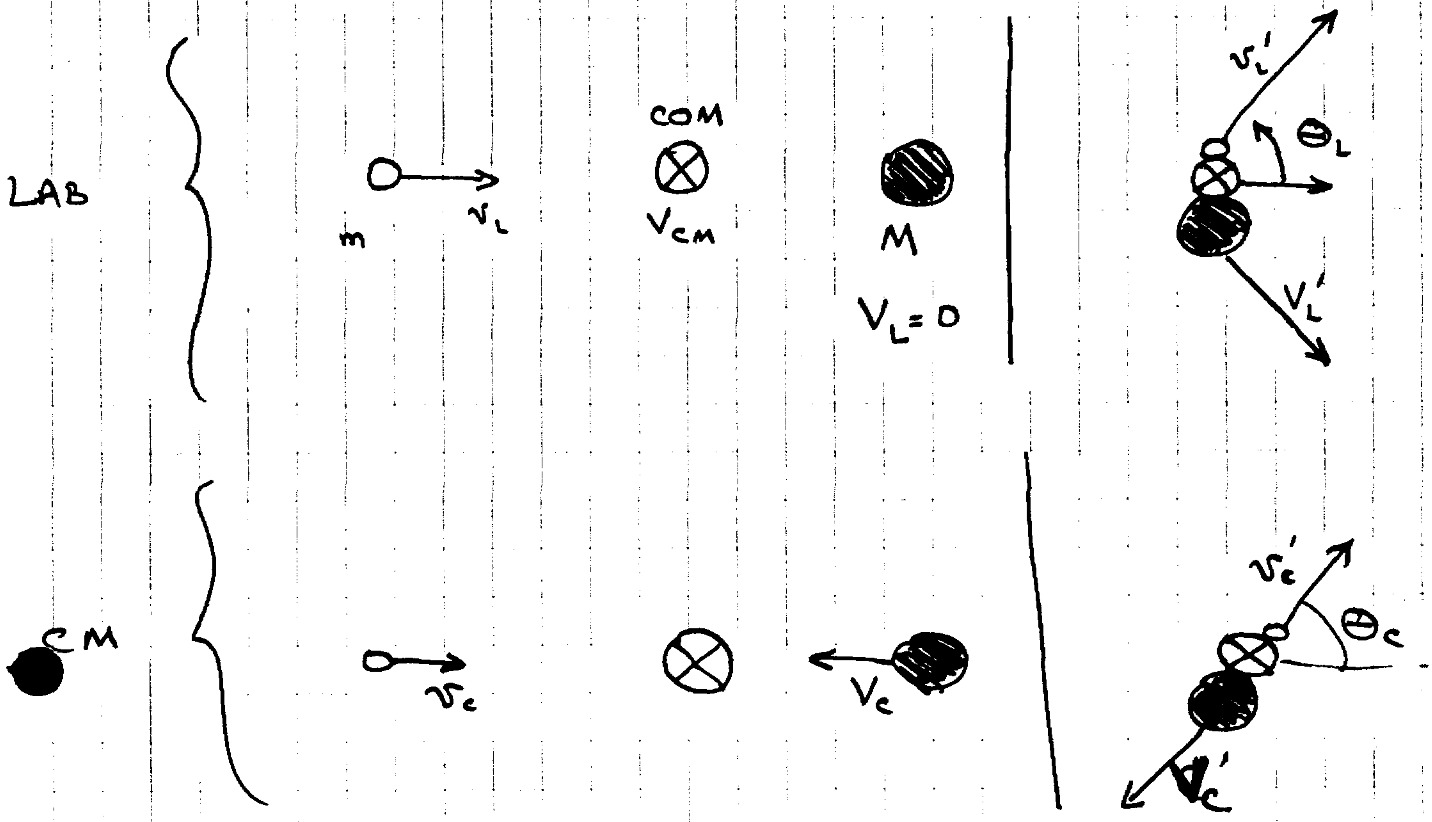
KINEMATICS

2 body collision
- analysis easier in CM
- event occurs in LAB



momentum: $(m+M)\, V_{cm} = m v_L + M V_L \Rightarrow V_{cm} = \frac{1}{(1+A)} v_L$

$$v_c = v_L - V_{cm} = \frac{A}{A+1} v_L$$

$$V_c = -V_{cm} = \frac{-1}{A+1} v_L$$

Kinetic energy:

