Chain Reactions

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Summary:
In the chapter Basic Definitions and Perspectives*, we spoke of fission and the chain reaction that is fundamental to the study of reactor physics. But we spoke of it from a phenomenological point of view. Here we introduce the exponential nature of reactors and take a more quantitative approach. This helps to set the scene for reactor modelling*

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1 Introduction

1.1 Overview

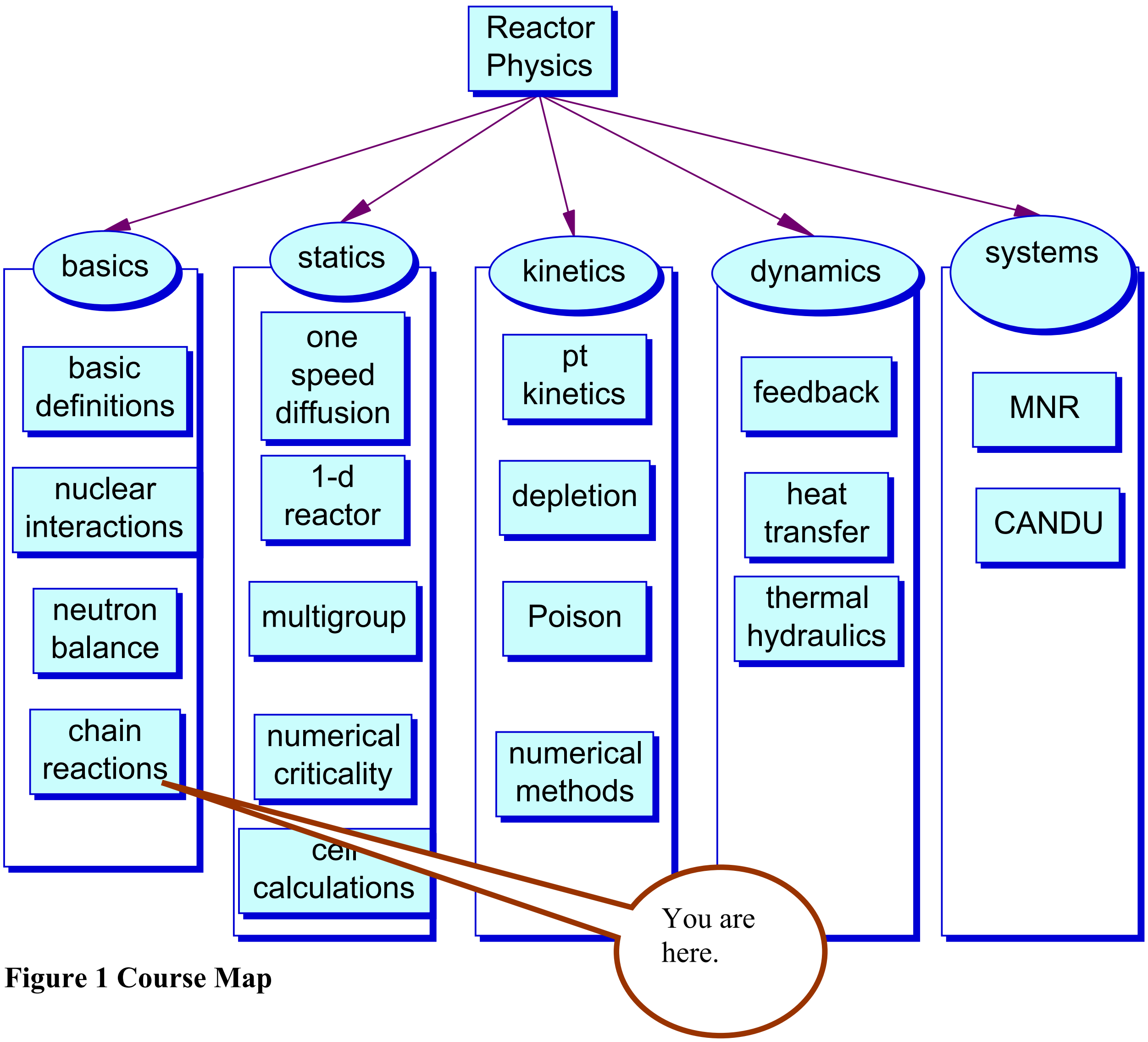

Figure 1 Course Map

1.2 Learning Outcomes

The goal of this chapter is for the student to understand:

