ENGINEERING PHYSICS 4D3/6D3

DAY CLASS Dr. Wm. Garland

DURATION: 50 minutes

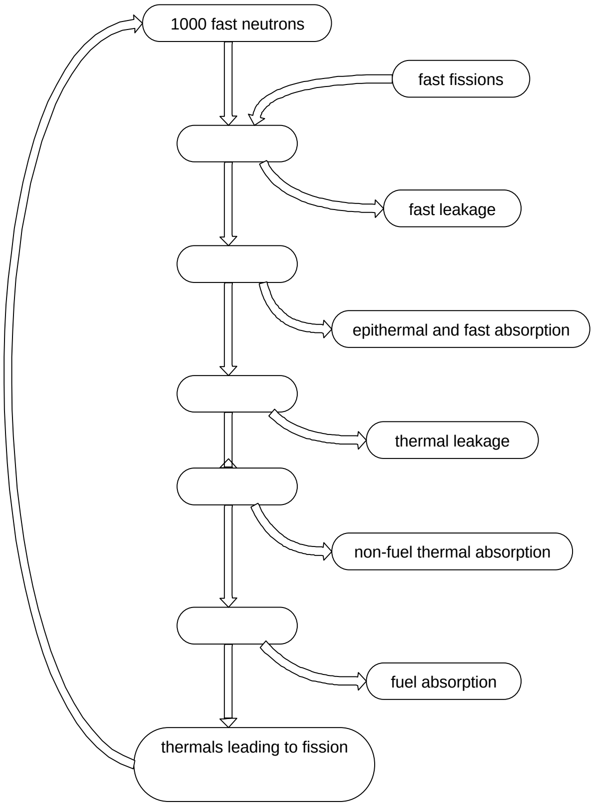
McMASTER UNIVERSITY MIDTERM EXAMINATION October 31, 2001

Special Instructions:

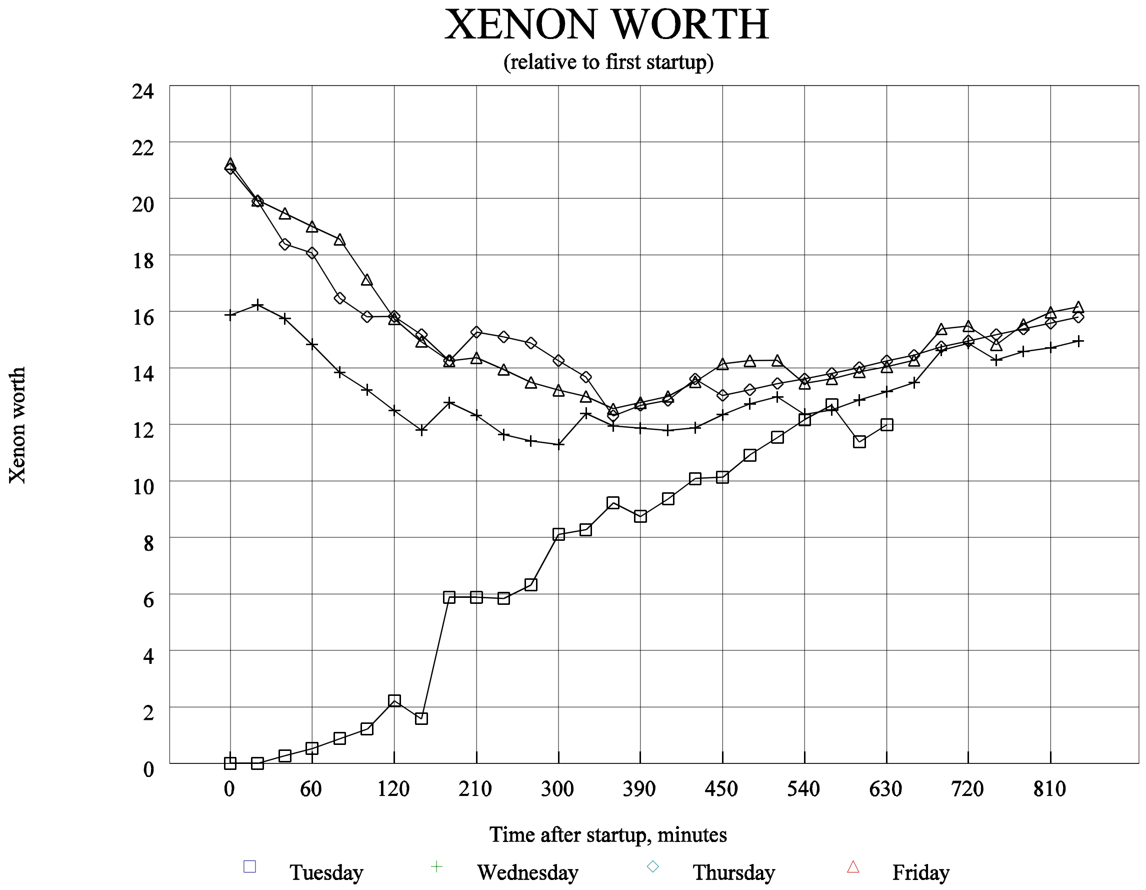
1. Closed Book. All calculators and up to 6 single sided 8 ½" by 11" crib sheets are permitted.
2. Do all questions.
3. The value of each question is as indicated.
   TOTAL Value: 100 marks