LAB: $E_L = \frac{1}{2} m v_L^2 = \frac{1}{2} m (v_L')^2 + \frac{1}{2} M (V_L')^2$

CM: $E_c = \frac{1}{2} m v_c^2 + \frac{1}{2} M V_c^2 = \frac{1}{2} \mu v_L^2$

$\left(\mu \equiv \frac{mM}{m+M} \right)$ REDUCED MASS

$= \frac{1}{2} m (v_c')^2 + \frac{1}{2} M (V_c')^2$

Momentum:

LAB: $(m+M) V_{cm} = m v_L + M V_L$

CM: $m v_c + M V_c = 0 = m v_c' + M V_c'$

$\Rightarrow V_c = -\frac{m v_c}{M}$ $\qquad V_c' = -\frac{m}{M} v_c'$

$$E_c = \frac{1}{2} m v_c^2 + \frac{1}{2} M V_c^2 = \left[\frac{1}{2} m \left(\frac{A}{A+1} \right)^2 + \frac{1}{2} M \left(\frac{1}{A+1} \right)^2 \right] v_L^2$$

$$= \left[\frac{1}{2} \frac{m M^2}{m^2 (M/m + 1)^2} + \frac{1}{2} M \left(\frac{1}{M/m+1} \right)^2 \right] v_L^2$$

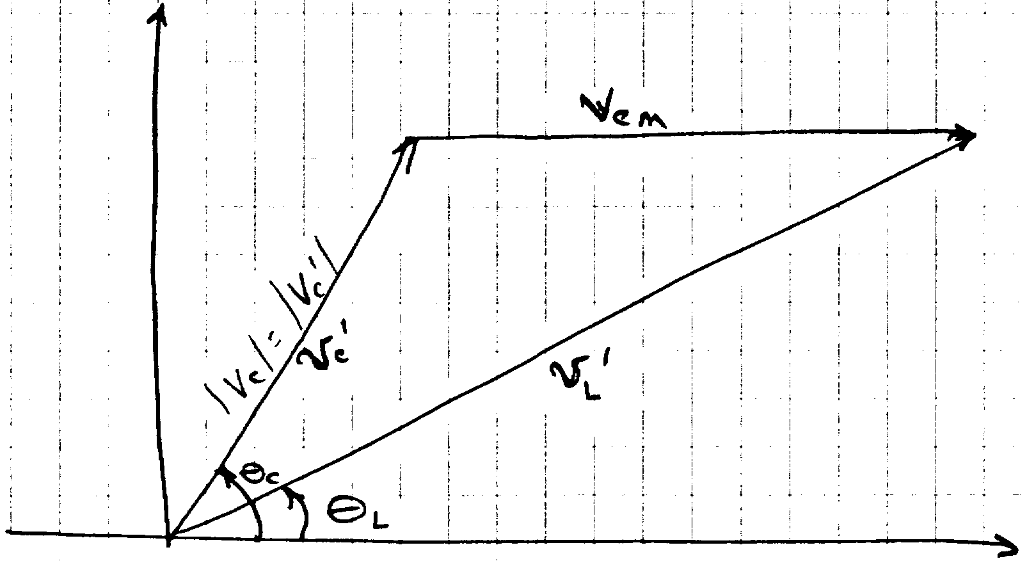
$$= \frac{1}{2} mM \left(\frac{M}{(M+m)^2} + \frac{m}{(M+m)^2} \right) v_L^2$$

$$= \frac{1}{2} \frac{mM}{(M+m)} v_L^2$$

$$= \frac{1}{2} m (v_c')^2 + \frac{1}{2} M \left(\frac{m}{M} \right)^2 (v_c')^2 = \frac{1}{2} m (v_c')^2 \left[1 + \frac{m}{M} \right]$$

$$= \frac{1}{2} m (v_c)^2 \left[1 + \frac{m}{M} \right]$$

$\therefore |v_c| = |v_c'|$, $|V_c| = |V_c'|$



$$v_L' \sin\theta_L = v_c' \sin\theta_c$$

$$v_L' \cos\theta_L = v_c' \cos\theta_c + V_{cm}$$

$$\therefore \boxed{\tan\theta_L = \frac{v_c' \sin\theta_c}{v_c' \cos\theta_c + V_{cm}} = \frac{\sin\theta_c}{\cos\theta_c + 1/A}}$$

used to relate angles in LAB vs CM.

