Fuel - Coolant Heat Transfer

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Summary:
The temperature distribution in the fuel and the heat transfer between the fuel and the coolant is examined, leading to the development of the axial coolant temperature. The governing equations are developed and solved in the steady state for pin geometry, although extension to other geometries is straightforward. Critical Heat Flux and Critical Power Ratio are introduced conceptually but not investigated in any detail. The importance of heat transfer limitations to power output is emphasized.

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1 Introduction

1.1 Overview

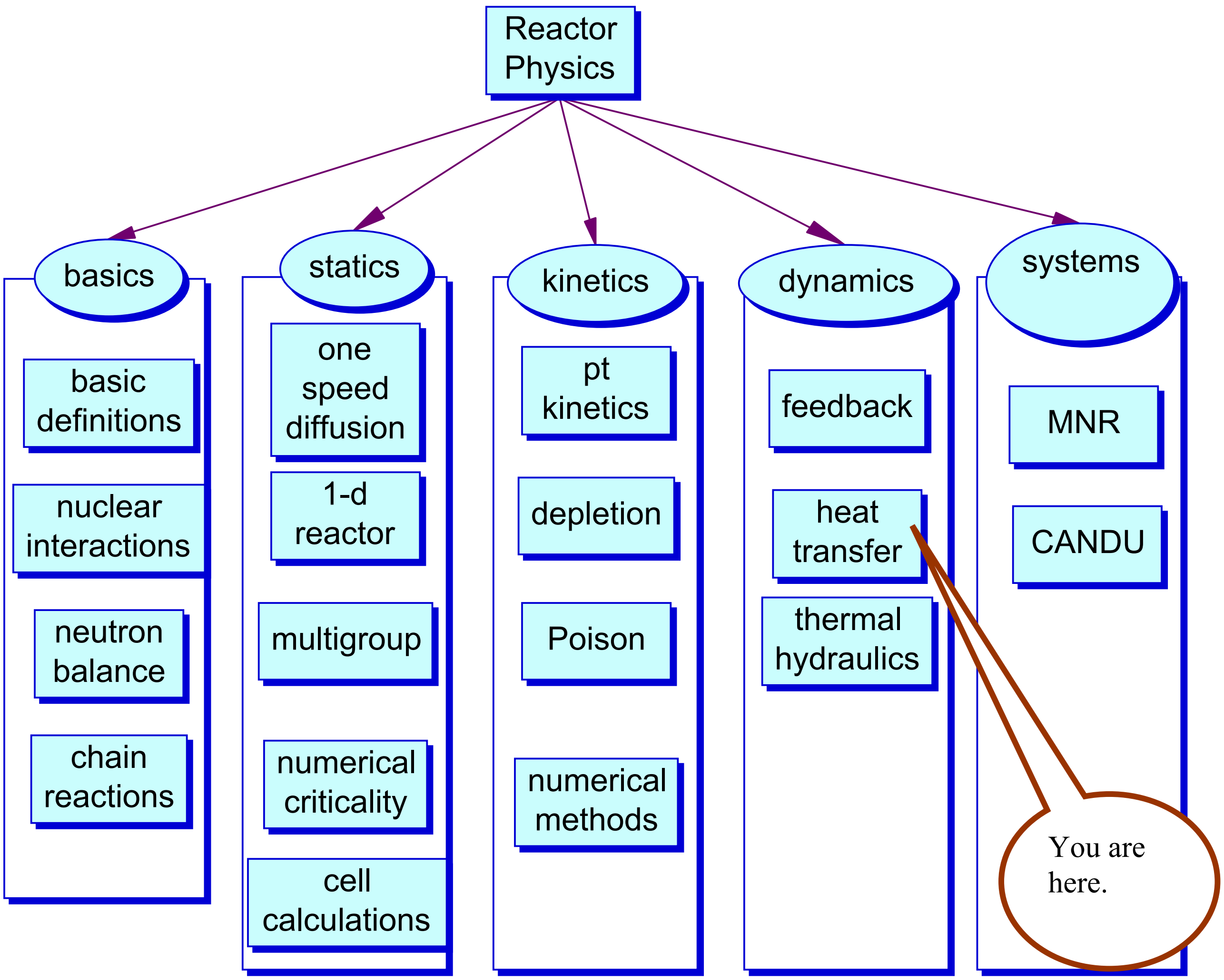


Figure 1 Course Map.

The interface between the fuel and the coolant is centrally important to reactor design since it is here that the limit to power output occurs. Nuclear fission can provide a virtually unlimited heat generation rate, far more than can be transported away by the coolant. Herein we investigate the heat transfer at the fuel site so that this limitation can be factored into the reactor design. The key concepts covered are (see figure 5.1):

- heat is generated in the fuel at a rate proportional to reactor power
- heat is conducted to the coolant as the coolant flows along the fuel
- in the steady state, the fuel temperature is just sufficiently greater than the coolant temperature to transfer the heat generated - ie, fuel temperature "floats" on coolant temperature

- too much power will cause the fuel temperature to be too high and a heat transfer crisis will occur
- heat transfer is a key limiting factor to power output.

Figure 2 shows a typical fuel pin and the key equations for the temperature profiles. In this chapter we will investigate the underlying heat transfer phenomena and develop the temperature profile equations so that we can quantify the heat transfer issues that arise in transferring the fission heat out of the fuel meat and in keeping the fuel cool.