THIS EXAMINATION PAPER INCLUDES 2 PAGES AND 3 QUESTIONS. YOU ARE RESPONSIBLE FOR ENSURING THAT YOUR COPY OF THE PAPER IS COMPLETE. BRING ANY DISCREPANCY TO THE ATTENTION OF YOUR INVIGILATOR.

1. [35 marks] Using the figure as a guide, deduce how many fast neutron escape from a critical reactor from the following facts. When this reactor is critical, per unit time:
   1. Every thermal fission produces on average 2.4331 fast neutrons
   2. 25 fast neutrons are produced by fast fissions
   3. The ratio of the fission and absorption macroscopic cross sections for the fuel is 0.4835
   4. 50 thermal neutrons are absorbed elsewhere than in the fuel
   5. 20 thermal neutrons escape from the reactor
   6. 100 non-thermal neutrons are absorbed in the reactor.

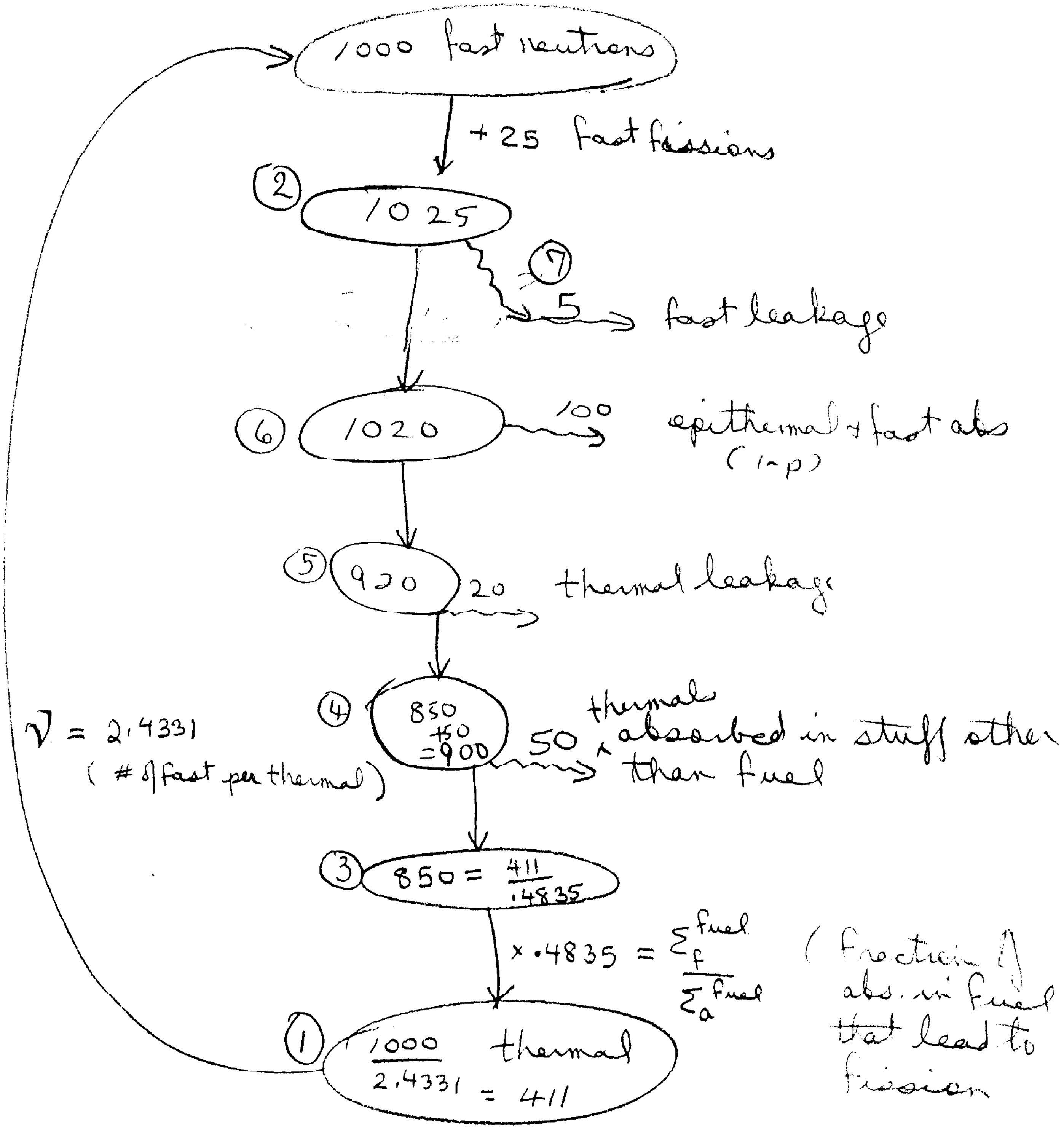


2. [35 marks] The figure shows xenon concentration in the MNR core as a function of time after startup, determined from steady state control rod positions. Each startup follows a shutdown of 8 hours except Tuesday, where the shutdown was 80 hours.
a) What is happening on Tuesday?
b) What is happening on the other days?
c) Some three months after a fuel change, startup during the latter part of the week becomes increasingly difficult. Why?
Explain with the use of equations where appropriate.



3. [30 marks] Write down the multi-group diffusion equations for the following case: steady state, 3 groups, no upscatter, fission neutrons born only in the fastest energy group, fission only occurs in the lowest energy group.

- END -


1000 fast neutrons
+25 fast fissions
② 1025
⑦ 5 fast leakage
⑥ 1020
100 epithermal + fast abs
(1-p)
⑤ 920
20 thermal leakage
④ 850 +50 =900
50 thermals absorbed in stuff other than fuel
③ 850 = 411/.4835
× .4835 = Σ_f^fuel / Σ_a^fuel
(fraction of abs. in fuel ~~that~~ lead to fission
① 1000/2.4331 = 411 thermal
ν = 2.4331
(# of fast per thermal)

① If have 1000 fast neutrons & 2.4331 fast are produced for every thermal fission, then must have $\frac{1000}{2.4331}$ thermal fissions

② Add 25 fast neutrons due to fast fission.

③ Need $\frac{411}{.4835}$ thermal neutrons abs. in fuel if only .4835 (as a fraction) ~~lead~~ to fission.
$= 850$

④ 50 lost in non-fuel abs. ∴ must have had $850 + 50 = 900$ thermals.

⑤ Further 20 thermal leak ∴ must have had 920 $(= 900 + 20)$ ~~before that~~

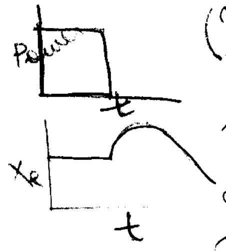
⑥ 100 non-thermals are abs. ∴ must have had 1020 before that loss.

⑦ ~~Deduce that~~ 5 must have leaked out as fast neutrons.

2. a) On Tuesday, the reactor is being started up after an 80 hr. shutdown. This is sufficient time for the xenon to have decayed away. So on startup, there is little Xe. During the day, the xenon builds up in the core and will eventually saturate as the Xe produced by fissioning & I decay equals the Xe burned off by neutron absorption, ie

$$\frac{dx}{dt} = \gamma_x \Sigma_f \phi + \underbrace{\lambda_I I - \lambda_x X - \sigma_a^x \phi X}_{\text{small initially but grows over time.}}$$

b) On the other days, there is initially significant amounts of Xe since it rises after shutdown $[\phi = 0, (\lambda_I I - \lambda_x X) > 0, \therefore \frac{dx}{dt} > 0]$. When the reactor is started up (ϕ increases), there is a net loss of Xe initially due to the $-\sigma_a^x \phi X$ burnoff term. But I production also increases as in:



$$\frac{dI}{dt} = \gamma_I \Sigma_f \phi - \lambda_I I.$$

This I later decays to Xe & thus the Xe level rises to some saturation value. Note ~~that~~ saturation Xe levels were not reached by the end of Tuesday since the Wed. max peak < Thurs/Fri.

c) After 3 months, k of the core has gone down due to fuel burnup. The core now has limited poison override capability to counter the 20 mk or so Xe poison that builds up overnight. Also, at constant power, $P = w_f \Sigma_f \phi$, $\Sigma_f \downarrow \therefore \phi \uparrow \therefore X_\infty \uparrow \therefore \Sigma_{poison} \uparrow$ as fuel is burned up.

3. $$\frac{1}{v_g}\frac{\partial \phi_g}{\partial t} = \nabla\cdot D_g \nabla \phi_g - \Sigma_{ag}\phi_g - \Sigma_{sg}\phi_g + \sum_{g'=1}^{G} \Sigma_{sg'g}\,\phi_{g'} + \chi_g \sum_{g'=1}^{G} \nu_{g'}\Sigma_{fg'}\,\phi_{g'}$$

$G = 3$

$\chi_2 = \chi_3 = 0 \;,\; \chi_1 = 1$ ⇐ all fission neutrons born in Group 1

$\nu_1\Sigma_{f_1} = \nu_2\Sigma_{f_2} = 0 \;,\; \nu_3\Sigma_{f_3} \neq 0$ ⇐ fission caused by thermal neutrons only.

$\Sigma_{sg'g} = 0$ for $g' > g$ ⇐ no upscatter.

$\frac{\partial \phi_g}{\partial t} = 0$ ⇐ steady state

Define Σ_{rg} (removal) $\equiv \Sigma_{ag} + \Sigma_{sg} - \Sigma_{sgg}$

i.e.

$$\begin{pmatrix} -\nabla\cdot D_1\nabla + \Sigma_{r_1} & 0 & 0 \\ -\Sigma_{s12} & -\nabla\cdot D_2\nabla + \Sigma_{r_2} & 0 \\ -\Sigma_{s13} & -\Sigma_{s23} & -\nabla\cdot D_3\nabla + \Sigma_{r_3} \end{pmatrix} \begin{pmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \end{pmatrix}$$

$$= \begin{pmatrix} 0 & 0 & \nu_3\Sigma_{f3} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \end{pmatrix}$$

We can write the scattering terms in a number of ways:

$$g=1: \quad 0 = \nabla\cdot D_1\nabla\phi_1 \overbrace{- \Sigma_{a_1}\phi_1 - \Sigma_{s_1}\phi_1 + \Sigma_{s_{11}}\phi_1}^{\equiv\, -\Sigma_{r_1}\phi_1} + \cancelto{0}{\Sigma_{s_{21}}}\phi_2 + \cancelto{0}{\Sigma_{s_{31}}}\phi_3 + \nu_3\Sigma_{f_3}\phi_3$$

these cancel

$$\Sigma_{s_1} = \left(\Sigma_{s_{11}} + \Sigma_{s_{12}} + \Sigma_{s_{13}}\right)$$

$$g=2: \quad 0 = \nabla\cdot D_2\nabla\phi_2 - \Sigma_{a_2}\phi_2 - \Sigma_{s_2}\phi_2 + \Sigma_{s_{12}}\phi_1 + \Sigma_{s_{22}}\phi_2 + \cancelto{0}{\Sigma_{s_{32}}}\phi_3$$

$$\equiv -\Sigma_{r_2}\phi_2$$

these cancel

$$\Sigma_{s_2} = \left(\cancelto{0}{\Sigma_{s_{21}}} + \Sigma_{s_{22}} + \Sigma_{s_{23}}\right)$$

$$= \nabla\cdot D_2\nabla\phi_2 - \Sigma_{a_2}\phi_2 - \Sigma_{s_{23}}\phi_2 + \Sigma_{s_{12}}\phi_1$$

$$\equiv -\Sigma_{r_3}\phi_3$$

$$g=3: \quad 0 = \nabla\cdot D_3\nabla\phi_3 - \Sigma_{a_3}\phi_3 - \Sigma_{s_3}\phi_3 + \Sigma_{s_{13}}\phi_1 + \Sigma_{23}\phi_2 + \Sigma_{33}\phi_3$$

these cancel

$$\Sigma_{s_3} = \left(\cancelto{0}{\Sigma_{s_{31}}} + \cancelto{0}{\Sigma_{s_{32}}} + \Sigma_{s_{33}}\right)$$