Reactor Physics: The General Diffusion Equations Revisited

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Summary:
The general multigroup neutron diffusion equations, precursor equations, poison equations and fuel depletion equations are summarized. In order to emphasize the dynamic nature of the neutron flux and related variables, the numerical solution to these general space-time equations is revisited by first focusing on the transient portion and then adding the spatial diffusion.

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1 Introduction

1.1 Overview

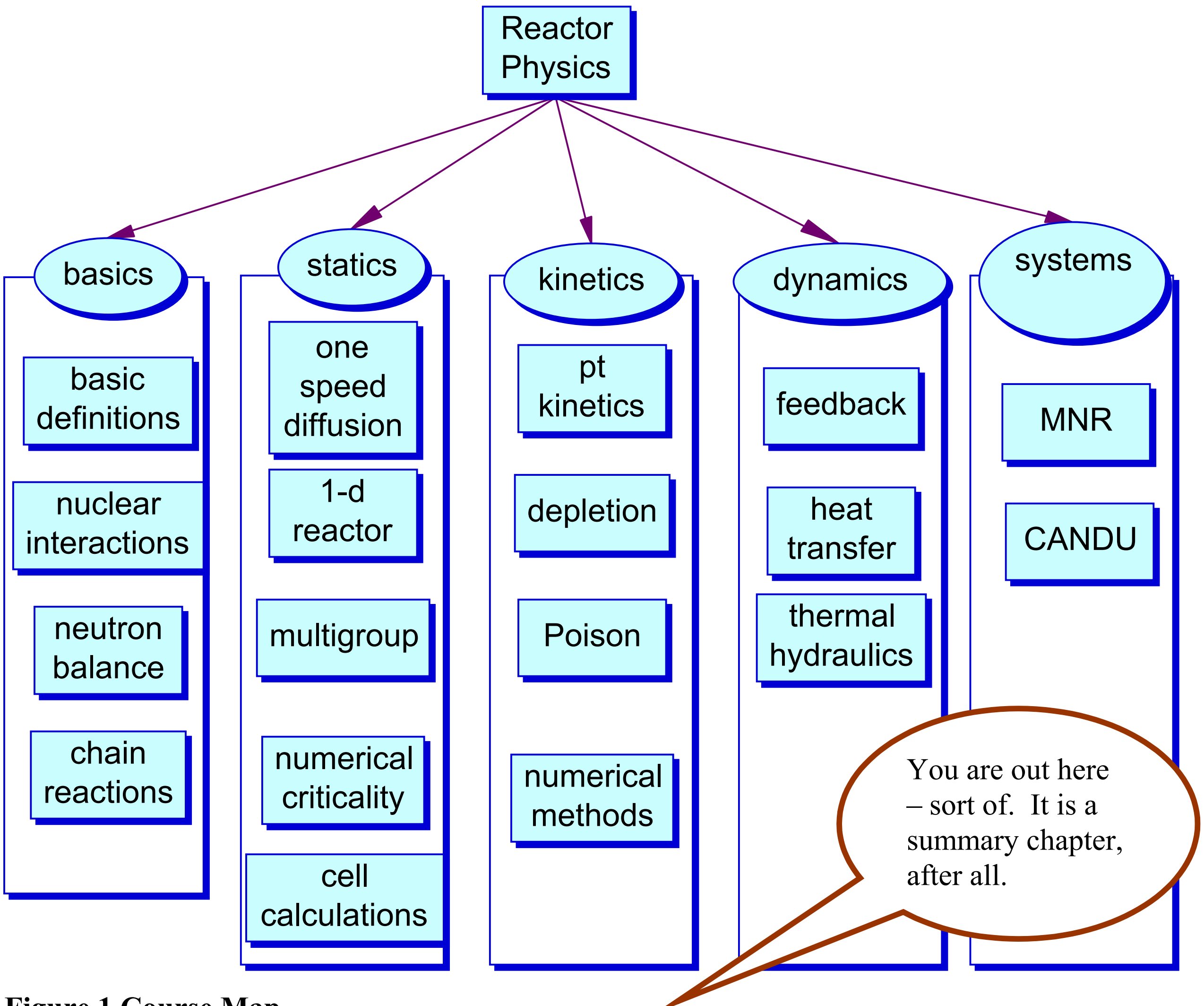

Figure 1 Course Map.

1.2 Learning Outcomes

The goal of this chapter is for the student to understand:

- How the general equations can be interpreted as a combination of amplitude evolution driven by a balance of processes distributed over energy and space.