- The exponential nature of chain reactions
- The key variables of the chain reaction process
- An approximate value for the key variables
- How to control the process, qualitatively

2 The Multiplication Factor, k

The multiplication factor, k, is defined as:

$$
\begin{aligned}
&k \equiv \text{muliplication factor} = \frac{\text{\# of neutrons in generation n}}{\text{\# of neutrons in generation n-1}} \\
&k < 1 \quad \text{subcritical} \\
&k = 1 \quad \text{critical} \\
&k > 1 \quad \text{supercritical}
\end{aligned} \tag{1}
$$

The primary objective of nuclear operations is to safely operate at constant power for the most part. This implies that we want to keep k =1. This implies the need for control of the nuclear fission reaction. It is unfortunate that the name 'critical' is associated with steady state (k=1). When a reactor is critical, it is a status quo situation. It does not imply that there is a crisis.

At any rate, there is an alternate definition of k:

$$
k \equiv \frac{\text{rate of neutron production}}{\text{rate of neutron loss}} \equiv \frac{P(t)}{L(t)} \tag{2}
$$

We define the neutron lifetime, ℓ:

$$
\ell \equiv \frac{\text{total neutron population}}{\text{rate of loss}} \equiv \frac{n(t)}{L(t)} \tag{3}
$$

Performing a simple balance of production and loss, we derive a simple neutron balance equation

$$
\begin{aligned}
\frac{dn}{dt} &= P(t) - L(t) \\
&= (k-1)L(t) \\
&= \frac{(k-1)}{\ell} n(t)
\end{aligned} \tag{4}
$$

This is easy to solve:

$$
n(t) = n_0 e^{\left(\frac{k-1}{\ell}\right)t} = n_0 e^{t/T}, \text{ where } T = \frac{\ell}{k-1} \equiv \text{period} \tag{5}
$$

If the period were about 0.10 seconds, then the amplification factor is

$$
e^{1/0.1} = 22{,}000 \leftarrow \text{this is the growth in 1 second!} \tag{6}
$$

Obviously, we have to ensure that k~1 to keep the period long, ie, to keep things happening slowly. We shall see in the chapter on *Point Kinetics* that even a k of 1.007 can lead to an undesirably huge rate of increase in neutron, and hence power, level.

3 The Four Factor Formula and the Neutron Life Cycle

The multiplication factor, k, is an overall measure of reactor kinetics. Because neutrons migrate around the reactor core readily, whether the overall population of neutrons is increasing or decreasing is a systems issue in the same manner that human population is a systems issue. However, just like in human dynamics, the overall situation is a result of local effects. Prior to ready access to powerful computers like we have today, analytical solutions to the neutron balance equation were sought on the basis of simplifications that were rooted in the key processes that were taken place. Thus the essence is captured, if not the detailed accuracy that large code-based simulations offer.

We define the **four factor formula**:

$$k_\infty = ,\eta f p \tag{7}$$

- the fast fission factor

$$\begin{aligned} , &= \frac{\text{total \# of fission neutrons (from fast and thermal neutrons)}}{\text{\# from thermal neutrons}} \\ &= \text{fast fission factor} \\ &\sim 1.03 \end{aligned} \tag{8}$$

-η

$$\begin{aligned} \eta &= \frac{\text{average \# of neutrons produced}}{\text{\# of neutrons absorbed in fuel}} \\ &= \frac{\nu\sigma_f}{\sigma_a} \quad \left(\text{or } \frac{\sum\limits_j \nu_j \Sigma_f^{\ j}}{\sum\limits_j \Sigma_a^{\ j}} \text{ for a mixture} \right) \\ &\sim 2.0 \rightarrow 2.5 \text{ (see figure xxx)} \end{aligned} \tag{9}$$

