ENGINEERING PHYSICS 4D3/6D3

DAY CLASS — Dr. Wm. Garland

DURATION: 50 minutes

McMASTER UNIVERSITY MIDTERM EXAMINATION — November 9, 2000

Special Instructions:

1. Closed Book. All calculators and up to 8 single sided 8 ½" by 11" crib sheets are permitted.
2. Do all questions.
3. The value of each question is as indicated. TOTAL Value: 30 marks

THIS EXAMINATION PAPER INCLUDES 1 PAGE AND 3 QUESTIONS. YOU ARE RESPONSIBLE FOR ENSURING THAT YOUR COPY OF THE PAPER IS COMPLETE. BRING ANY DISCREPANCY TO THE ATTENTION OF YOUR INVIGILATOR.

1. [10 marks] Given the material properties for a homogeneous fuel water mixture, ie specified Σ_a, $\nu\Sigma_f$ and D, a **cubic** reactor is proposed. It is calculated (using the simple one speed diffusion model) to go critical at dimensions H x H x H, where H is the side length.
   a. What is the one speed steady state diffusion equation for the cubic reactor?
   b. Given a flux shape of
   $$\phi(x,y,z) = \phi_0 \cos(\tfrac{\pi x}{H})\cos(\tfrac{\pi y}{H})\cos(\tfrac{\pi z}{H})$$
   what is the criticality condition?
   c. What are the geometric and material bucklings? Are they equal?

2. [10 marks] Given the material properties for a homogeneous fuel water mixture, ie specified Σ_a, $\nu\Sigma_f$ and D, a **cylindrical** reactor is proposed. It is calculated (using the simple one speed diffusion model) to go critical at dimensions R x H, where R is the radius and H is the height. The cylindrical reactor has the optimal shape that you determined in a recent assignment:
   $$\frac{\text{Radius}}{\text{Height}} = \frac{2.405}{\sqrt{2}\pi}$$
   a. What is the one speed steady state diffusion equation for the cylindrical reactor?
   b. Given a flux shape of
   $$\phi(r,z) = \phi_0 J_0(\tfrac{\nu_0 r}{H})\cos(\tfrac{\pi z}{H}), \text{where } \nu_0 = 2.405$$
   what is the criticality condition?
   c. What are the geometric and material bucklings? Are they equal?

3. [10 marks] For the two reactor models presented in the previous questions (one cubic in shape, the other cylindrical in shape):
   a. Compare the bucklings. Are they equal?
   b. Does the cubic reactor require more or less fuel water mixture than a cylindrical shaped reactor for criticality?
   c. If both reactors were operating at the same total power level, compare the flux levels. Are they equal?
   d. Compare the rates at which both reactors burn up fuel. Are they equal?

---The End---

Midterm, Nov 10, 2000

Solution

1. a) $\frac{1}{v}\cancel{\frac{\partial \phi}{\partial t}}_0 = \nabla \cdot D \nabla \phi + (\nu \Sigma_f - \Sigma_a)\phi \Rightarrow D\nabla^2 \phi + (\nu\Sigma_f - \Sigma_a)\phi = 0$

D ← not a function of space

$$\Rightarrow \nabla^2 \phi + \left(\frac{\nu\Sigma_f - \Sigma_a}{D}\right)\phi = 0 \quad \Rightarrow \nabla^2\phi + B^2\phi = 0 \quad (1)$$

B^2 ← Buckling

where $\nabla^2\phi = \frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2} + \frac{\partial^2\phi}{\partial z^2}$

b) $\phi = \phi_0 \cos\left(\frac{\pi x}{H}\right)\cos\left(\frac{\pi y}{H}\right)\cos\left(\frac{\pi z}{H}\right)$

Sub into (1): $-\left(\frac{\pi}{H}\right)^2 - \left(\frac{\pi}{H}\right)^2 - \left(\frac{\pi}{H}\right)^2 + B^2 = 0$

$\therefore B^2 = 3\left(\frac{\pi}{H}\right)^2 = \frac{\nu\Sigma_f - \Sigma_a}{D}$ = criticality condition

c) B_g^2 = geometric buckling $= \left(\frac{3\pi}{H}\right)^2$

B_m^2 = material buckling $= \frac{\nu\Sigma_f - \Sigma_a}{D}$

} equal at criticality

2. a) as per 1. above except $\nabla^2\phi = \frac{1}{r}\frac{\partial}{\partial r} r \frac{\partial\phi}{\partial r} + \frac{\partial^2\phi}{\partial z^2}$

$\nabla^2\phi + B^2\phi = 0$ as before

$B^2 = \frac{\nu\Sigma_f - \Sigma_a}{D}$ as before.

b) $\phi = \phi_0 J_0\left(\frac{\nu_0 r}{H}\right)\cos\left(\frac{\pi z}{H}\right)$, $\nu_0 = 2.405$

Sub. in as before: $-\left(\frac{\nu_0}{R}\right)^2 - \left(\frac{\pi}{H}\right)^2 + B^2 = 0$

$$\therefore B^2 = \left(\frac{\nu_0}{R}\right)^2 + \left(\frac{\pi}{H}\right)^2 = +\left(\frac{\nu_0}{\frac{\nu_0 H}{\sqrt{2}\pi}}\right)^2 + \left(\frac{\pi}{H}\right)^2 = +\left(\frac{\sqrt{2}\pi}{H}\right)^2 + \left(\frac{\pi}{H}\right)^2 = 3\left(\frac{\pi}{H}\right)^2$$

$= \frac{\nu\Sigma_f - \Sigma_a}{D}$ ⟸ criticality condition

c. $B_g^2 = \left(\frac{\nu_0}{R}\right)^2 + \left(\frac{\pi}{H}\right)^2 = 3\left(\frac{\pi}{H}\right)^2$
$B_m^2 = \frac{\nu\Sigma_f - \Sigma_a}{D}$
} equal at criticality

3. Cubic reactor vs cylindrical reactor

a) Both are governed by $\nabla^2\phi + B^2\phi = 0$
Both are critical. Both have same B_m^2.
$B_m^2 = B_g^2$ ∴ all bucklings are equal.

b) Volume of cubic reactor = H^3
" " cylindrical reactor = $\pi R^2 H$
$= \pi \left(\frac{2.405}{\sqrt{2}\pi}\right)^2 H^3 = \frac{(2.405)^2}{2\pi} H^3$
$= 0.92 H^3$
∴ cylindrical reactor is smaller.

c) Power = $\underbrace{W_f \nu \Sigma_f}_{\text{constant}} \phi \times \text{Volume}$
∴ flux in cylindrical reactor is higher than in cubic reactor since Volume of cylinder < Volume of cube

d) Fuel burnup rate $\propto \frac{\text{interaction rate}}{\text{Volume}} \times$ volume.
$= \Sigma_f \phi \times \text{Volume} \propto$ Power
∴ fuel burnup rate = the same for both reactors.

— end —