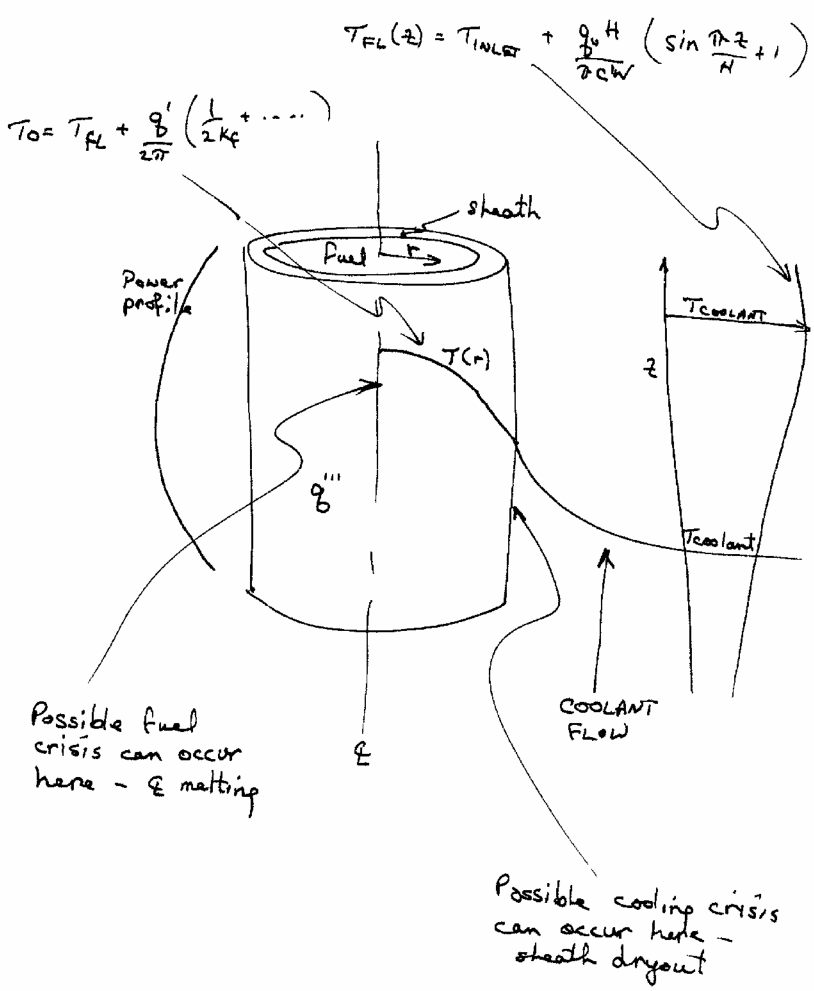


Figure 2 Overview of fuel heat transfer.

1.2 Learning Outcomes

The goal of this chapter is for the student to understand:

- Know WHAT (basic comprehension)
    - definitions of terms
    - physical layout of reactor channel
    - typical values
    - units
    - key physical phenomena
        - conduction
        - convection
        - dryout
        - centreline melting
        - CHF
        - CPR
- Know HOW (ability to do)
    - model heat conduction in solids
    - model heat convection to fluids
    - calculate temperature distribution
        - radial
        - axial
- Know WHY(high level understanding)
    - heat transfer as a limiting factor for power output
    - heat transfer dependence on parameters
    - crisis prevention

2 General Heat Conduction Equation

For a solid, the general energy thermal energy balance equation of an arbitrary volume, V, is:

$$\oiiint_V \frac{\partial(\rho e)}{\partial t} dV = \oiiint_V q'''(\underline{r},t)dV - \oiint_S \underline{q}''(\underline{r},t)\bullet\hat{\underline{n}}dS \qquad (2.1)$$

where ρ is the material density, e is the internal energy, V is the volume, S is the surface area, q''' is the volumetric heat generation, $\underline{q}''$ is the heat flux vector, and $\hat{\underline{n}}$ is the unit vector on the surface. We replace the internal energy with temperature, T, times the heat capacity, c. Using Gauss' Law to convert the surface integral to a volume integral and dropping the volume integral everywhere:

$$\frac{\partial(\rho c T)}{\partial t} = q'''(\underline{r},t) - \nabla\bullet\underline{q}''(\underline{r},t) \qquad (2.2)$$

We further need a relation to specify the heat flux in terms of temperature. In a solid, Fourier's law of thermal conduction applies:

$$\underline{q}''(\underline{r},t) = -k\nabla T(\underline{r},t) \qquad (2.3)$$

where and k is the thermal conductivity. This gives the usable form:

$$\frac{\partial(\rho c T)}{\partial t} = q'''(\underline{r},t) + \nabla \bullet k \nabla T(\underline{r},t) \qquad (2.4)$$

The parameters have the following units:

ρ	kg/m3
c	J/(kg K)
k	J/(m K-sec)
q"	J/(m2-sec) = W/m2
q"'	J/(m3-sec) = W/m3
T	K
α	defined as k/ c = m2/sec.

Exercises:

WHAT: Explain the meaning of each term in equation 2.1.

HOW: Derive equation 2.4 for the clad region.
Derive the steady state version of equation 2.4.

WHY: Why is a high fuel conductivity a good thing?

