Reactor Physics: The One Group Reactor Model

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Summary:

The one group reactor model is introduced for simple geometries and solved analytically and numerically for the steady state. The concept of criticality is introduced and discussed. The transient case is discussed briefly in the context of criticality.

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1 Introduction

1.1 Overview

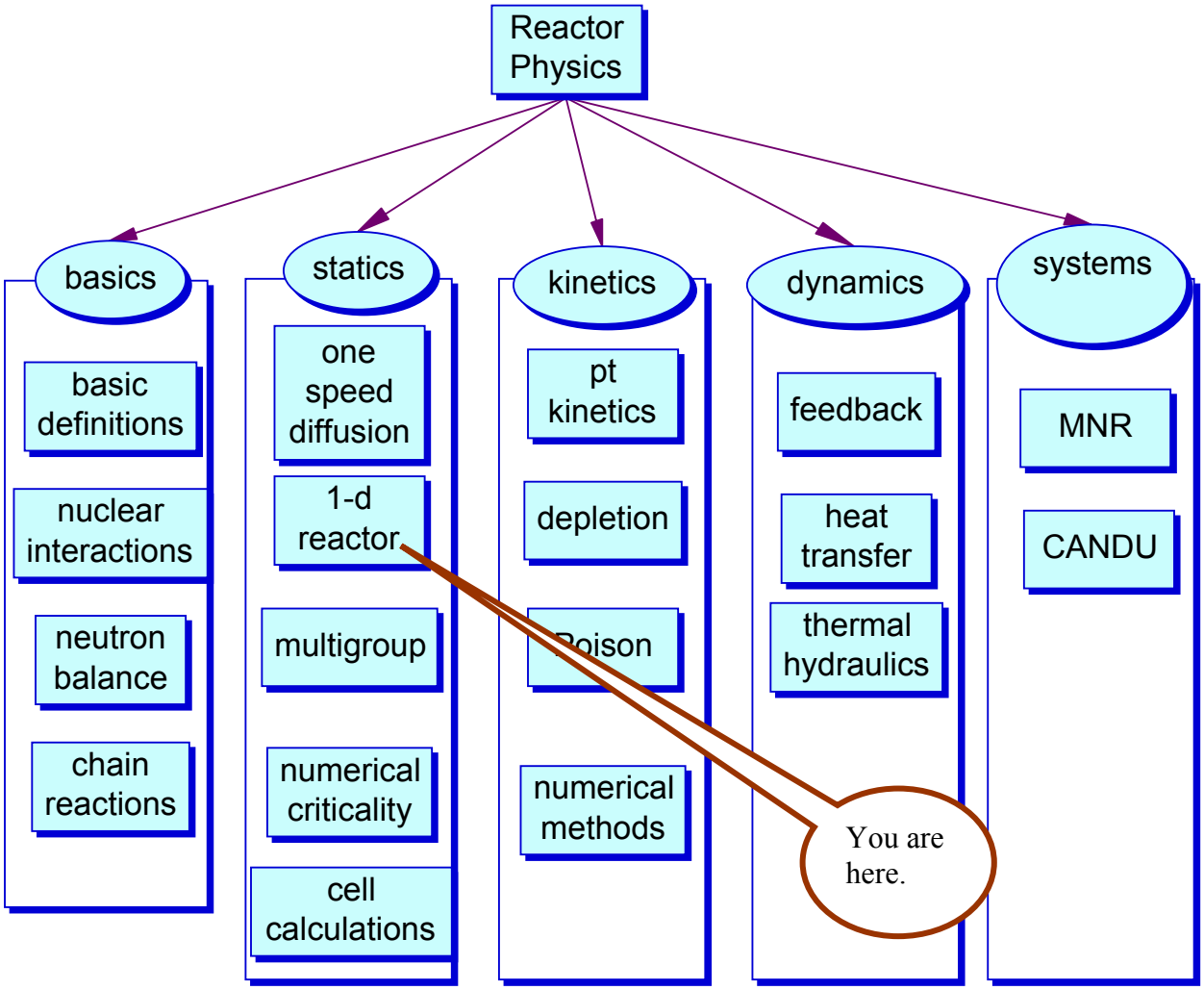

Figure 1 Course Map

1.2 Learning Outcomes

The goal of this chapter is for the student to understand:

- Simple reactor systems
- The notion of criticality, what it means both physically and mathematically
- How to analytically solve for the steady state flux for simple geometries
- How to numerically solve the steady state for more arbitrarily complex geometries.

