Solution

Student Name : Mortimer Snerd **ID:** 123456 Page 1 of 3

ENGINEERING PHYSICS 4D3/6D3

DAY CLASS Dr. Wm. Garland
DURATION: 50 minutes
McMASTER UNIVERSITY MIDTERM EXAMINATION October 18, 2005

Special Instructions:

1. Closed Book. All calculators and up to 6 single sided 8 ½" by 11" crib sheets are permitted.
2. Do all questions.
3. The value of each question is as indicated. TOTAL Value: 100 marks

THIS EXAMINATION PAPER INCLUDES 3 PAGES AND 3 QUESTIONS. YOU ARE RESPONSIBLE FOR ENSURING THAT YOUR COPY OF THE PAPER IS COMPLETE. BRING ANY DISCREPANCY TO THE ATTENTION OF YOUR INVIGILATOR.

1. [40 marks] Consider a cubic homogeneous reactor of dimension a x a x a in an ocean of water. For a 1-group neutron diffusion model in steady state:

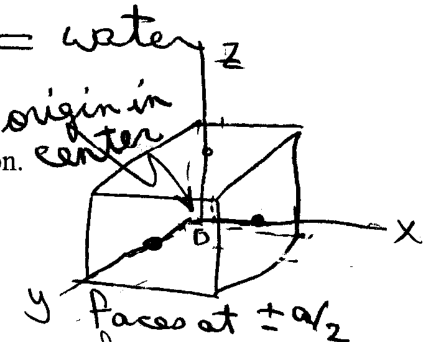
(Same as midterm 2003 Question 4)

a. State the relevant one group neutron balance equations for the reactor and the ocean. Sketch the situation to clarify notation and layout.

$$0 = D^c\left(\frac{\partial^2\phi^c}{\partial x^2}+\frac{\partial^2\phi^c}{\partial y^2}+\frac{\partial^2\phi^c}{\partial z^2}\right)+(\nu\Sigma_f^c-\Sigma_a^c)\phi^c \Leftarrow \text{Reactor core}$$

$$0 = D^w\left(\frac{\partial^2\phi^w}{\partial x^2}+\frac{\partial^2\phi^w}{\partial y^2}+\frac{\partial^2\phi^w}{\partial z^2}\right)-\Sigma_a^w\phi^w \Leftarrow \text{water}$$

b. State the required boundary conditions for an analytical solution.



12 B.C's required (+ ϕ at (0,0,0) to set amplitude)

$$\phi^c\Big|_{x=a/2} = \phi^w\Big|_{x=a/2} \Leftarrow \text{also for } y+z \Big\}\ \underline{3\ \text{B.C's.}}$$

$$J_x^c = -D^c\frac{\partial\phi^c}{\partial x}\Big|_{x=a/2} = J_x^w = -D\frac{\partial\phi^w}{\partial x}\Big|_{x=a/2} \Leftarrow \text{also for } y+z \Big\}\ \underline{3\ \text{B.C.'s}}$$

$$\phi^w\Big|_{x=\infty} = 0$$
(also for y+z)
↑ 3 B.C.

$$\frac{\partial\phi^c}{\partial x}\Big|_{x=0} = 0$$
(also for y+z)
↑ 3 B.C

c. If you were solving this numerically, what boundary conditions would you use?

Just set $\phi\Big|_{x=max_x} = 0$, same for x+y, where max is the outside edge of the numerical grid. (6 faces = 6 B.C.) This is consistent with 1 Finite difference equation of 2nd order in 3 dimensions

d. Why are the B.C.s different for the numerical and analytical cases?

Analytically, you have 2 equations to solve, each with its own parameters + you have to match up the solutions at the interface – ie you have more boundaries to consider.

Numerically, you just have one equation:

$$0 = \nabla\cdot D(\underline{r})\nabla\phi(\underline{r}) + (\nu\Sigma_f(\underline{r}) - \Sigma_f(\underline{r}))\phi(\underline{r})$$

The only 'boundaries' are the outside edges.

2. [30 marks] A bare cubic reactor can be characterized by one group neutron diffusion. The reactor is composed of a lattice of fuel channels with D_2O coolant surrounded by D_2O moderator like in a CANDU reactor, but we will model it as a homogeneous mixture. For this setup assume D = 6.0 cm., $\Sigma_a = 6.0\text{x}10^{-4}$ cm.$^{-1}$, critical height = width = length = 100 cm.

   a. What is $\nu\Sigma_f$?