- the thermal utilization

$$\begin{aligned} f &= \text{thermal utilization} = \frac{\Sigma_a^{fuel}}{\Sigma_a^{fuel} + \Sigma_a^{mod}} \\ &\simeq 0.7 \end{aligned} \tag{10}$$

or more correctly,

$$f = \frac{\text{volume}_{fuel}\, \Sigma_a^{fuel}\, \phi_{fuel}}{\sum\limits_i \text{volume}_i\, \Sigma_i\, \phi_i} = \frac{1}{1 + \dfrac{\text{volume}_{mod}\, \Sigma_{mod}\, \phi_{mod}}{\text{volume}_{fuel}\, \Sigma_a^{fuel}\, \phi_{fuel}}} \tag{11}$$

where mod = moderator and other non-fuel material. In addition,

$$\frac{\phi_{mod}}{\phi_{fuel}} \equiv \text{thermal disadvantage factor} \tag{12}$$

- the resonance escape probability

p = resonance escape probability ~ 0.9.

Putting that all together:

$$k = \overbrace{\frac{\text{total \#fissions}}{\text{\# thermal fissions}}}^{\varepsilon} \text{ x } \overbrace{\frac{\text{\# thermal neutrons produced}}{\text{\#\# absorbed in fuel}}}^{\eta} \text{ x } \overbrace{\frac{\text{\# absorbed in fuel}}{\text{total thermal absorption}}}^{f} \text{ x } \overbrace{\frac{\text{thermal absorption}}{\text{total absorption}}}^{p}$$

$$= \frac{\text{total fission}}{\text{total absorption}}.$$

Figure 2 shows the four factors in relation to the neutron life cycle.