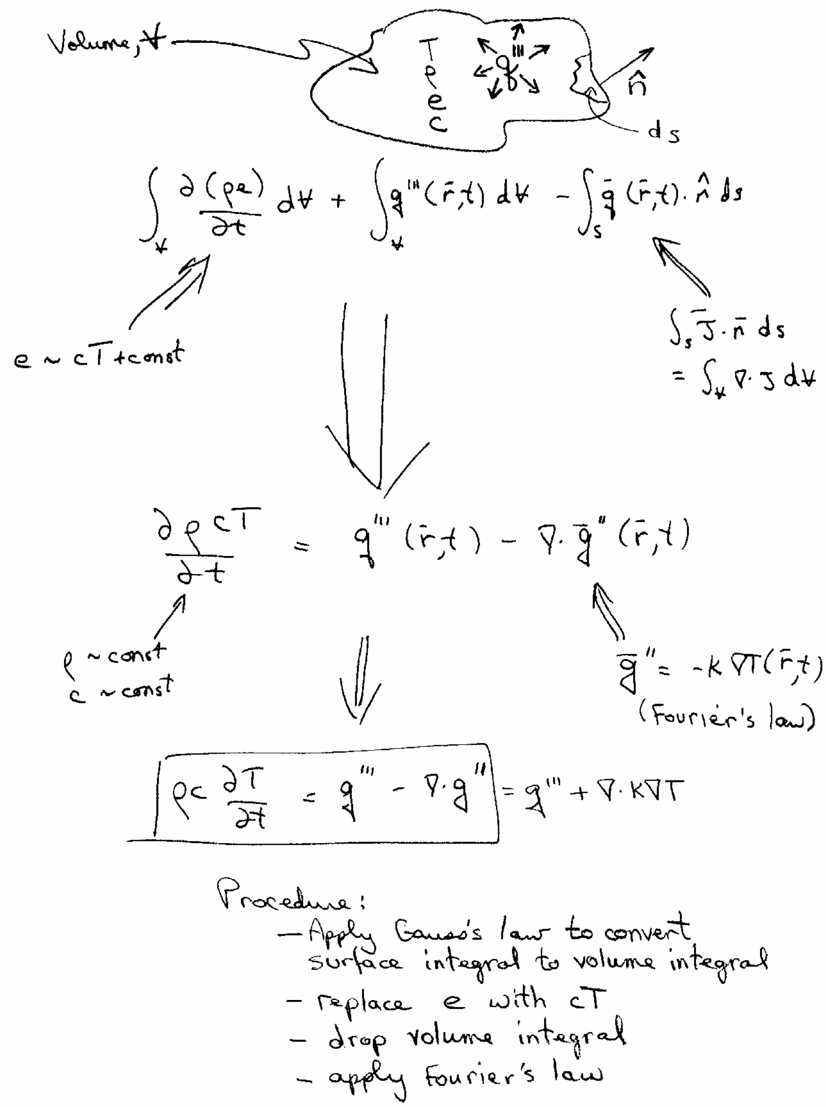


Figure 3 Heat conduction in a solid.

3 Definitions and Assumptions

Fuel pins are typically long and thin. The fuel meat is covered by a metallic sheath or clad to protect the fuel from erosion and corrosion and to prevent fission products from entering the coolant. Heat is generated in the fuel at a rate proportional to the fissioning rate. Heat conduction through solids is driven by temperature gradients. In a long thin pencil or pin, such as in figure 4, the radial temperature gradients are much larger than the axial gradients. Hence, axial heat conduction is usually ignored.

There is a small spatial variation in the heat generation but for our purposes we will assume a uniform heat generation rate, q'''. The linear power density, q', is defined as the heat generated per unit length of fuel (of radius r_f):

$$q' = \int_0^{r_f} q''' \cdot 2\pi r \, dr = \pi r_f^2 q''' \text{ watts / cm.} \tag{3.1}$$

At steady state, all the heat generated in the fuel volume must be transported out through the fuel surface. The heat flux through a surface area, q', is:

$$\int_S q'' dA = q'' \cdot 2\pi r_f = q' \tag{3.2}$$

where q" is the heat flux per unit surface area. Thus:

$$q'' = \frac{q'}{2\pi r_f} = \frac{q''' \pi r_f^2}{2\pi r_f} = q''' \frac{r_f}{2} \tag{3.3}$$

for uniform q'''.

<u>Exercises:</u>

WHAT: Explain the physical mechanisms.

HOW: Explain how is the heat generation balanced with heat conduction?

WHY: Why are fuel pins used in power reactors? Wouldn't plates be better?