$k = 1 = \frac{\nu\Sigma_f/\Sigma_a}{1+B^2L^2}$ since critical. $\left(\text{or } B_g^2 = \frac{\nu\Sigma_f+\Sigma_a}{D}\right.$ (same thing) $\left.\right)$

$B^2 = 3(\pi/100)^2 = 0.002961$

$L^2 = D/\Sigma_a = 1\times10^{+4}\ \text{cm}^2$

$\therefore\ \nu\Sigma_f = (1+B^2L^2)\cdot\Sigma_a = 0.01837$

$= \boxed{1.837\times10^{-2}\ \text{cm}^{-1} = \nu\Sigma_f}$

   b. You, as reactor designer, are exploring alternate designs. If you replace the D_2O coolant with H_2O, the material properties change to D = 0.3 cm., $\Sigma_a = 0.03$ cm.$^{-1}$. What would be the new dimensions, keeping the same fuel and keeping it cubic in shape?

$$B^2 = \frac{\frac{\nu\Sigma_f}{\Sigma_a}-1}{L^2} = \frac{\frac{1.837\times10^{-2}}{.03}-1}{.3/.03} < 0\ !?$$

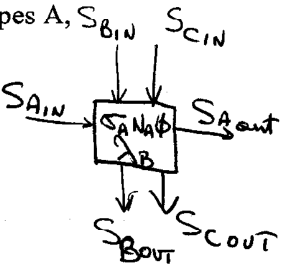
Uh-oh! Looks like we cannot make the reactor critical with the same fuel if we use H_2O, no matter what the dimensions.

∴ must increase the amount of fuel.

This is what happened for the new CANDU design (ACR) where switching to H_2O coolant forced the use of enriched fuel.

[Note: the values used in this problem are bogus and are not CANDU numbers]

3. [30 marks] Consider a processing facility for the production of an isotope B, consisting of a small container inside a reactor. The container is small enough and the absorption cross sections low enough that the neutron flux is uniform in the container. The isotope B is produced by an A(n, γ)B and B decays to C with a half life, $T_{1/2}$; there are no other significant nuclear reactions in the container. Isotopes A, B and C can be added and removed from the container at will.



a. State the governing balance equations for A, B and C.

$$\frac{\partial N_A(t)}{\partial t} = S_A(t) - \sigma_A N_A(t) \phi(t)$$

↖ sinks + sources of A

$$\frac{\partial N_B}{\partial t}(t) = S_B(t) + \sigma_A N_A \phi(t) - \lambda_B N_B(t)$$

$$\frac{\partial N_C}{\partial t}(t) = S_C(t) + \lambda_B N_B(t) \quad \left(\lambda_B = \frac{\ln 2}{T_{1/2}} \right)$$

b. Propose a numerical solution algorithm, showing the finite difference equations and a simple flow diagram showing how the algorithm works.

$$N_A^{t+\Delta t} = N_A^t + \Delta t \left(S_A - \sigma_A N_A \phi \right)$$

↖ N_A^t for explicit

$N_A^{t+\Delta t}$ for implicit

Let's use implicit:

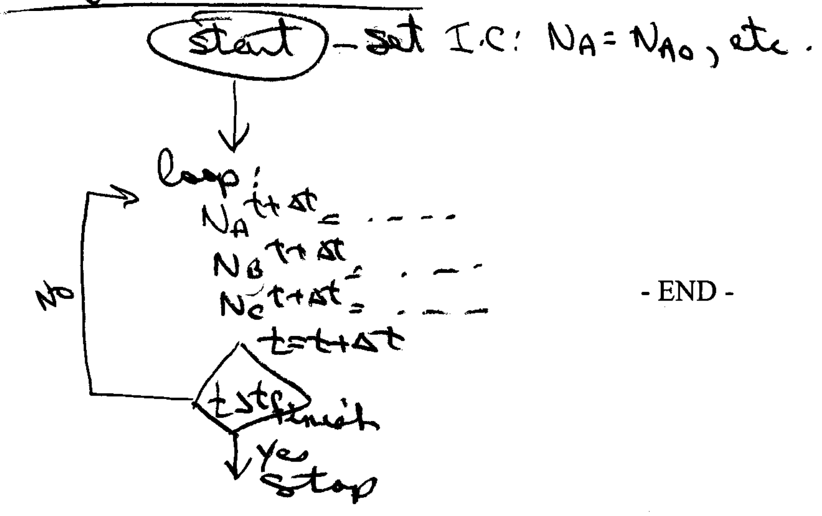
$$\Rightarrow N_A^{t+\Delta t} = \frac{N_A^t + \Delta t S_A}{(1 + \Delta t \sigma_A \phi)}$$

$$\sim N_B^{t+\Delta t} = \frac{N_B^t + \Delta t S_B + \sigma_A N_A^t \phi}{(1 + \Delta t \lambda_B)}$$

$$\sim N_C^{t+\Delta t} = N_C^t + \Delta t \left(S_B + \lambda_B N_B \right)$$

ϕ, S_A, S_B, S_C are all input values. In general, they could be functions of t.

algorithm flowchart



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