2 The One Group Model

$$S_f(\mathbf{r},t) \Rightarrow \nu \Sigma_f \phi(\mathbf{r},t) \tag{1}$$

$$\frac{1}{v}\frac{\partial}{\partial t}\phi(\mathbf{r},t) = \nabla \cdot D(\mathbf{r})\nabla\phi(\mathbf{r},t) - \Sigma_a(\mathbf{r})\phi(\mathbf{r},t) + \nu\Sigma_f(\mathbf{r})\phi(\mathbf{r},t) \tag{2}$$

Features:

- This is the time dependent, one speed model where we ignore the delayed neutrons.
- The absorption term, Σ_a is a local parameter that may vary in space, depending on the material at hand (moderator, structure, coolant, fuel, etc.).
- The fuel fission cross section is part of the fuel absorption cross section, ie: $\Sigma_a^{fuel} = \Sigma_\gamma^{fuel} + \Sigma_f^{fuel}$.
- We treat D and Σ as constant over time since typically they do not vary substantially over the short time span that the flux can vary significantly in time. The details of the problem at hand will guide you.

Example: We look at a simple one dimensional slab reactor, a case that readily yields an analytical solution. The reactor is composed of a homogeneous mixture of fuel and moderator. The slab thickness is **a** centimeters. We assume the dimensions represent the extrapolated distances (at which the flux goes to zero).

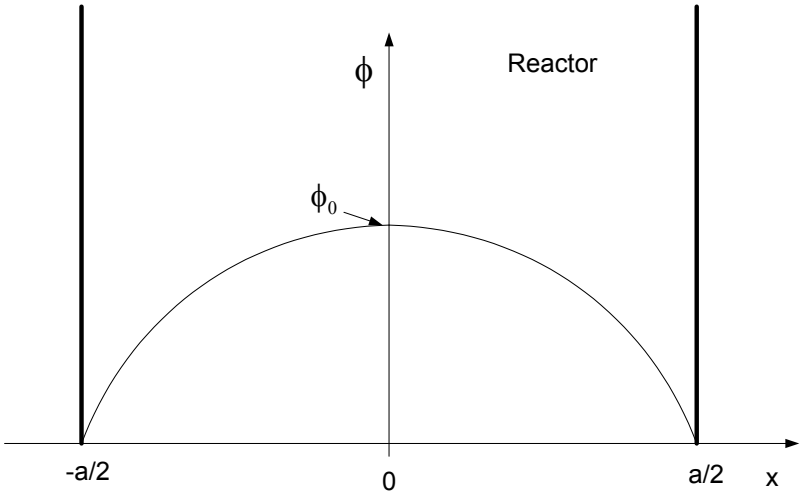

Figure 2 A simple slab reactor

The steady state equation for constant coefficients is:

$$D\frac{\partial^2\phi(x)}{\partial x^2} + (-\Sigma_a + \nu\Sigma_f)\phi(x) = 0 \tag{3}$$

The solution is of the form:

$$\phi = \phi_0 \cos(Bx) \tag{4}$$

and for the flux to go to zero at the extrapolated boundaries, we must have

$$B = \frac{\pi}{a} \tag{5}$$

Substituting equation 4 into 3 we find that, after dropping the common $\phi_0 \cos$ factor:

$$-D B^2 - \Sigma_a + \nu \Sigma_f = 0 \quad \Rightarrow \quad D B^2 = \nu \Sigma_f - \Sigma_a$$

and (6)

$$B_g^2 = \left(\pi / a\right)^2 = \frac{\nu \Sigma_f - \Sigma_a}{D} = B_m^2$$

where the subscript g refers to the geometric buckling and the subscript m refers to the material buckling.

This is the criticality condition.

Referring back to equation 3 and using $\frac{\nu \Sigma_f - \Sigma_a}{D} = B^2$, we find:

$$\frac{\partial^2 \phi(x)}{\partial x^2} + B^2 \phi(x) = 0$$

You will see this form used quite often.

In 3-D,

$$D\left(\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} + \frac{\partial^2 \phi}{\partial z^2}\right) + (-\Sigma_a + \nu \Sigma_f)\phi(x) = 0$$

And the solution is of the form:

$$\phi = \phi_0 \cos(B_x x) \cos(B_y y) \cos(B_z z)$$

Which leads to the criticality condition:

$$B_g^2 = B_x^2 + B_y^2 + B_z^2 = \left(\frac{\pi}{a}\right)^2 + \left(\frac{\pi}{b}\right)^2 + \left(\frac{\pi}{c}\right)^2 = \frac{\nu \Sigma_f - \Sigma_a}{D} = B_m^2$$

where a, b and c are the slab dimensions in the 3 directions. Note that the basic equality of geometric and material buckling holds even though the exact form of the equality is case dependent.