2 Summary of the General Diffusion Based Equations

We had the general multigroup neutron diffusion equations:

$$\frac{1}{v_g}\frac{\partial}{\partial t}\phi_g(\underline{r},t) = \underbrace{\nabla\cdot D_g(\underline{r})\nabla\phi_g(\underline{r},t)}_{\text{leakage}} - \underbrace{\Sigma_{ag}(\underline{r})\phi_g(\underline{r},t)}_{\text{loss by absorption}} - \underbrace{\Sigma_{sg}(r)\phi_g(\underline{r},t)}_{\text{removal by scattering}} + \underbrace{\sum_{g'=1}^{G}\Sigma_{sg'g}(r)\phi_{g'}(\underline{r},t)}_{\text{scattering into group g}}$$

$$+ \underbrace{\chi_g^P}_{\substack{\text{fraction}\\\text{appearing}\\\text{in group g}}} \underbrace{(1-\beta)}_{\substack{\text{prompt}\\\text{fraction}}} \underbrace{\sum_{g'=1}^{G}\nu_{g'}\Sigma_{fg'}(\underline{r})\phi_{g'}(\underline{r},t)}_{\text{total fission production}} + \underbrace{\chi_g^C}_{\substack{\text{fraction}\\\text{appearing}\\\text{in group g}}} \underbrace{\sum_{i=1}^{N}\lambda_i C_i(\underline{r},t)}_{\substack{\text{total}\\\text{delayed}\\\text{source}}} + \underbrace{S_g^{ext}}_{\substack{\text{external}\\\text{source}}} \tag{2.1}$$

$$\frac{\partial}{\partial t}C_i(\underline{r},t) = -\lambda_i C_i(\underline{r},t) + \underbrace{\beta_i}_{\substack{\text{i}^{\text{th}}\text{ part}\\\text{of delayed}\\\text{neutrons,}\\\text{split by}\\\text{decay}\\\text{constants}}} \underbrace{\sum_{g'=1}^{G}\nu_{g'}\Sigma_{fg'}(\underline{r})\phi_{g'}(\underline{r},t)}_{\text{total number of fission neutrons}} \tag{2.2}$$

$$\frac{\partial I(\mathbf{r},t)}{\partial t} = \underbrace{\gamma_I\sum_{g'=1}^{G}\nu_{g'}\Sigma_{fg'}(\underline{r})\phi_{g'}(\underline{r},t)}_{\text{fission production}} - \underbrace{\lambda_I I(\mathbf{r},t)}_{\text{decay}}$$

$$\frac{\partial X(\mathbf{r},t)}{\partial t} = \underbrace{\gamma_X\sum_{g'=1}^{G}\nu_{g'}\Sigma_{fg'}(\underline{r})\phi_{g'}(\underline{r},t)}_{\text{fission production}} + \underbrace{\lambda_I I(\mathbf{r},t)}_{\text{decay source}} - \underbrace{\lambda_X X(\mathbf{r},t)}_{\text{decay}} - \underbrace{\sum_{g'=1}^{G}\sigma_{ag'}^{X}(\underline{r})\phi_{g'}(\underline{r},t)X(\mathbf{r},t)}_{\text{absorption loss}} \tag{2.3}$$

$$\frac{dN_A}{dt} = \underbrace{-\lambda_A N_A}_{\text{decay}} - \underbrace{\left[\sum_{g'=1}^{G}\sigma_{ag'}^{A}(\underline{r})\phi_{g'}(\underline{r},t)\right]N_A}_{\text{neutron capture}} + \underbrace{\lambda_B N_B}_{\text{parent decay}} + \underbrace{\left[\sum_{g'=1}^{G}\sigma_{\gamma g'}^{C}(\underline{r})\phi_{g'}(\underline{r},t)\right]N_C}_{\text{capture transmutation}} + \underbrace{F(t)}_{\text{fuel loading}} \tag{2.4}$$

3 Transient Modelling

We revisit the above general equations in three stages. First we consider an one speed infinite reactor with delayed precursors that we can focus on the transient behaviour. Then we add the energy effects via the multigroup equations so that we can see the dynamics of the group interchange. Finally, we add the spatial diffusion to see how the local amplitude changes in ϕ lead to a net spatial migration of the neutrons.

