

# SOLUTION

DAY CLASS

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DURATION: 20 minutes

McMASTER UNIVERSITY QUIZ #1

2005-09-22

**Special Instructions:**

1. Closed Book. All calculators and up to 6 single sided 8 1/2" by 11" crib sheets are permitted.
2. The value of each part is as indicated. TOTAL Value: 100 marks

**THIS EXAMINATION PAPER INCLUDES 1 PAGE AND 1 MULTI-PART QUESTION. YOU ARE RESPONSIBLE FOR ENSURING THAT YOUR COPY OF THE PAPER IS COMPLETE. BRING ANY DISCREPANCY TO THE ATTENTION OF YOUR INVIGILATOR.**

1.

- a. [20 marks] State the general one speed neutron differential balance equation written in terms of flux, current and space dependent material properties and in general spatial coordinates as developed in the course so far. Do a units check on each term of the equation.

$$\frac{1}{v} \frac{\partial \phi}{\partial t} = -\nabla \cdot \mathbf{J} - \Sigma_a \phi + S$$

$\left[ \frac{1}{\text{cm}} \right] \left[ \frac{\#}{\text{cm}^2 \cdot \text{s}} \right] = \left[ \frac{\#}{\text{cm}^3 \cdot \text{s}} \right] = \left[ \frac{1}{\text{cm}} \frac{\#}{\text{cm}^2 \cdot \text{s}} \right] \quad \left[ \frac{1}{\text{cm}} \frac{\#}{\text{cm}^2 \cdot \text{s}} \right] \quad \left[ \frac{\#}{\text{cm}^3 \cdot \text{s}} \right]$

- b. [20 marks] What is the appropriate simplified form assuming that neutron movement can be modelled as a diffusion process? When would this approximation not be reasonable?

$$\frac{1}{v} \frac{\partial \phi}{\partial t} = +\nabla \cdot D \nabla \phi - \Sigma_a \phi + S \quad (\text{ie } \mathbf{J} \approx -D \nabla \phi)$$

Approx. not valid near boundaries and other discontinuities where  $\Sigma + D$  (hence  $\phi$ ) vary greatly. Other assumptions tend to cancel.

- c. [20 marks] What is the differential equation for the one spatial dimension case? How many boundary and initial conditions are needed?

$$\frac{1}{v} \frac{\partial \phi}{\partial t} = \frac{\partial}{\partial x} D \frac{\partial \phi}{\partial x} - \Sigma_a \phi + S$$

Need 2 B.C.,  
1 I.C.

- d. [20 marks] What is the differential equation for the case of steady state and an infinite planar source in an infinite medium, as derived in class? What are the appropriate boundary and initial conditions?

$$0 = D \frac{\partial^2 \phi}{\partial x^2} - \Sigma_a \phi + S \delta(x)$$

assuming  $D + \Sigma_a$  uniform in space.

No I.C. 2 B.C.:  $J|_{x=0} = S/2, \phi(\infty) = 0.$

- e. [20 marks] What is the transient differential equation for the case of complete spatial uniformity (source uniformly distributed in space, homogeneous medium)? What are the boundary conditions and initial conditions? What is the steady state flux for this case?

$$\frac{1}{v} \frac{\partial \phi}{\partial t} = \nabla \cdot \mathbf{J} - \Sigma_a \phi + S$$

0

since no spatial dependency  $\Rightarrow$  no B.C. needed, -END-

I.C. =  $\phi(t=0) = \phi_0$

In steady state:  $0 = 0 - \Sigma_a \phi + S \Rightarrow$

$$\phi = S / \Sigma_a \quad \left[ \frac{\#}{\text{cm}^2 \cdot \text{s}} \right]$$