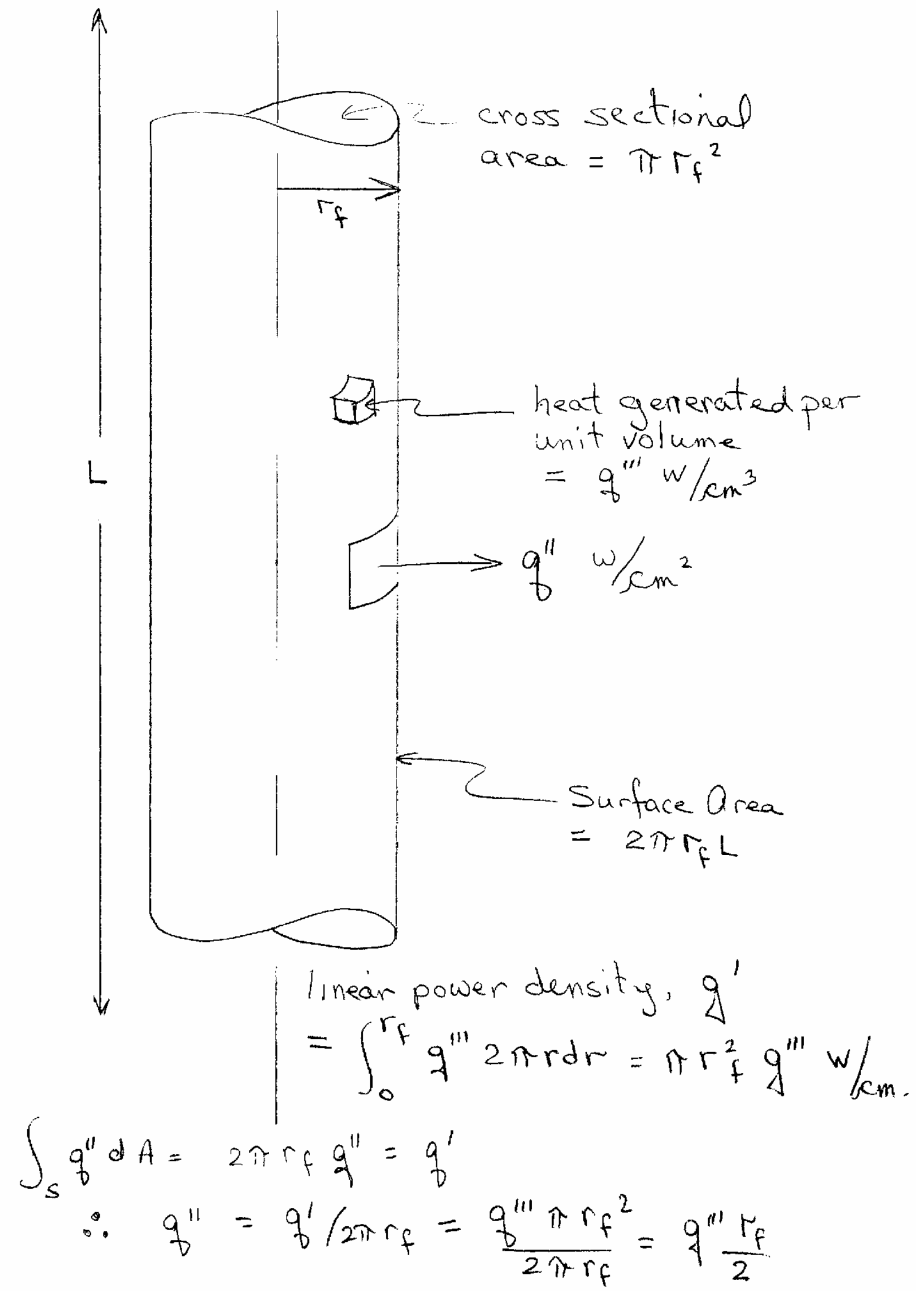
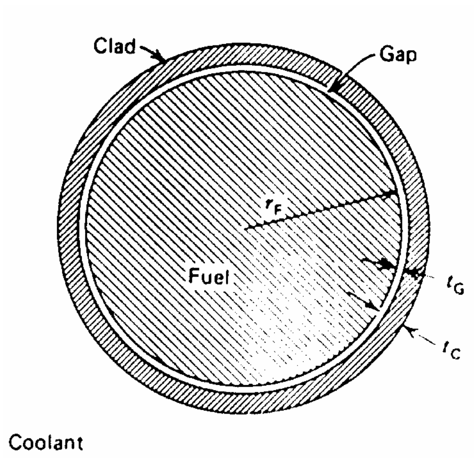


Figure 4 q''' vs q'' vs q'

4 Radial Heat Transfer



Consider a typical cylindrical fuel pin composed of fuel meat surrounded by a metal clad, as shown in figure 5. There is usually a small gap between the fuel and the clad which offers substantial resistance to heat transfer. The flowing coolant surrounds the pin. We will look at the fuel, gap, clad and coolant separately to develop the temperature profile in each material. Then, we will combine the equations to give the full fuel to coolant temperature profile. It is sufficient for our purposes to focus on the steady state.

Figure 5 Radial fuel pin geometry [Source: DUD76, figure 12-3]

4.1 Fuel Meat

Equation 2.4 in the steady state is:

$$-\nabla \bullet k \nabla T(\underline{r}) = q'''(\underline{r}) \tag{4.1}$$

The pins are much longer than their diameter, hence, axial heat conduction can be ignored. In radial coordinates:

$$-\frac{1}{r}\frac{d}{dr}\left(k_f r \frac{dT}{dr}\right) = q'''(\underline{r}) \tag{4.2}$$

If we assume a uniform heat generation rate, q''', this can be directly integrated to give:

$$k_f r \frac{dT}{dr} = -\frac{r^2}{2} q''' \tag{4.3}$$

The constant of integration is zero since the temperature gradient at r=0 is zero. The thermal conductivity, k, is a strong function of T in fuel. Hence, the subsequent integration of equation 4.3 is:

$$\int_{T_0}^{T_F} k_f(T)dT = -\frac{r_F^2}{4} q''' \equiv \overline{k_f}\left(T_F - T_0\right) \tag{4.4}$$

where the subscript 0 indicates the centre point and the subscript F indicates the fuel meat radius. Since $T=T_0$ at r=0, the constant of integration is again zero. Finally we have:

$$\Delta T_{fuel} \equiv T_F - T_0 = \frac{r_F^2}{4\overline{\overline{k_f}}} q''' = \frac{q'}{4\pi \overline{\overline{k_f}}} \tag{4.5}$$

Note that we get the same T for a given q' no matter what the fuel radius. For UO_2 ceramic, k_f is typically 0.02 - 0.03 W/cm-K. At a q' of 500 W/cm, the ΔT is about 1400 °C.