3.1 The Infinite One Speed Reactor

Dropping the spatial and energy dependencies of equation 2.1, we have an infinite, one speed case:

$$\frac{1}{v}\frac{\partial\phi}{\partial t} = -\Sigma_a\phi + (1-\beta)\nu\Sigma_f\phi + \sum_{i=1}^{N}\lambda_i C_i + S^{ext} \tag{3.1}$$

$$\frac{\partial}{\partial t}C_i = -\lambda_i C_i + \beta_i \nu \Sigma_f \phi \tag{3.2}$$

To bring the reactor to criticality (ie, steady state) we could apply a 1/k fudge factor to the fission source term to simulate a variable fuel loading. Alternatively, and more realistically, the absorption cross section can be adjusted as needed, simulating a control rod.

In finite difference form:

$$\frac{1}{v}\frac{\left(\phi^{t+\Delta t} - \phi^{t}\right)}{\Delta t} = -\Sigma_a\phi + (1-\beta)\nu\Sigma_f\phi + \sum_{i=1}^{N}\lambda_i C_i + S^{ext} \tag{3.3}$$

$$\frac{C_i^{t+\Delta t} - C_i^{t}}{\Delta t} = -\lambda_i C_i + \beta_i \nu \Sigma_f \phi \tag{3.4}$$

We can numerically solve for $\phi_p^{t+\Delta t}$ and $C_i^{t+\Delta t}$ in the usual manner. We can play the usual explicit / implicit tricks as desired.

So the flux and precursor transients can be calculated at will, given the initial flux and precursor levels, and give cross sections and other parameters as a function of time over the simulation (simulating rod withdrawal or whatever).

3.2 The Infinite Multigroup Reactor

To the above simulation, we add the energy effects:

$$\frac{1}{v_g}\frac{\partial}{\partial t}\phi_g(t) = -\Sigma_{ag}\phi_g(t) - \Sigma_{sg}\phi_g(t) + \sum_{g'=1}^{G}\Sigma_{sg'g}\phi_{g'}(t) + \chi_g^P(1-\beta)\sum_{g'=1}^{G}\nu_{g'}\Sigma_{fg'}\phi_{g'}(t) + \chi_g^C\sum_{i=1}^{N}\lambda_i C_i(t) + S_g^{ext} \quad (3.5)$$

$$\frac{\partial}{\partial t}C_i(t) = -\lambda_i C_i(t) + \beta_i\sum_{g'=1}^{G}\nu_{g'}\Sigma_{fg'}\phi_{g'}(t) \quad (3.6)$$

Now the flux levels are changing over time as before but there is now a separate (but interconnected) dynamic for each energy group. The fissions caused by thermal neutrons are producing fast neutrons which, by the moderation process, in turn feeds the thermal flux equation with source neutrons. Recall the flow in energy as discussed early in an earlier chapter, shown here as figure 2.

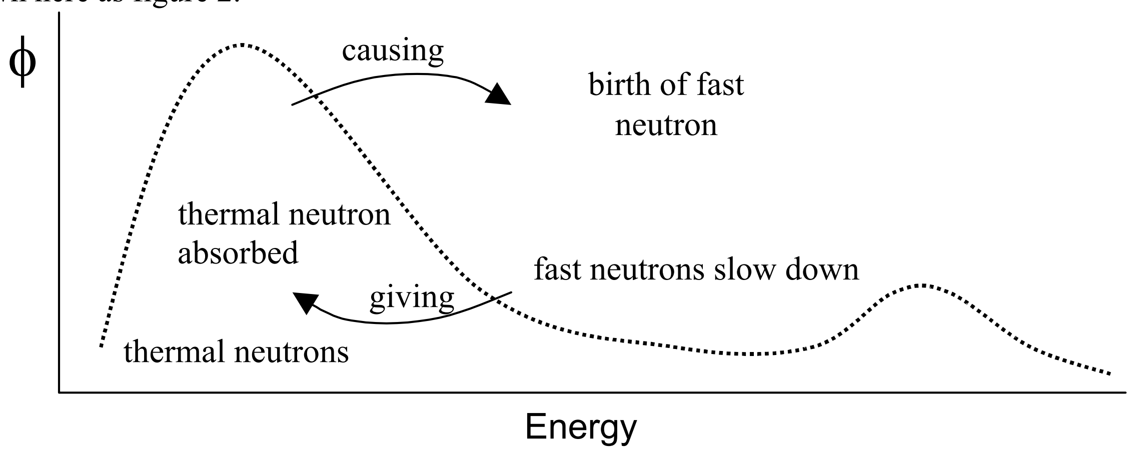

Figure 2 Thermal-fast exchange.

3.3 The Infinite Multigroup Reactor with Spatial Diffusion

So far we have considered an infinite, homogeneous reactor. The flux amplitude could change over time depending on control actions, etc. But all points in space behaved in an identical manner since there was nothing to distinguish one point from another. But what if the space was not homogeneous? Then different points in space could have different flux amplitudes and, consequently, different precursor levels, different poison levels, different fuel depletion levels, and so on. We are back to the full blown equations as presented in section 2. The different flux levels at different points in space leads to a net migration of neutrons from high flux level to lower flux levels and we have see how we can model this process by diffusion. So from this perspective, neutron diffusion is a consequence of amplitude changes that occur over time for the various energy dependent flux groups. Figure 3 tries to summarize these notions.

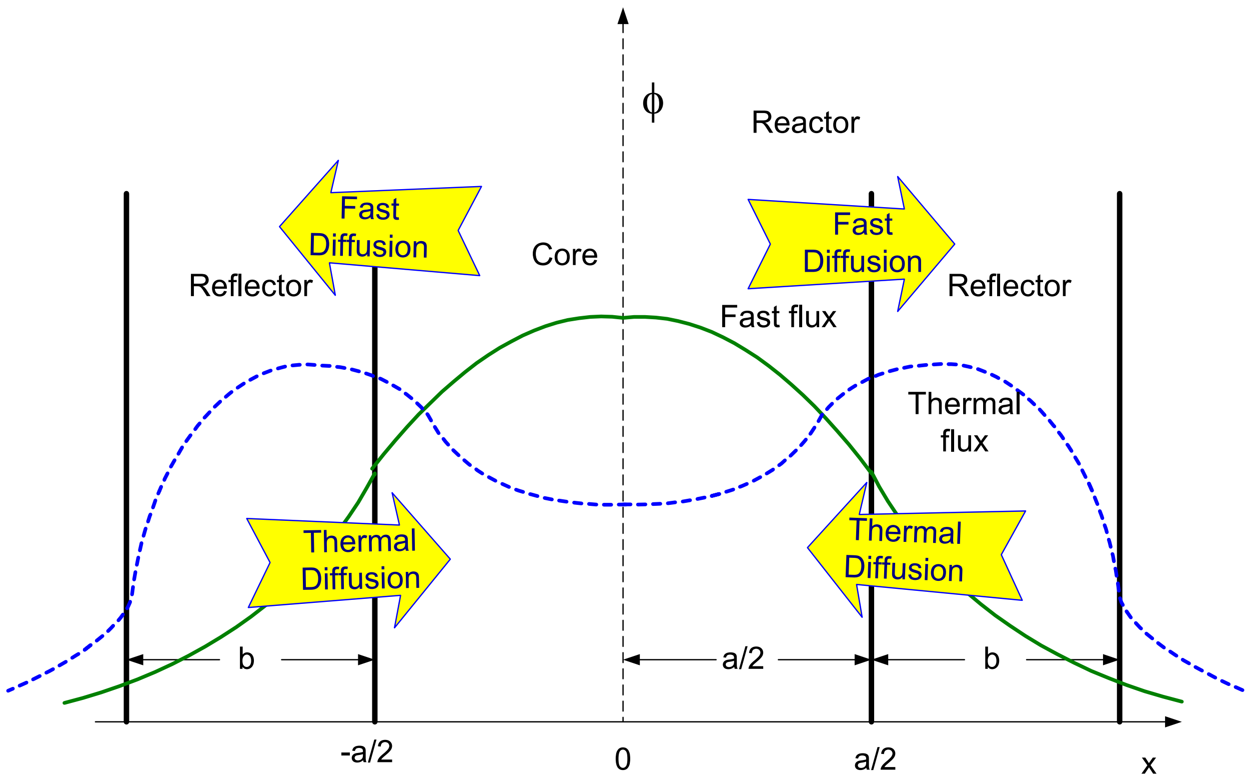

Figure 3 Diffusion occurs as a result to time and energy effects.

Numerical modeling of this complete set is no different, in principle, from the simpler cases covered already. The system is discretized in space, time and energy and solved by:

- for each time step:
    - for each energy group
        - for each point in space
            - update the flux
            - update the precursors
            - update the poisons
            - update the fuel depletion
    - update cross sections, control rod positions, etc.
- terminate on convergence or end time.

Of course, you can always take advantage of the time scale fudging technique (see Process Modelling) to avoid unnecessary calculations for fast changing variables that you are not interested in. For variables that are changing very slowly, you can choose to update them only periodically.

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