At this point you should be able to answer Questions 1, 2 and 3 at the end of this chapter.

It is important to stop for a moment and reflect on what this means. This is a statement of the balance between neutron production by fission and neutron loss by leakage and absorption. There is a unique slab thickness that ensures that the net production / loss is zero for the given material properties. Any other size will result in a net loss of gain of neutrons over time.

It is also import to realize that this critical size is the size that is **predicted** by the model. Reality will be something else.

3 Relating k to the Criticality Condition

We have $-D B^2 - \Sigma_a + \nu\Sigma_f = 0$. Since there is uncertainty in the material parameters and since $\nu\Sigma_f$ is the dominant source of error, we introduce a fudge factor to ensure criticality:

$$-D B^2 - \Sigma_a + \frac{\nu\Sigma_f}{k_{fudge}} = 0 \tag{7}$$

Rearranging:

$$\frac{\nu\Sigma_f}{k_{fudge}} = D B^2 + \Sigma_a \Rightarrow k_{fudge} = \frac{\nu\Sigma_f}{\Sigma_a + D B^2} = \frac{\nu\Sigma_f \big/ \Sigma_a}{1 + L^2 B^2} \tag{8}$$

This looks like the four factor formula with a leakage term after we manipulate the above equation a bit.

We can write

$$k = k_\infty P_{NL} = , \eta f p P_{NL} \tag{9}$$

where P_{NL} is the probability of non-leakage.

Now,

$$, \sim 1,\ p \sim 1 \text{ and } \eta f = \frac{\nu\Sigma_f^{fuel}}{\Sigma_a^{fuel}} \frac{\Sigma_a^{fuel}}{\Sigma_a^{total}} = \nu\Sigma_f \big/ \Sigma_a \tag{10}$$

And

$$\begin{aligned} P_{NL} &= \frac{\int \Sigma_a \phi dV}{\int \Sigma_a \phi dV + \int \nabla \bullet \mathbf{J} dV} = \frac{\int \Sigma_a \phi dV}{\int \Sigma_a \phi dV - \int D\nabla^2 \phi dV} \\ &= \frac{\int \Sigma_a \phi dV}{\int \Sigma_a \phi dV + \int DB^2 \phi dV} \sim \frac{1}{1 + L^2 B^2} \end{aligned} \tag{11}$$

So in equation 8, if we find that for a given size (ie B^2) and material properties (D, Σ_a, $\nu\Sigma_f$) that k_{fudge} must be, say, 1.1 to make the model critical, this is equivalent that the multiplication factor k = 1.1, ie the arrangement is supercritical as specified and the $\nu\Sigma_f$ term must be reduced or downgraded to bring the reactor model to the critical state.

[Note that in the above equation we used the relationship $D\nabla^2\phi + DB^2\phi(x) = 0$.]

4 Analytical Solution for the Transient case

We had the general one speed neutron balance equation:

$$\frac{1}{v}\frac{\partial}{\partial t}\phi(\mathbf{r},t) = \nabla\cdot D(\mathbf{r})\nabla\phi(\mathbf{r},t) - \Sigma_a(\mathbf{r})\phi(\mathbf{r},t) + \nu\Sigma_f(\mathbf{r})\phi(\mathbf{r},t) \tag{12}$$

For the homogeneous case in one dimension:

$$\frac{1}{v}\frac{\partial}{\partial t}\phi(x,t) = D\frac{\partial^2\phi(x,t)}{\partial x^2} - \Sigma_a\phi(x,t) + \nu\Sigma_f\phi(x,t) \tag{13}$$

We'll try to solve this using Separation of Variables. We define:

$$\phi(x,t) = \psi(x)T(t) \tag{14}$$

and substitute in to get:

$$\frac{1}{v}\psi\frac{\partial T}{\partial t} = TD\frac{\partial^2\psi}{\partial x^2} - \Sigma_a T\psi + \nu\Sigma_f T\psi \tag{15}$$

$$\therefore \frac{1}{T}\frac{\partial T}{\partial t} = \frac{v}{\psi}\left[D\frac{\partial^2\psi}{\partial x^2} + \left(\nu\Sigma_f - \Sigma_a\right)\psi\right] \equiv -\lambda = \text{constant} \tag{16}$$