4.2 Gap

Equation 2.4 in the steady state for the gap region is:

$$-\frac{1}{r}\frac{d}{dr}\left(k_G r\frac{dT}{dr}\right)=0 \tag{4.6}$$

This can be directly integrated to give:

$$k_G r\frac{dT}{dr}=\text{constant} \;\Rightarrow\; -k_G\frac{dT}{dr}=-\frac{\text{constant}}{r} \tag{4.7}$$

The constant of integration is determined by considering the heat flux, q" at the fuel - gap interface:

$$-k_G\frac{dT}{dr}\bigg|_{r=r_F}=q''=\frac{q'}{2\pi r_F} \tag{4.8}$$

Substituting equation 4.8 into 4.7, we get:

$$k_G r\frac{dT}{dr}=-\frac{q'}{2\pi} \tag{4.9}$$

$$\therefore k_G\frac{dT}{dr}=-\frac{q'}{2\pi r} \tag{4.10}$$

Integrating again we have:

$$k_G\Delta T_{Gap}\equiv k_G\left(T_F-T_C\right)=\frac{q'}{2\pi}\ln\left(\frac{r_F+t_G}{r_F}\right) \tag{4.11}$$

where the subscript C indicates the gap-clad interface. The boundary conditions $T=T_C$ at $r = r_F + t_G$ is incorporated into the above solution. Finally we have:

$$\Delta T_{Gap}=\frac{q'}{2\pi k_G}\ln\left(\frac{r_F+t_G}{r_F}\right)=\frac{q'}{2\pi r_F}\left(\frac{t_G}{k_G}\right) \text{ since } \ln(1+x)\approx x \tag{4.12}$$

The gap conductivity k_G is ~ 0.002 W/cm-K but it varies considerably with the amount of fission product gases. For a gap thickness of 0.005 cm, we get a ΔT_{GAP} of about 300 °C for a q' of 500 W/cm. Since the fuel will swell to touch the clad (although not perfectly since the surfaces have a finite roughness), an effective heat transfer coefficient, h_G is used:

$$h_G\left(\Delta T_{Gap}\right)=q'' \tag{4.13}$$

Thus:

$$\Delta T_{Gap}=\frac{q'}{2\pi r_F h_G} \tag{4.14}$$

A heat transfer coefficient of 0.5 - 1.1 W/cm^2-K gives a T_{Gap} less than 300 °C.

4.3 Clad

As per the gap region, the steady state equation for the clad region is:

$$-\frac{1}{r}\frac{d}{dr}\left(k_C r\frac{dT}{dr}\right)=0 \tag{4.15}$$

This is solved in the same manner as for the gap to give:

$$k_C\Delta T_{Clad}\equiv k_C\left(T_C-T_S\right)=\frac{q'}{2\pi}\ln\left(\frac{r_F+t_G+t_C}{r_F+t_G}\right) \tag{4.16}$$

where the subscript S indicates the clad-coolant surface interface. The boundary conditions T = T_S at r = $r_F + t_G + t_C$ is incorporated into the above solution. Finally we have:

$$\Delta T_{Clad}=\frac{q'}{2\pi k_C}\ln\left(\frac{r_F+t_G+t_C}{r_F+t_G}\right)=\frac{q'}{2\pi\left(r_F+t_G\right)}\left(\frac{t_G+t_C}{k_C}\right)\text{ since }\ln\left(1+x\right)\approx x \tag{4.17}$$

The clad conductivity k_C is ~ 0.11 W/cm-K giving a T_{Clad} of about 80 °C for a q' of 500 W/cm.

4.4 Coolant

From the clad to the coolant, the heat flux is determined by:

$$q''=h_S\left(T_S-T_{FL}\right) \tag{4.18}$$

T_{FL} is the bulk temperature of the coolant fluid. Thus, the temperature drop from the clad surface to the bulk fluid temperature is:

$$\Delta T_{Cool}=\frac{q''}{h_S}=\frac{q'}{2\pi h_S\left(r_F+t_C+t_G\right)} \tag{4.19}$$

A heat transfer coefficient of ~4.5 W/cm^2-K give a T_{Cool} of about 10 - 20 °C.

4.5 Overall Temperature Difference

Adding up all the temperature differences we find:

$$T_0-T_{FL}=\frac{q'}{2\pi}\left(\frac{1}{2\overline{k}_f}+\frac{1}{h_G r_f}+\frac{t_G+t_C}{k_C\left(r_F+t_C\right)}+\frac{1}{h_S\left(r_F+t_C+t_G\right)}\right) \tag{4.20}$$

Thus, given a bulk coolant temperature, the centre line fuel temperature floats on top of the coolant temperature by an amount that depends on the heat being generated and the various resistances to heat flow. For a given fuel design, most of the parameters are fixed under normal operation. The one exception is h_S. As illustrated in figure 6, h_S (defined as the slope of the q" - T curve) can vary considerably. If the surface temperature is too high, a vapour blanket forms at the surface and the heat cannot flow out of the fuel. In effect, h_S drops. This is the dreaded fuel cooling crisis that can occur if power regulation is lost, if a loss of coolant flow occurs or if a loss of coolant inventory occurs. The result of such a crisis is clad failure and release of fission products to the coolant system, and possibly to the turbine cycle and the atmosphere.