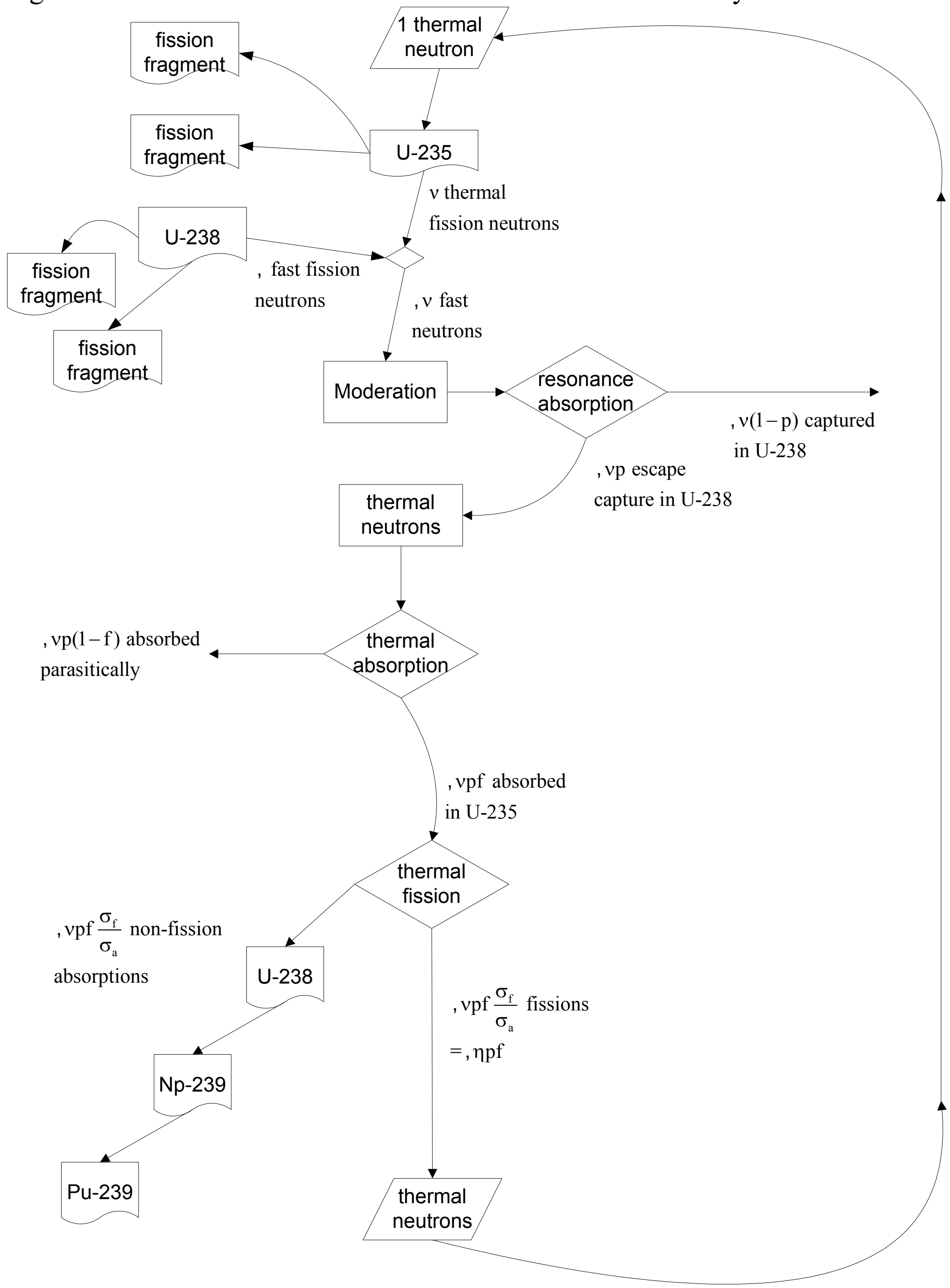

Figure 2 Four factor formula in relation to the neutron life cycle.

4 Conversion and Breeding

Natural uranium is 99.3% U-238. Typical enriched fuel used in power reactors is still usually over 95% U-238. Most reactors these days use low enriched fuel or LEU which by international agreement is held at < 20% U-235. So most fuel has a lot of U-238 in it. U-238 has absorption resonances in the mid energy range so it is a neutron parasite. However, all is not lost; U-238 has a finite probability of being converted to a fissile fuel when it absorbs a neutron. Hence it is called *fertile* and this process is called conversion. If more fuel results from the conversion process than is used in the process, then the process is called *breeding*. Most reactors convert to some degree. Breeding is possible and some demonstration reactors have breed. Obviously we need to have η>2 for breeding to be possible.

The U chain is as follows:

$$\begin{gathered}\underset{\text{fertile}}{U_{92}^{238}}(n,\gamma)U_{92}^{239}\xrightarrow[23\text{m}]{\beta^-}\underset{\text{Neptunium}}{Np_{93}^{239}}\xrightarrow[2.3\text{d}]{\beta^-}\underset{\text{fissile}}{Pu_{94}^{239}}\\ Pu_{94}^{239}+n\begin{array}{l}\nearrow \text{fission}\\ \searrow \underset{\text{Neptunium}}{Pu_{94}^{240}}\,(n,\gamma)\,\underset{\text{fissile}}{Pu_{94}^{241}}\end{array}\end{gathered}\tag{13}$$

The thorium chain is as follows:

$$\underset{\text{fertile}}{Th_{90}^{232}}(n,\gamma)Th_{90}^{233}\xrightarrow[2.2\text{m}]{\beta^-}\underset{\text{Protactinium}}{Pa_{91}^{233}}\xrightarrow[27.4\text{d}]{\beta^-}\underset{\text{fissile}}{U_{92}^{233}}\tag{14}$$

Fissile fuel are ones in which fission can be induced by neutrons of zero energy (U-233, U-235, Pu-239, Pu-241).

Fissionable fuels are ones in which fission can be induced only by neutrons of ~MeV energy (Th-232, U-238, Pu-240, Pu-242).

Fertile fuels are ones that can be transmuted into fissile fuels (Th-232, U-238).

In CANDU reactors, roughly 50% of the power comes from Pu even though there is no Pu in the fuel initially. Hence, in CANDU reactors, conversion of U-238 to Pu is a significant process.

5 Typical Spectra

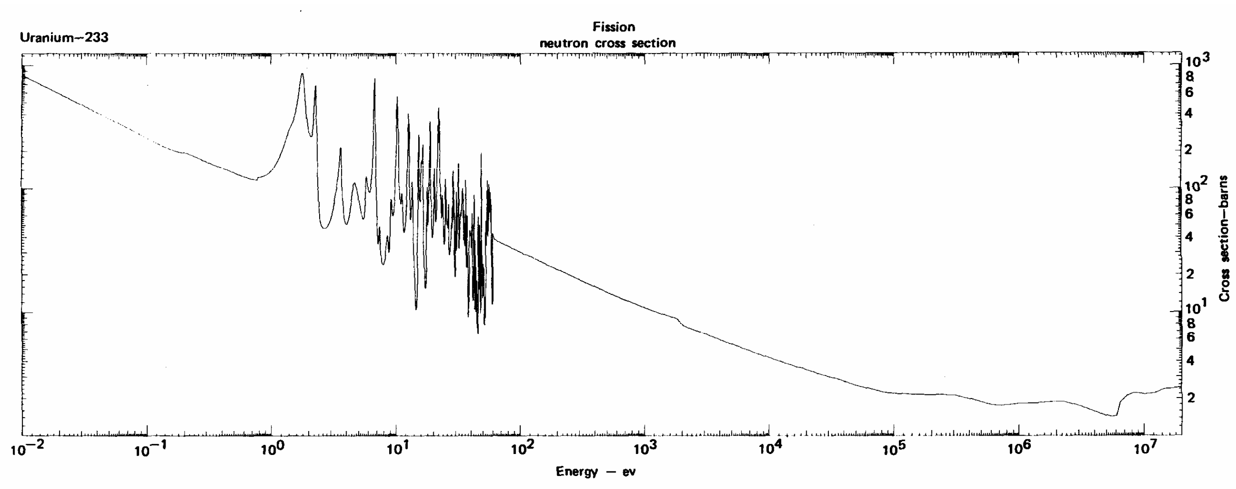

Figure 3 Fission cross section of U-233 [source: DUD1976, figure 2-17]

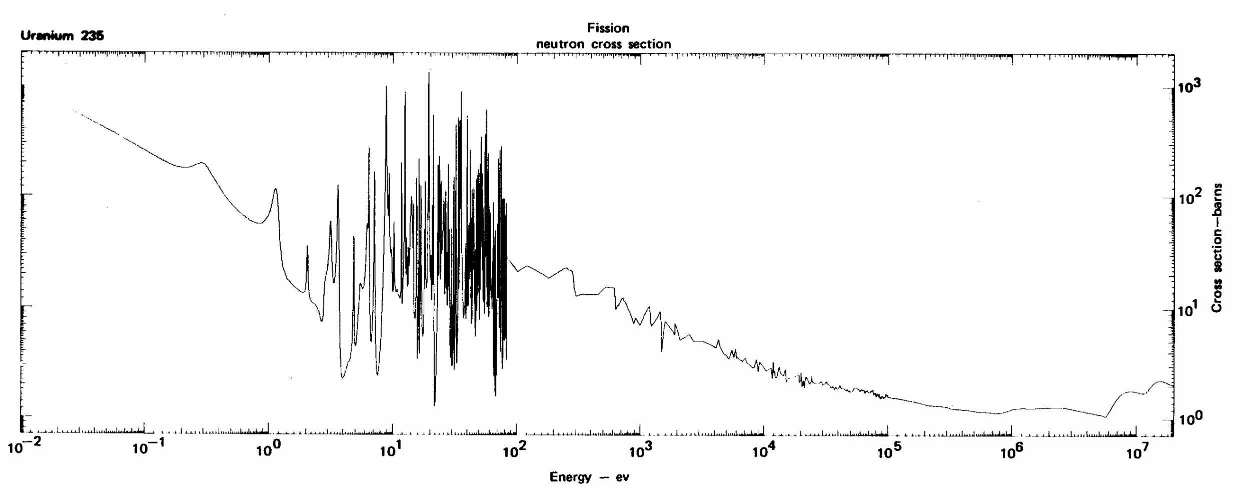

Figure 4 Fission cross section of U-235 [source: DUD1976, figure 2-17]

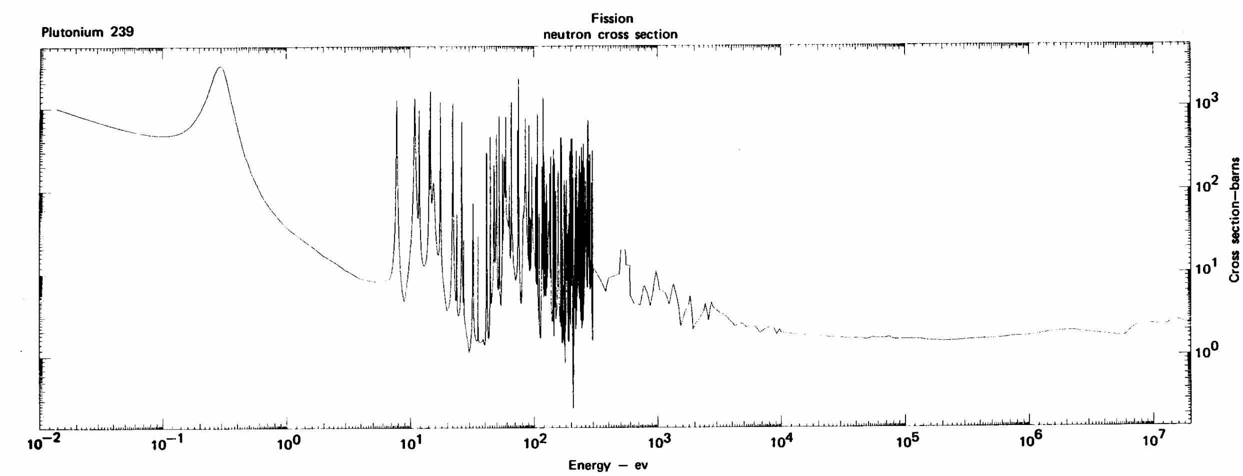

Figure 5 Fission cross section of P-239 [source: DUD1976, figure 2-17]

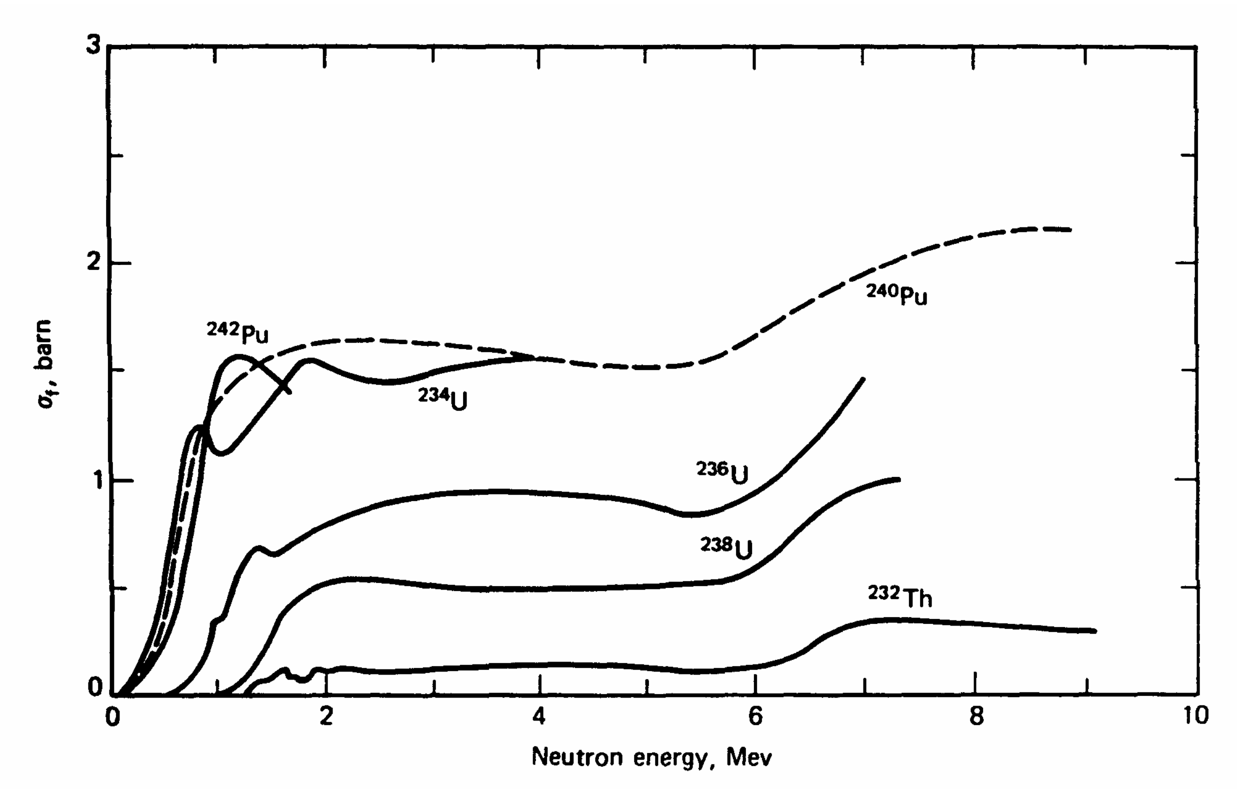

Figure 6 Fission cross sections of principal fissionable isotopes [source: DUD1976, figure 2-18]

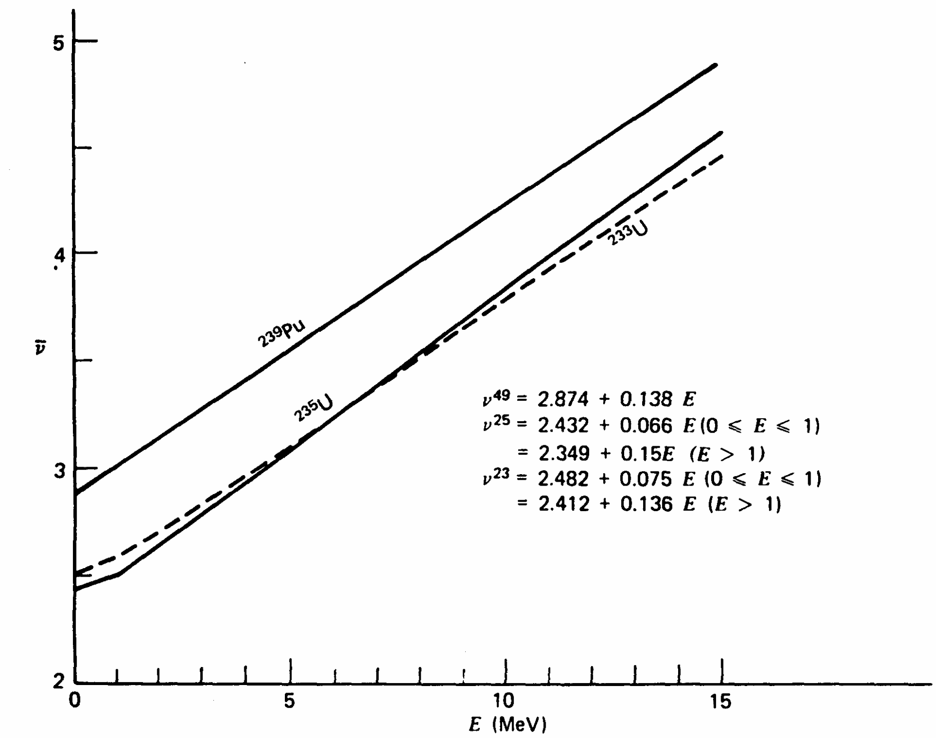

Figure 7 Average neutron number per fission, ν, as a function of energy [source: DUD1976, figure 2-20]

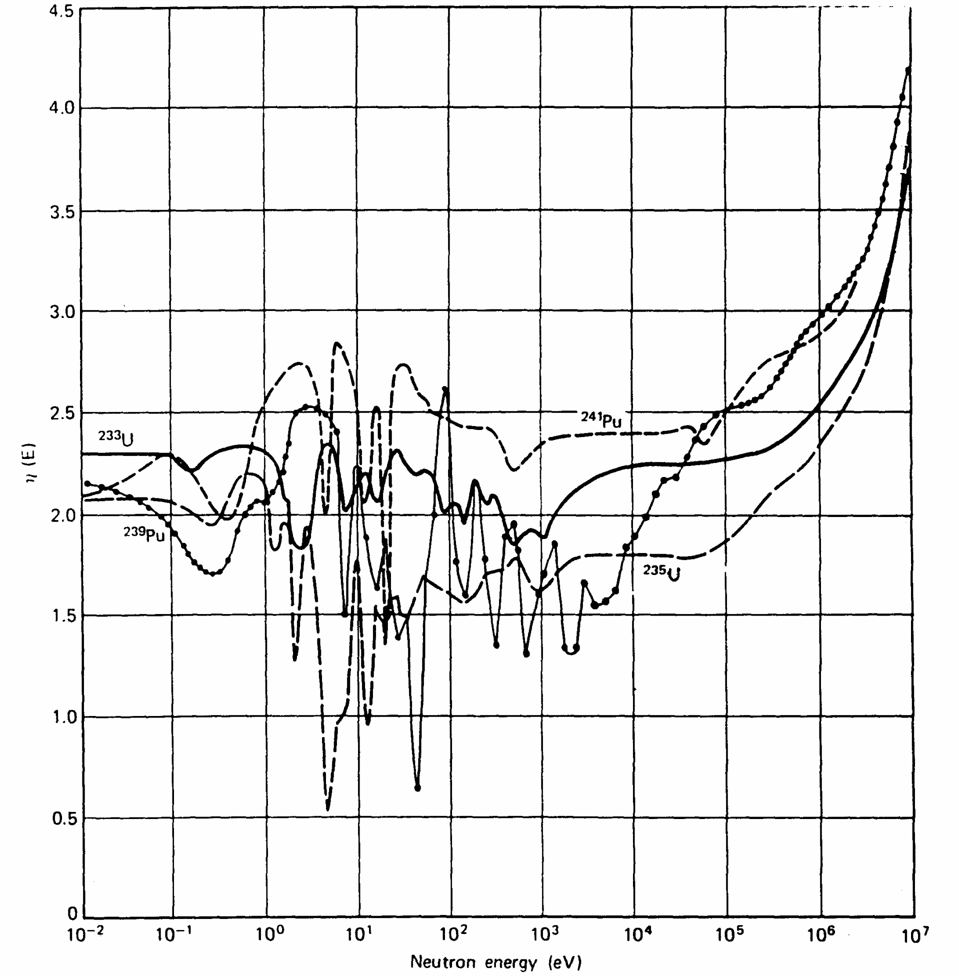

Figure 8 Variation of η with energy for U-233, U-235, Pu-239 and Pu-241 [source: DUD1976, figure 2-25]

6 Control of the Chain Reaction

Reactor control is generally divided into two classes: kinetics and dynamics.

6.1 Reactor Kinetics

Reactor kinetics refers to the manipulation of parameters that affect k and to the subsequent direct response of the reactor system. Examples are:

- Absorber rods or shim movements to compensate for fuel burnup.
- Safety scram rods to rapidly shutdown the chain reaction.
- Control rods to provide real-time control to keep k = 1 or to manoeuvre up and down in power.

6.2 Reactor Dynamics

Reactor dynamics refers to the more indirect feedback mechanisms due to power level effects and other overall systems effects such as:

- Temperature feedback.
- Void feedback.
- Pump speed control (affects water density and temperature).

6.3 Core design

As you might expect, the transient response of the reactor to the above direct and indirect changes in basic parameters is highly dependent on the design details of the reactor. Sample issues are:

- Where should the control rods be placed for maximum effectiveness?
- Will the power go up or down if a void is introduced into the reactor?
- Will the power go up or down if core temperature goes up?
- How often should the reactor be refueled?
- and so on...

To be able to answer these and the many other design, analysis and operational questions, we need to model the reactor with some precision. Hand waving will not do. And so, with an understanding of the basic phenomena and of modeling techniques, we are ready to jump into the deep end.

About this document:

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