ENGINEERING PHYSICS 4D3/6D3

DAY CLASS — Dr. Wm. Garland

DURATION: 20 minutes

McMASTER UNIVERSITY QUIZ #1 — October 20, 1999

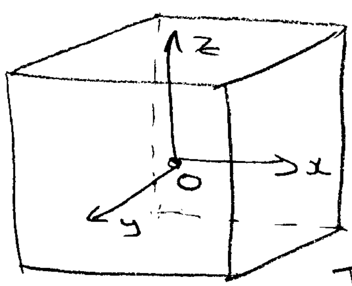
Special Instructions: Closed Book. All calculators and up to 8 single sided 8 ½" by 11" crib sheets are permitted.

THIS EXAMINATION PAPER INCLUDES 1 PAGE AND 1 QUESTION.

1. State the one-group transient neutron diffusion equation and the appropriate initial and boundary conditions for the following cases:
   a. [30 marks] A finite, three-dimensional homogeneous rectangular reactor of dimensions 100 cm x 100 x 100 cm (length x width x height) surrounded by air (treat air as a vacuum).
   b. [70 marks] An infinite array of identical, homogeneous fuel cells in water. The fuel cells each have a cross section 10 cm x 10 cm and are very long (>> 10 cm). The spacing between the fuel assemblies is 10 cm.

Sol'n

(a)



$$\frac{1}{v}\frac{\partial \phi}{\partial t} = D\left(\frac{\partial^2 \phi}{\partial x^2} + D\frac{\partial^2 \phi}{\partial y^2} + D\frac{\partial^2 \phi}{\partial z^2}\right) + (\nu\Sigma_f - \Sigma_a)\phi$$

where $\phi = \phi(x, y, z, t)$

I.C: $\phi(x, y, z, t) = \text{Given}$

BC: $\phi = 0$ at each of the six faces, ie: $\phi(\pm 50, y, z, t) = 0$ (1)(2)

(assume dimensions include the extrapolated lengths) $\phi(x, \pm 50, z, t) = 0$ (3)(4)

$\phi(x, y, \pm 50, t) = 0$ (5)(6)

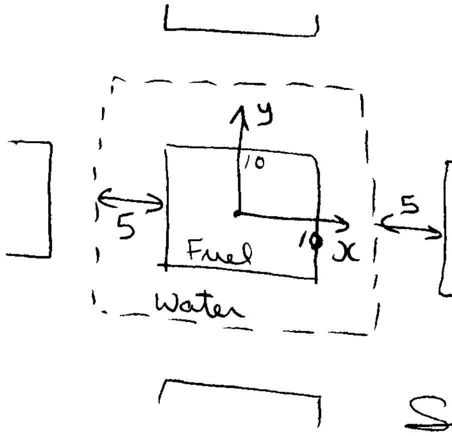
Or we could take advantage of symmetry and use:

$\frac{\partial \phi}{\partial x} = \frac{\partial \phi}{\partial y} = \frac{\partial \phi}{\partial z} = 0$ at origin for all time (1)(2)(3)

$\phi(50, y, z, t) = 0 = \phi(x, 50, z, t) = \phi(x, y, 50, t)$ (4)(5)(6)

There are 6 B.C. + 1 I.C.

(b)



Fuel Region

$$\frac{1}{v}\frac{\partial \phi^F}{\partial t} = D^F\left(\frac{\partial^2 \phi^F}{\partial x^2} + \frac{\partial^2 \phi^F}{\partial y^2}\right) + (\nu^F \Sigma_f^F - \Sigma_a^F)\phi^F$$

Water Region

$$\frac{1}{v}\frac{\partial \phi^W}{\partial t} = D^W\left(\frac{\partial^2 \phi^W}{\partial x^2} + \frac{\partial^2 \phi^W}{\partial z^2}\right) - \Sigma_a^W \phi^W$$

Symmetric about the x + y axes.
Ignore Z since $\frac{\partial^2 \phi}{\partial z^2} \sim 0$

$\phi = \phi(x, y, t)$

I.C.:

$\phi^F(x, y, 0) = \text{Given}$
$\phi^W(x, y, 0) = \text{Given}$

There are 8 B.C. + 2 I.C.

B.C.:

(1) $\left.\frac{\partial \phi^F}{\partial x}\right|_{0,y,t} =$ (2) $\left.\frac{\partial \phi^F}{\partial y}\right|_{x,0,t} = 0$ (ie flux slope = 0 at origin)

(3) $\phi^F(10, y, t) = \phi^W(10, y, t)$
(4) $\phi^F(x, 10, t) = \phi^W(x, 10, t)$
(fluxes at fuel-water interfaces = 0 for all time)

(5) $J_y^F(10, y, t) = J_y^W(10, y, t)$
(6) $J_x^F(x, 10, t) = J_x^W(x, 10, t)$
(currents at fuel-water interfaces = 0 for all time)

where $J_y^F = -D^F \frac{\partial \phi^F}{\partial y}$ + $J_x^F = -D^F \frac{\partial \phi^F}{\partial x}$

$J_y^W = -D^W \frac{\partial \phi^W}{\partial y}$ + $J_x^W = -D^W \frac{\partial \phi^W}{\partial x}$

(7) $J_y^W(15, y, t) = 0$
(8) $J_x^W(y, 15, t) = 0$
Reflection at the cell faces

If you didn't assume symmetry, you'd need to write extra water equations and generate a host of ϕ + J BC's to match.