ENGINEERING PHYSICS 4D3/6D3

DAY CLASS — Dr. Wm. Garland

DURATION: 20 minutes

McMASTER UNIVERSITY QUIZ #1 — October 16, 2000

Special Instructions: Closed Book. All calculators and up to 8 single sided 8 ½" by 11" crib sheets are permitted.

THIS EXAMINATION PAPER INCLUDES 1 PAGE AND 1 QUESTION.

1. For the one-group transient neutron diffusion model of a one dimensional homogeneous bare slab reactor
   a. [25 marks] State the neutron balance equation and the appropriate initial and boundary conditions.
   b. [50 marks] Derive the stability criteria for the explicit numerical scheme.
   c. [25 marks] Show how this condition is relaxed when an implicit scheme is used.

Sol'n

a) $\frac{1}{v}\frac{\partial \phi}{\partial t} = D\frac{\partial^2 \phi}{\partial x^2} + (\nu\Sigma_f - \Sigma_a)\phi$, $\phi(\pm a/2, t) = 0$

$\phi(x,0) = \text{given}$

b) In finite difference form (explicit):

$$\frac{1}{v}\frac{\phi_p^{t+\Delta t} - \phi_p^t}{\Delta t} = \frac{D}{\Delta x^2}(\phi_w^t - 2\phi_p^t + \phi_E^t) + (\nu\Sigma_f - \Sigma_a)\phi_p^t$$

$$\therefore \phi_p^{t+\Delta t} = \frac{v\Delta t D}{\Delta x^2}(\phi_w^t + \phi_E^t) + \underbrace{\left(1 - v\Delta t\left(\frac{2D}{\Delta x^2} + \nu\Sigma_f - \Sigma_a\right)\right)}_{\text{gives instability if} < 0}\phi_p^t$$

$\therefore$ for stability,

$$\Delta t \le \frac{\Delta x^2}{v(2D - \nu\Sigma_f \Delta x^2 + \Sigma_a \Delta x^2)}$$

c) If the implicit form is used:

$$\phi_p^{t+\Delta t} = \frac{v\Delta t D}{\Delta x^2}(\phi_w^{t+\Delta t} + \phi_E^{t+\Delta t}) + \phi_p^t + \left(-2\frac{v\Delta t}{\Delta x^2} + \nu\Sigma_f - \Sigma_a\right)\phi_p^{t+\Delta t}$$

$$\therefore \phi_p^{t+\Delta t} = \frac{\frac{v\Delta t D}{\Delta x^2}(\phi_w^{t+\Delta t} + \phi_E^{t+\Delta t}) + \phi_p^t}{1 + v\Delta t\left(2D/\Delta x^2 - \nu\Sigma_f + \Sigma_a\right)}$$

This is well behaved even for large Δt.
(Watch out for positive reactivity cases, though where $\nu\Sigma_f$ big).