The LHS (a function of T only) has the solution:

$$T(t) = T(0)e^{-\lambda t} \tag{17}$$

while the RHS (a function of ψ only) can be rewritten as follows:

$$D\frac{\partial^2\psi}{\partial x^2} + \left(\nu\Sigma_f - \Sigma_a\right)\psi = -\frac{\lambda}{v}\psi = 0 \text{ when critical} \tag{18}$$

ie:

$$D\frac{\partial^2\psi}{\partial x^2} + \left(\frac{\lambda}{v} + \nu\Sigma_f - \Sigma_a\right)\psi = 0 \Rightarrow \frac{\partial^2\psi}{\partial x^2} + B^2\psi = 0 \tag{19}$$

Thus

$$\frac{\lambda}{v} + \nu\Sigma_f - \Sigma_a = DB^2 \Rightarrow \lambda = v\left(\Sigma_a + DB^2 - \nu\Sigma_f\right) \tag{20}$$

Now the flux goes to zero at the boundaries of the slab, $x = \pm\frac{a}{2}$, and we try a cosine solution:

$$\psi_n(x) = \cos\left(B_n x\right), \quad B_n^2 = \left(\frac{\pi n}{a}\right)^2 \tag{21}$$

This is an eigenvalue problem and we see that the eigenvalue,

$$\lambda \Rightarrow \lambda_n = v\left(\Sigma_a + DB_n^2 - \nu\Sigma_f\right) \tag{22}$$

The solution is then

$$\phi(x,t) = \sum_{n\ \text{odd}} A_n e^{-\lambda_n t}\cos\left(\frac{n\pi x}{a}\right) \tag{23}$$

where A_n is the amplitude function of the n^{th} term. We use the initial flux and the orthogonal nature of the cosine function to determine A_n. The initial condition is

$$\phi(x,0) = \sum_{n\ odd} A_n \cos\left(\frac{n\pi x}{a}\right) \tag{24}$$

We multiply both sides by $\cos\left(\frac{m\pi x}{a}\right)$ and integrate over the slab:

$$\begin{aligned}\int_{-\frac{a}{2}}^{+\frac{a}{2}} \phi(x,0)\cos\left(\frac{m\pi x}{a}\right)dx &= \sum_{n\ odd}\int_{-\frac{a}{2}}^{+\frac{a}{2}} A_n \cos\left(\frac{n\pi x}{a}\right)\cos\left(\frac{m\pi x}{a}\right)dx \\ &= \frac{a}{2}A_m\end{aligned} \tag{25}$$

$$\therefore A_n = \frac{2}{a}\int_{-\frac{a}{2}}^{+\frac{a}{2}} \phi(x,0)\cos\left(\frac{n\pi x}{a}\right)dx \tag{26}$$

For long times,

$$\phi(x,t) \sim A_1 e^{-\lambda_1 t}\cos\left(\frac{\pi x}{a}\right) = A_1 e^{-\lambda_1 t}\cos B_1 x \tag{27}$$

since **the higher order modes (larger n) decay faster than the lower order modes (low n)**, ie

$$\lambda_1 < \lambda_2 < \lambda_3 < \cdots \tag{28}$$

because

$$B_1^2 < B_2^2 < B_3^2 < \cdots \tag{29}$$

[Note that we ignore the even terms in the summation because they give non-zero terms at the boundaries, $x = \pm\frac{a}{2}$.]

5 Criticality

We had found that $\lambda \Rightarrow \lambda_n = v\left(\Sigma_a + DB_n^2 - v\Sigma_f\right)$ for the transient case. We also found that the slowest decaying mode was that associated with the fundamental eigenvalue, λ_1. It follows that in the steady state,

$$\lambda_1 = 0 = v\left(\Sigma_a + DB_1^2 - v\Sigma_f\right) \tag{30}$$

ie, only the fundamental cosine mode remains. All the other higher order modes have died away. Thus

$$B_1^2 = \underbrace{B_g^2}_{\text{geometric buckling}} = \underbrace{\frac{v\Sigma_f - \Sigma_a}{D}}_{\text{material buckling}} \tag{31}$$

To summarize:

$$\begin{aligned} B_m^2 > B_g^2 &\Rightarrow \lambda_1 < 0 \ \text{ supercritical} \\ B_m^2 = B_g^2 &\Rightarrow \lambda_1 = 0 \ \text{ critical} \\ \underbrace{B_m^2}_{\text{fission source}} < \underbrace{B_g^2}_{\text{leakage}} &\Rightarrow \lambda_1 > 0 \ \text{ subcritical} \end{aligned} \tag{32}$$