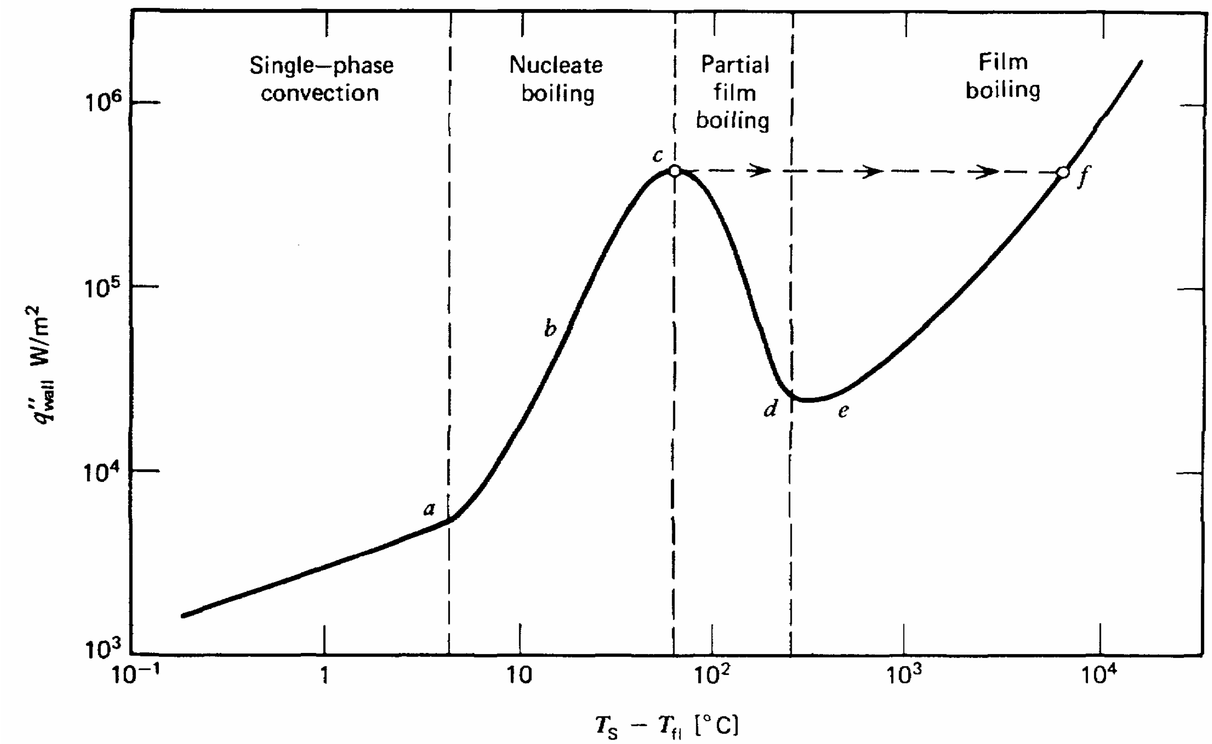


Figure 6 Heat flux vs. ΔT for pool-boiling heat transfer [Source DUD76, figure 12-9]

5 General Thermal Energy Equation

To determine the axial temperature distribution in the coolant, we need to consider the axial heat transport mechanisms. For this we need the general form of the thermal energy equation:

$$\oiiint_V \frac{\partial(\rho e)}{\partial t} dV = -\oiint_S \rho e\, \underline{v} \bullet \hat{n}\, ds + \oiiint_V q'''(\underline{r},t)\, dV - \oiint_S \underline{q}''(\underline{r},t) \bullet \hat{n}\, ds + \oiiint_V \tau : \nabla \underline{v} dV - \oiiint_V P \nabla \bullet \underline{v} dV$$

(4.21)

where the last two terms are the viscous heat dissipation (friction heating) and pressure work terms, respectively (don't sweat it, we'll be dropping these terms anyway). The first term on the right hand side of equation 4.21 represents the flow of energy through the surfaces, i.e., energy transport. This can be rearranged in terms of enthalpy (h = e + P/ρ):

$$\oiiint_V \frac{\partial(\rho h - P)}{\partial t} dV = -\oiint_S \rho h\, \underline{v} \bullet \hat{n}\, ds + \oiiint_V q'''(\underline{r},t)\, dV - \oiint_S \underline{q}''(\underline{r},t) \bullet \hat{n}\, ds \qquad (4.22)$$

$+P\underline{v}$ and friction terms that are relatively small and tend to cancel.

So this is the general energy balance equation. It looks a bit daunting but it simplifies greatly when we apply it to the case at hand: turbulent flow along a heated surface.

5.1 Axial Temperature Distribution

In typical power reactors, $\partial P/\partial t \ll \partial \rho h/\partial t$ so that term can be ignored. For the steady state situation, the energy balance on a lump of fluid coolant of length dz surrounding the fuel pin (see figure 7) is, thus:

$$A dz \frac{\partial(\rho h)}{\partial t} = 0 = (A\rho h v)\big|_z - (A\rho h v)\big|_{z+dz} + q''(z) 2\pi r_F dz \qquad (4.23)$$