$$\frac{E'}{E} = \frac{\cancel{\tfrac{1}{2}} m (v_L')^2}{\cancel{\tfrac{1}{2}} m (v_L)^2} = \frac{A(v_c' \cos\theta_c + V_{cm})^2 / \cos^2\theta_L}{v_L^2}$$

$$= \frac{\left[\left(\frac{A}{A+1}\right)\cos\theta_c + \left(\frac{1}{1+A}\right)\right]^2 \cancel{v_L^2}}{\cos^2\theta_L \; \cancel{v_L^2}}$$

$$= \frac{(A\cos\theta_c + 1)^2}{(A+1)^2 \cos^2\theta_L} = \frac{A^2\cos^2\theta_c + 1 + 2A\cos\theta_c}{(A+1)^2 * \cos^2\theta_L}$$

$$\overset{?}{\longrightarrow} \quad \frac{A^2 + 1 + 2A\cos\theta_c}{(A+1)^2}$$

$$\therefore E' \equiv \boxed{E_f = \frac{(1+\alpha) + (1-\alpha)\cos\theta_c}{2} E_i}$$

$$\boxed{\text{where } \alpha \equiv \left(\frac{A-1}{A+1}\right)^2}$$

$$\frac{A^2 + \cancel{2A} + 1 + A^2 - \cancel{2A} + 1}{2(A^2+1)}$$

$$\frac{(A+1)^2 + (A-1)^2 + [(A+1)^2 - (A-1)^2] \cos\theta_c}{2(A+1)^2}$$

Notes: $\Theta_c = 0 \Rightarrow E_f = E_i$

$\Theta_c = 180° \Rightarrow E_f = \alpha E_i$

for H , $\alpha = 0$

U , $\alpha = \left(\frac{237}{239}\right)^2 = .9833$

E_f always $< E_i$

$E_f \in (\alpha E_i \to E_i)$

Max. energy loss : $(1-\alpha) E_i$

$$\int_{\alpha E_i}^{E_i} P(E_i \to E_f)\, dE_f = 1$$

$$P(E_i \to E_f) \times (E_i - \alpha E_i) = 1$$

$$\therefore P(E_i \to E_f) = \frac{1}{(1-\alpha)E_i}$$

$$\bar{E}_f \equiv \int_{\alpha E_i}^{E_i} E_f \, P(E_i \to E_f)\, dE_f = \frac{1}{(1-\alpha)E_i} \left.\frac{E_f^2}{2}\right|_{\alpha E_i}^{E_i}$$

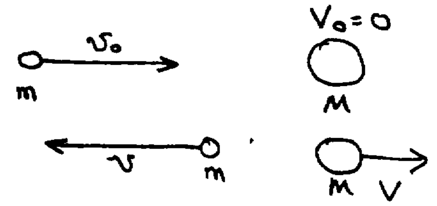
$$= \frac{1}{2} \frac{1-\alpha^2}{1-\alpha} E_i$$

$$= \frac{(1+\alpha)E_i}{2}$$

$$\sigma_s(E_i \to E_f) = \frac{\sigma_s(E_i)}{(1-\alpha)E_i} \qquad \alpha E_i \le E_f \le E_i$$

$$= 0 \qquad \text{otherwise.}$$

Consider a head-on collision (elastic):



Conservation of energy: $\frac{1}{2} m v_0^2 = \frac{1}{2} M V^2 + \frac{1}{2} m v^2$ — ①

Conservation of momentum: $m v_0 = MV - mv$ — ②

From ①, $v_0^2 - v^2 = (v_0 + v)(v_0 - v) = \frac{M V^2}{m}$ — ③

From ② $(v_0 + v) = \frac{MV}{m}$ $\therefore$ $v_0 - v = V$ — ④

$\therefore$ from ④ and ② $m v_0 = M(v_0 - v) - mv$

solving for v/v_0 gives $\boxed{\frac{v}{v_0} = \frac{\left(\frac{M}{m} - 1\right)}{\left(\frac{M}{m} + 1\right)}}$

Hence for neutron $\boxed{\frac{E}{E_0} = \frac{\frac{1}{2} m v^2}{\frac{1}{2} m v_0^2} = \frac{\left(\frac{M}{m} - 1\right)^2}{\left(\frac{M}{m} + 1\right)^2}}$

where m = 1amu for neutron

Note: if M large, E is not very much smaller than E_0.