6 Criticality for General Bare Geometries

$$\frac{1}{v}\frac{\partial}{\partial t}\phi(\mathbf{r},t) = \nabla\cdot D(\mathbf{r})\nabla\phi(\mathbf{r},t) - \Sigma_a(\mathbf{r})\phi(\mathbf{r},t) + \nu\Sigma_f(\mathbf{r})\phi(\mathbf{r},t) \tag{33}$$

In the steady state with homogeneous properties, we have:

$$\nabla^2\phi(\mathbf{r},t) + \left(\frac{\nu\Sigma_f - \Sigma_a}{D}\right)\phi(\mathbf{r},t) = 0$$

or

$$\nabla^2\phi(\mathbf{r},t) + \underbrace{\left(\frac{k_{\infty}-1}{L^2}\right)}_{\equiv B^2}\phi(\mathbf{r},t) = 0, \text{ where } L^2 \equiv \frac{D}{\Sigma_a} \text{ and } k_{\infty} \equiv \frac{\nu\Sigma_f}{\Sigma_a} \tag{34}$$

The boundary condition is zero flux at the extrapolated boundary (surface):

$$\phi(\tilde{\mathbf{r}}_s) = 0 \tag{35}$$

Consider, for example a reactor made in the shape of a right cylinder as shown in figure 3.

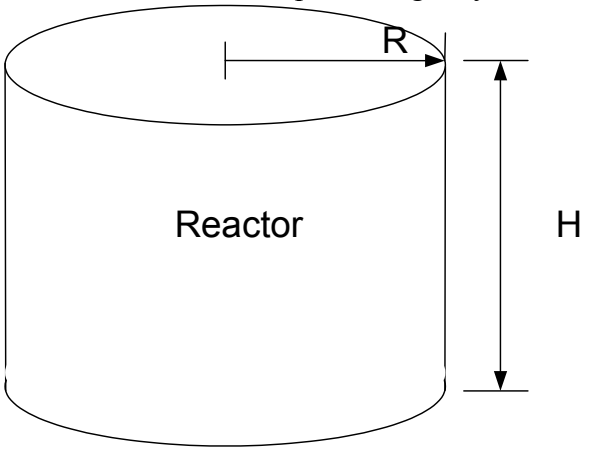

Figure 3 Cylindrical reactor.

The one speed neutron balance equation is:

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\phi}{\partial r}\right) + \frac{\partial^2\phi}{\partial z^2} + B^2\phi = 0 \tag{36}$$

B.C.:

$$\begin{aligned}\phi(\tilde{R},z) &= 0\\ \phi(r,\pm\tilde{H}/2) &= 0\end{aligned} \tag{37}$$

Let:

$$\phi(r,z) = \mathbb{R}(r)\mathbb{Z}(z) \tag{38}$$

Substituting into equation 36:

$$\frac{d}{dr}\left(r\frac{d\mathbb{R})}{dr}\right)+\alpha^2\mathbb{R}=0$$
$$\frac{d^2\mathbb{Z}}{dr^2}+\lambda^2\mathbb{Z}=0 \tag{39}$$

The solution to the radial equation is:

$$\mathbb{R}=AJ_0(\alpha r)+\cancel{C}Y_0(\alpha r) \tag{40}$$

C must be 0 because $Y_0(r)\to\infty$ as $r\to 0$. Since the flux must go to zero at the extrapolated boundary, $\alpha=\frac{2.405}{\tilde{R}}$, the first root of J_0.

The solution to the axial equation is:

$$\mathbb{Z}=\cos(\lambda z) \tag{41}$$

where $\lambda^2=(\pi/H)^2$ since the flux goes to zero at the extrapolated boundaries.
Thus the full solution is:

$$\phi=AJ_0\left(2.405r\cancel{/}\tilde{R}\right)\cos\left(\pi z\cancel{/}\tilde{H}\right) \tag{42}$$

The criticality condition is:

$$\underbrace{\left(\frac{2.405}{R}\right)^2+\left(\frac{\pi}{H}\right)^2}_{B_g^2}=\underbrace{\frac{\nu\Sigma_f-\Sigma_a}{D}}_{B_m^2}$$

7 Reflected Reactor Geometries

Now let's look at a slab reactor with a reflector on each side. As before:

$$\frac{1}{v}\frac{\partial}{\partial t}\phi(\mathbf{r},t) = \nabla\cdot D(\mathbf{r})\nabla\phi(\mathbf{r},t) - \Sigma_a(\mathbf{r})\phi(\mathbf{r},t) + \nu\Sigma_f(\mathbf{r})\phi(\mathbf{r},t) \tag{43}$$

Assuming steady state, homogeneous properties within each region and one dimension:

$$D\frac{\partial^2\phi(x)}{\partial x^2} + (-\Sigma_a + \nu\Sigma_f)\phi(x) = 0 \tag{44}$$

This will apply to both the core region and the reflector region, see figure 4.

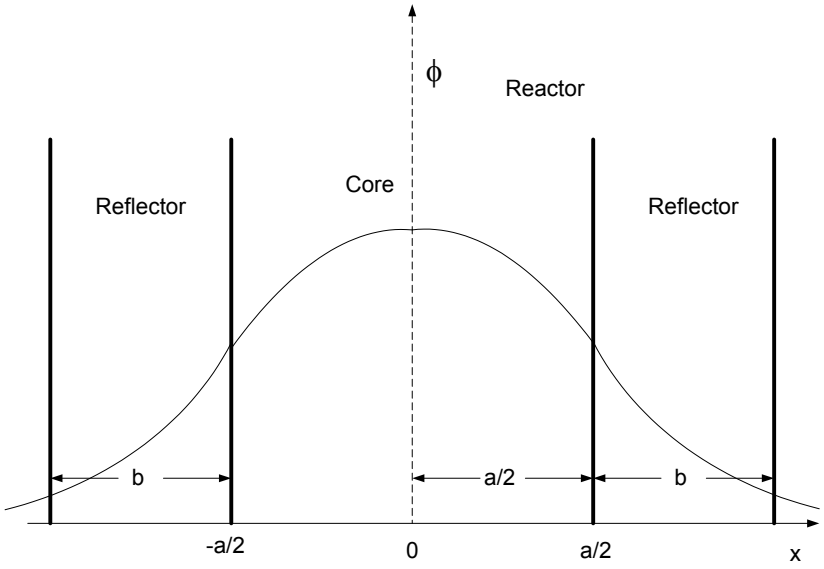

Figure 4 Reflected slab reactor

Governing equations:

$$\begin{aligned}\text{Core: } D^C\frac{d^2\phi^C(x)}{dx^2} + (-\Sigma_a^C + \nu^C\Sigma_f^C)\phi^C(x) = 0\\ \text{Reflector: } D^R\frac{d^2\phi^R(x)}{dx^2} - \Sigma_a^R\phi^R(x) = 0\end{aligned} \tag{45}$$

B.C.:

$$\begin{aligned}&\phi^R\left(a/2 + b\right) = 0\\ &\phi^C\left(a/2\right) = \phi^R\left(a/2\right)\\ &J^C\left(a/2\right) = J^R\left(a/2\right)\\ &\text{symmetry}\end{aligned} \tag{46}$$

We will just solve for x>0 since, by symmetry, the solution is the same for x<0. We try:

$$\phi^C = A^C \cos\left(B_m^C x\right), \text{ where } \left(B_m^C\right)^2 = \frac{\nu\Sigma_f^C - \Sigma_a^C}{D^C} \tag{47}$$

And

$$\begin{aligned}\phi^R &= A^R \sinh\left[\frac{a/2 + b - x}{L^R}\right], \text{ where } \left(L^R\right)^2 = \frac{D^R}{\Sigma_a^R} \\ &= 0 \text{ at } x = a/2 + b \text{ since } \sinh t = \frac{e^{+t} - e^{+t}}{2}\end{aligned} \tag{48}$$

Applying the boundary conditions:

$$\phi \text{ equal at interface} \Rightarrow A^C \cos\left(\frac{B_m^C a}{2}\right) = A^R \sinh\left(\frac{b}{L^R}\right) \tag{49}$$

$$J \text{ equal at interface} \Rightarrow D^C B_m^C A^C \sin\left(\frac{B_m^C a}{2}\right) = \frac{D^R}{L^R} A^R \cosh\left(\frac{b}{L^R}\right) \tag{50}$$

We divide equation 50 by 49 to eliminate the unknown A coefficients to give the **criticality condition**:

$$D^C B_m^C \tan\left(\frac{B_m^C a}{2}\right) = \frac{D^R}{L^R} \coth\left(\frac{b}{L^R}\right) \tag{51}$$

This relates the geometry of the reactor to the material properties of the reactor. To graphically illustrate this, we plot the LHS and the RHS against $\frac{B_m^C a}{2}$ in figure 5. Where the LHS and the RHS are equal, the reactor is critical. This occurs at multiple values of $\frac{B_m^C a}{2}$ but the lowest value (ie smallest core size, a) occurs at just less than $\pi/2$. If you recall, the bare reactor went critical at $B_m = \pi/a$ or, equally, $B_m a/2 = \pi/2$. Thus the core size for a reflected reactor is smaller than a non-reflected reactor, resulted in a fuel "savings".

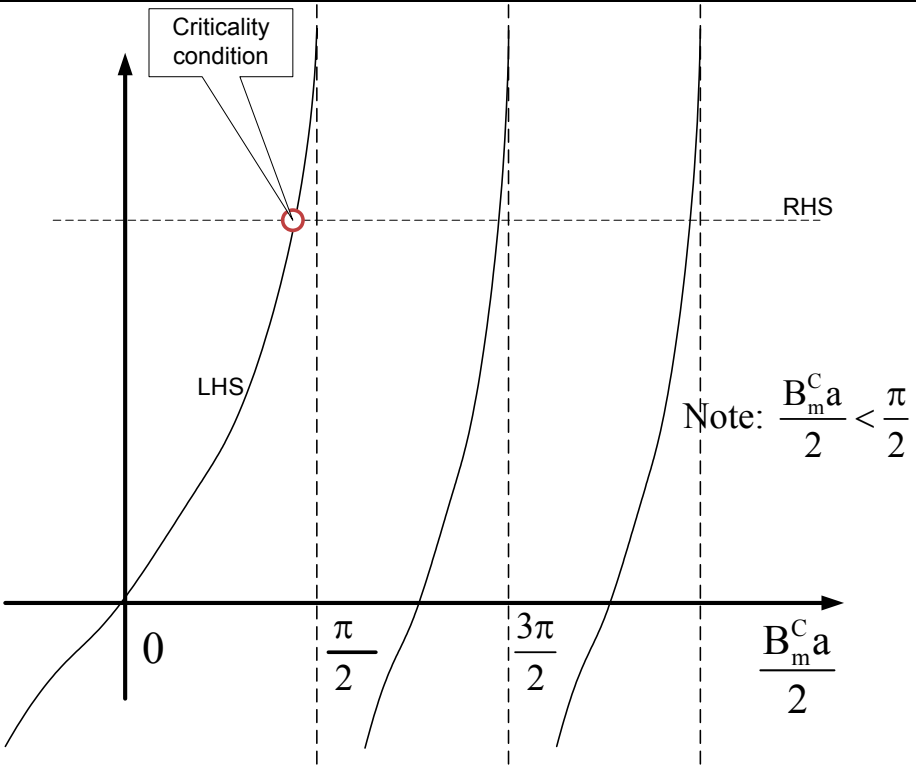

Figure 5 Criticality for a reflected slab reactor.

8 Reactor Criticality Calculations

As you no doubt noticed from the previous analytical solutions, any real reactor model would be too complex to be solved analytically. So we resort to numerical means.

You will recall that we had:

$$\frac{1}{v}\frac{\partial}{\partial t}\phi(\mathbf{r},t) = \nabla\cdot D(\mathbf{r})\nabla\phi(\mathbf{r},t) - \Sigma_a(\mathbf{r})\phi(\mathbf{r},t) + \nu\Sigma_f(\mathbf{r})\phi(\mathbf{r},t) \tag{52}$$

If we postulate that

$$\phi(x,t) = e^{\lambda t}\psi(x) \tag{53}$$

then

$$\frac{\lambda}{v}\psi(\mathbf{r},t) = \nabla\cdot D(\mathbf{r})\nabla\psi(\mathbf{r}) - \Sigma_a(\mathbf{r})\psi(\mathbf{r}) + \nu\Sigma_f(\mathbf{r})\psi(\mathbf{r}) \tag{54}$$

and we get a steady state solution when $\lambda = 0$. Therefore, the basic idea is to adjust the equaton parameters until $\lambda = 0$. But which parameters and how to adjust them?

Generally, you start with a fixed geometry and basic design, ie the materials are known. Thus, the free variable is the fuel, ie reactivity. Therefore, vary ν or, equally, introduce a fudge factor:

$$-\nabla\cdot D(\mathbf{r})\nabla\phi(\mathbf{r}) + \Sigma_a(\mathbf{r})\phi(\mathbf{r}) = \underbrace{\frac{\nu}{k}\Sigma_f(\mathbf{r})\phi(\mathbf{r})}_{\text{source term}}$$

$$\Downarrow \tag{55}$$

$$M\phi = \frac{1}{k}F\phi$$

where M and F are operators. The fudge factor is k. If you rearrange the equation, solving for k as we did in section 3 of this chapter, you will see that k takes on a form like the multiplication factor k that we introduced earlier. The k generated satisfies:

$$k_{fudge} = \frac{\nu\Sigma_f / \Sigma_a}{1 + \underbrace{L^2 B^2}_{\text{from the } \nabla\cdot D\nabla\phi \text{ term}}} \tag{56}$$