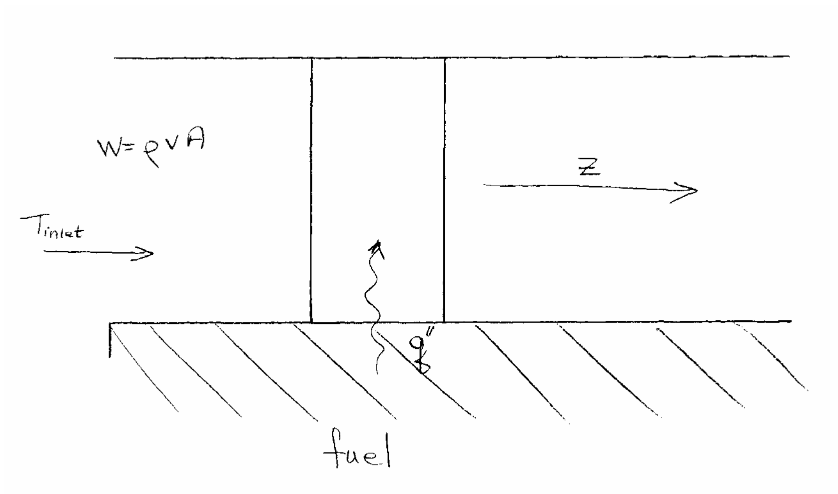


Figure 7 Axial Energy Balance

Since there is no heat generation in the coolant itself (apart from some minor turbulence heating), q''' = 0. Defining the mass flow as W = Aρv (kg/sec) and converting q'' to q', we have:

$$W\left(h_{z+dz} - h\big|_z\right) = q'(z) dz \qquad (4.24)$$

We note that W is constant along the channel length since mass is neither created nor destroyed. Also note that the heat flux is a function of axial position since the power generation axial distribution in a reactor is not uniform. To a first approximation it is a cosine shape. In single phase, then:

$$W c\, dT = q'(z) dz = q_0 \cos\left(\pi \frac{z}{H}\right) dz \qquad (4.25)$$

where H is the channel length, z = 0 at the channel midpoint and c is the fluid heat capacity. Integrating from the channel entry (z = -H/2) to the channel outlet (z = + H/2) gives:

$$T_{FL}(z) - T_{Inlet} = \frac{q_0 H}{\pi c W}\left(\sin\left(\frac{\pi z}{H}\right) + 1\right) \text{ where } z \in \left(-H/2, +H/2\right) \qquad (4.26)$$

This is plotted in figure 8.

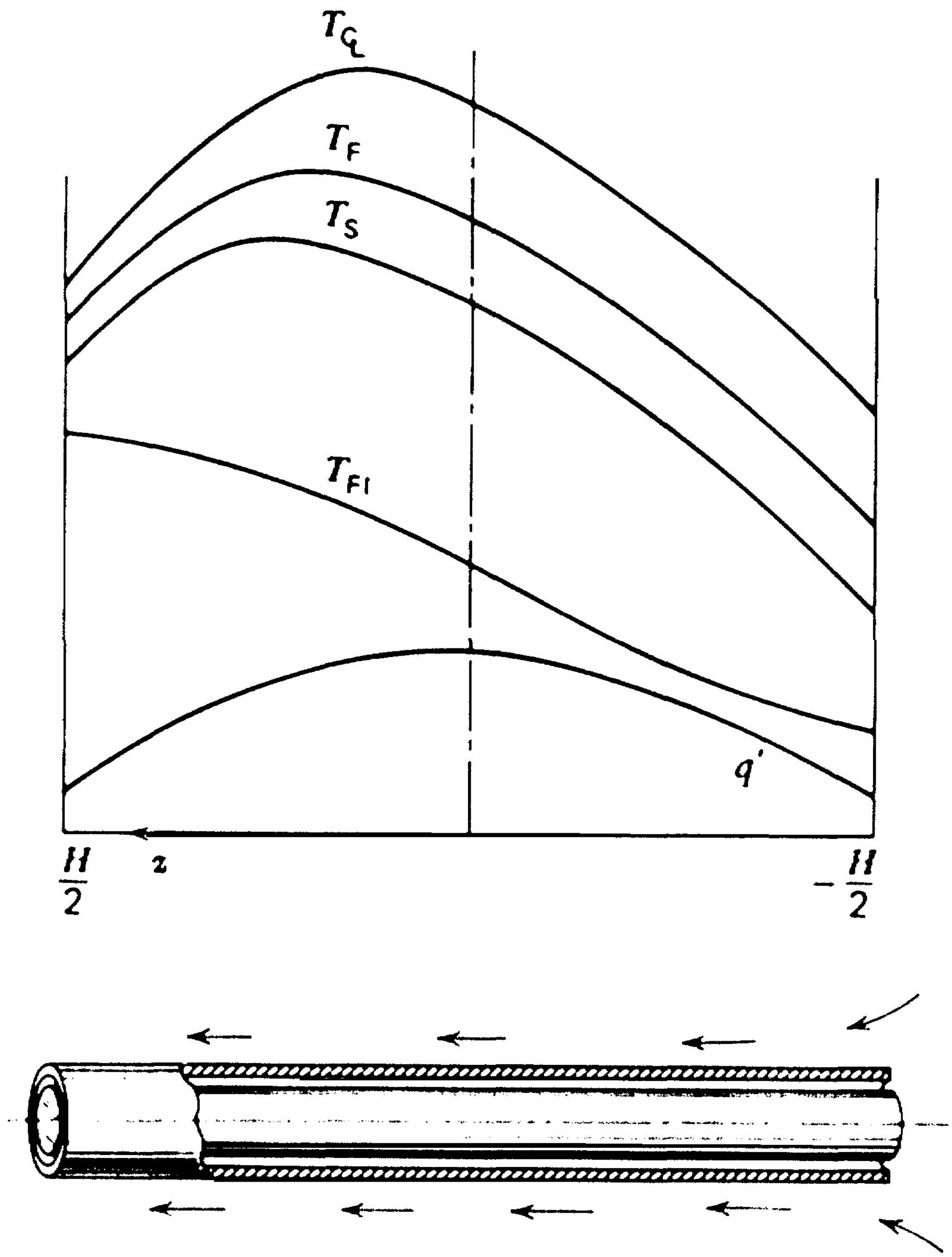


Figure 8 Axial temperature profile [Source: DUD76, figure 12-8]