Ex: 1) neutron striking U^{238}, $E \simeq 99\% \, E_0$.

2) neutron striking neutron, $E = 0$

$\therefore$ best neutron moderator is another neutron or Hydrogen nucleus

In practice, $E \neq 0$ for example (2) above because collision can be other than head-on.

Average Log Decrement, ξ

$$\xi \equiv \overline{\Delta \ln E} = \overline{\ln E_0 - \ln E}$$

For $\underset{(M)}{A} > 10$ $\quad \xi \simeq \frac{2}{A + 2/3}$, $\xi \simeq 1.0$ for ${}_1H^1$

C = # of collisions to reduce fission neutron to thermal energy.

$$C = \frac{\ln(E_f/E_{th})}{\xi}$$

	${}_1H^1$	${}_6C^{12}$	${}_{92}U^{238}$
C	18	110	>2100

In slowing down from 2 Mev to 0.025 ev, the neutron must pass through those energies at which the absorption cross section for U^{238} is extremely high.
If a reactor ~~was~~ made containing $U^{235} + U^{238}$ alone, it would require more than 2100 collisions to slow the neutrons down fr[om] fission energy to the thermal energy necessary to continue the chain. It would take many collisions to slow down past the region of resonance peaks of the U^{238}. This would probably result in the capture of the neutrons by the U^{238}, leaving few neutrons to cause thermal fission.
If, on the other hand, a lighter material was used as a moderator fewer collisions would be required and the chances of slowing down to thermal energy without absorption would be greatly increased.

In order for a material to act as a good moderator in slowing dow[n] neutrons from fission energies to thermal energies, it should have a high value of ξ and a high probability that the material will scatter the neutrons (high Σ_s) and a low absorption probability, Σ_a.

$\therefore$ Define moderating ratio $\overset{MR}{=} \equiv \frac{\Sigma_s \xi}{\Sigma_a}$, Slowing down power = SDP = Σ_s

Material	SDP	MR
H_2O	1.36	65
D_2O	0.18	4860 (pure), 2085 (0.25% H_2O)
Be	0.160	150
C	0.063	170

Slowing Down, Diffusion lengths and Fermi Age

The distance travelled by the neutron before thermalizing is also important.

Total path length = $C \lambda_s$ if scattering is random. But there is a preferential forward scattering of the collisions which changes the average distance travelled between collisions.

We define λ_t, the transport mean free path:

$$\lambda_t = \frac{\lambda_s}{1 - \overline{\cos\theta}} \qquad \text{where } \overline{\cos\theta} \simeq \frac{2}{3M}$$

↖ scattering ∡ ↖ target mass

$\lambda_t \geq \lambda_s$ and $\frac{\lambda_t}{\lambda_s} \uparrow$ as $M \downarrow$ ie more forward scattering for small targets.

The fast diffusion length, L_f, is determined experimentally but can be approximated by

$$L_f^2 \simeq \frac{\lambda_t \lambda_s C}{3} \qquad \text{(for fast neutrons)}$$

(λ for fast neutrons)

$L_f \equiv$ average distance travelled in slowing down

Fermi age $\equiv \tau = L_f^2$

Thermal diffusion length, L is the distance travelled from the time the neutrons have become thermalized to the time they are absorbed. This is also experimentally determined but can be approximated by:

$$L^2 = \frac{\lambda_t \lambda_a}{3} = \frac{1}{3 \Sigma_t \Sigma_a}$$

where λ + Σ are for thermal energies

The leakage from a reactor is partially dependent on L_f + L. A neutron that has to travel a long distance before slowing down or one that travels a long distance after becoming thermal has a greater chance of leaking from a reactor.