The numerical algorithm for a fixed design is as follow:

1. Set up a spatial grid as we did for the fixed source cases in *Reactor Physics: Numerical Methods*.

2. Guess $k^{(0)}$ and $\phi^{(0)}$ for the initial or zeroth iteration.

3. Calculate the source term $\frac{1}{k}F\phi^{(0)}$.

4. Solve as per the previous numerical methods (G-S or SOR, etc) with this known source. This involves an iteration on ϕ to get a converged spatial distribution of ϕ for this fixed source estimate. This generates $\phi^1(r)$. This is the **inner loop**. Note that this flux distribution is based on a guess of the source term which is a function of ϕ. Mathematically:

$$\begin{aligned} M\phi^{(1)} &= \frac{1}{k^{(0)}} F\phi^{(0)} = \frac{S^{(0)}}{k^{(0)}} \Rightarrow \phi^{(1)} \\ S^{(1)} &= F\phi^{(1)} \end{aligned} \tag{57}$$

5. Now we need to update the source and the value of the fudge factor k. The source is easy – just recalculate $F\phi$ as per equation 55 or 57 using the latest flux values. But what about k? We'll see in a minute that $k^1 = k^0 S^1 / S^0$.
6. With an updated source term, we now re-calculate the flux as above. This is the outer loop. The algorithm can be summarized:

$$\begin{aligned} &\text{Guess } \phi^{(0)} \text{and } k^{(0)} \\ &\text{[outer loop] on i} \\ &M\phi^{(i+1)} = \frac{1}{k^{(i)}} F\phi^{(i)} = \frac{S^{(i)}}{k^{(i)}} \Rightarrow \text{inner loop } \Rightarrow \phi^{(i+1)} \\ &\text{update } S^{(i+1)} = F\phi^{(i+1)} \\ &\text{update } k^{(i+1)} = k^{(i)} S^{(i+1)} / S^{(i)} \\ &\text{[outer loop] until flux converges} \end{aligned} \tag{58}$$

If we are clever, we can combine the inner and outer iterations by updating k and S every time we make and inner loop sweep. There is hardly any sense in converging the inner loop to a fine tolerance only to make big changes in the fudge factor and source term and then re-compute the flux again to a fine tolerance. It is far better to update k and S as you go.

Now, on to how to update k. We note that:

$$k \equiv \frac{\text{fission sources}}{\text{sinks}} = \frac{F\phi}{M\phi} \tag{59}$$

Thus

$$k^{(i+1)} \equiv \frac{\int_{\text{volume}} F\phi^{(i+1)} dV}{\int_{\text{volume}} M\phi^{(i+1)} dV} \tag{60}$$

But in our procedure:

$$M\phi^{(i+1)} = \frac{1}{k^{(i)}} F\phi^{(i)} = \frac{S^{(i)}}{k^{(i)}} \tag{61}$$

Therefore:

$$k^{(i+1)} \equiv \frac{\int_{volume} S^{(i+1)} dV}{\frac{1}{k^{(i)}} \int_{volume} S^{(i)} dV} \tag{62}$$

Q.E.D.

We can improve on this by looking ahead based on the past trend (extrapolation).

What does adjusting the k mean physically? Let's say that the converged value of k that you get for a particular case is 1.1. That means that the source term, $\nu \Sigma_f(\mathbf{r})\phi(\mathbf{r})$, was too big, causing the reactor to be supercritical. To make the reactor critical, it had to be reduced by 10%. Then, and only then, did the source and sink terms balance. Obviously, we could have make the reactor critical in a number of other ways, such as adjusting the absorption term or changing the size of the reactor. But since we typically start with a physical reactor of given (or proposed) dimensions and materials, it is convenient to control the criticality in the way we did. If you run into a case where it makes more sense to adjust another term, by all means, go for it. The overall procedure is similar and should work just as well. Just make sure that you are not attempting to find criticality by adjusting a term that is of little influence on the equation set. For instance, a control rod (absorber) at the very edge of a reactor sees few neutrons so it would be ineffective in affecting overall criticality. Use your common sense.

9 Questions

1. It would be folly to expect that the simple one speed model would really be all that accurate. What are the sources of errors?

2. How sensitive is the critical size to errors in cross sections?

3. How would you lock the model onto reality? What commissioning tests can you think of? How would you perform them?

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