5.2 Axial Quality Distribution

Equation 4.24 can be used to calculate the axial quality distribution by noting that:

$$h = h_{f_{SAT}} + xh_{fg} \tag{4.27}$$

where x is the weight fraction of steam in a two-phase mixture, h_{fSAT} is the saturated liquid enthalpy and h_{fg} is the latent heat of vaporization. Thus:

$$W\left(h(z) - h_{Inlet}\right) = \int_{-\frac{H}{2}}^{+\frac{H}{2}} q'(z)dz \tag{4.28}$$

If the axial position of the start of bulk boiling (the point where $h(z) = h_{fSAT}$) is defined as z_{BB}:

$$W\left(h(z) - h_{f_{SAT}}\right) = \int_{z_{BB}}^{z} q'(z)dz = x(z)W\,h_{fg} \tag{4.29}$$

The quality as a function of axial position is, finally:

$$x(z) = \frac{1}{W\,h_{fg}} \int_{z_{BB}}^{z} q'(z)dz \tag{4.30}$$

5.3 Critical Heat Flux

The local quality is of central importance to the margin to dryout in a reactor channel since x is the one parameter that was experimentally found to relate to centre line melting and sheath dryout, two phenomena that serve as indicators of the onset of a heat transfer crisis. Figure 9 shows a typical plot of heat flux and quality as a function of axial position. Industry has determined experimental correlations but, for our purposes, figure 9also shows a sketch of the Critical Heat Flux (CHF) as a function of local quality. Re-plotting actual heat flux vs. quality on the same graph allows an estimation of the margin to dryout or centre line melting. If the channel power were to increase, this curve will move up and to the right, approaching the CHF curve. The channel power that causes the two curves to touch is the limiting or critical channel power. The Critical Power Ratio, or CPR, is defined as the ratio of this critical power or heat flux and the nominal power or heat flux.

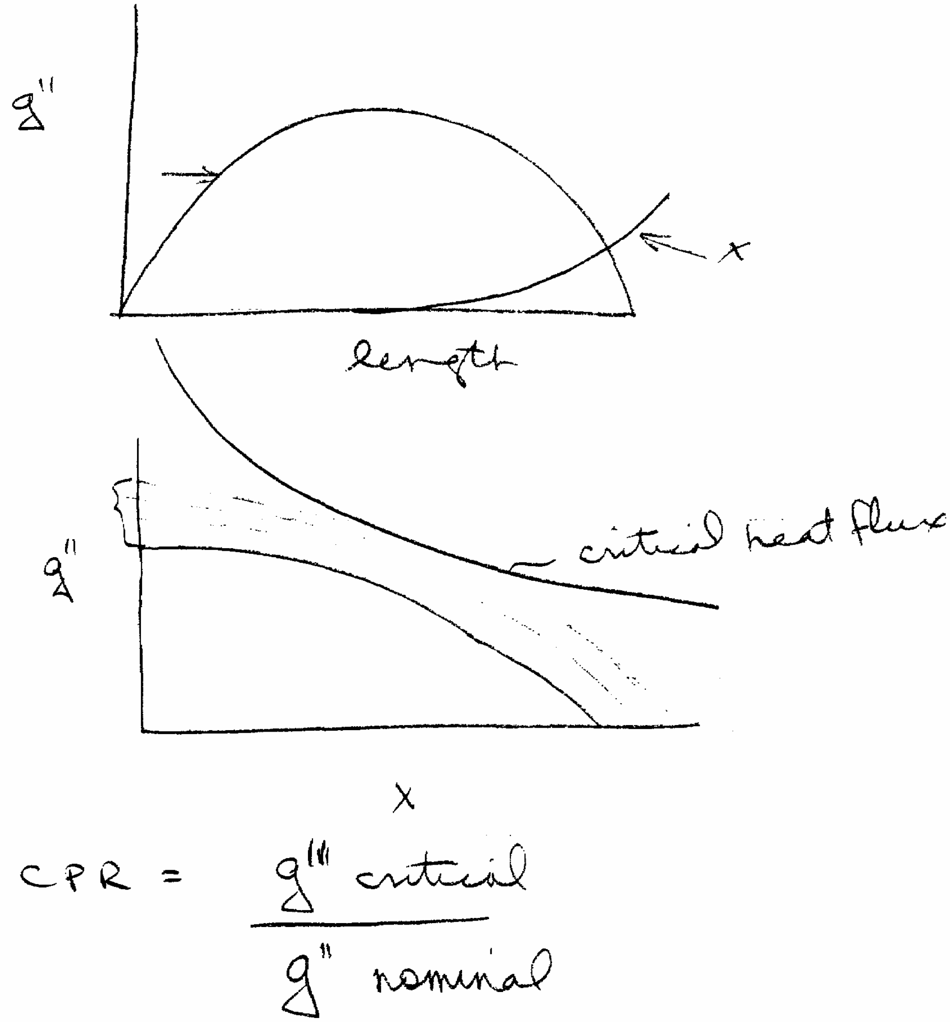


Figure 9 CHF and CPR

6 Summary

This chapter has dealt with the heat transfer situation in the fuel channel. Heat flux limitations here set the limit for plant power output and so are of critical importance to power reactor design.

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Revision history:
Revision 1.0, 2002.07.29, initial creation from hand written notes.
Revision 1.1, 2004.08.17, minor typos.

Source document archive location: See page footer.

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Notes: