Solution

ENGINEERING PHYSICS 4D3/6D3

DAY CLASS — Dr. Wm. Garland
DURATION: 50 minutes
McMASTER UNIVERSITY MIDTERM EXAMINATION — November 17, 1998

Special Instructions:

1. Closed Book. All calculators and up to 6 single sided 8 ½" by 11" crib sheets are permitted.
2. Do all questions. Place your answers on the exam sheets; use additional pages if necessary.
3. The value of each part is as indicated. TOTAL Value: 100 marks

THIS EXAMINATION PAPER INCLUDES 8 PAGES AND 1 MULTI-PART QUESTION. YOU ARE RESPONSIBLE FOR ENSURING THAT YOUR COPY OF THE PAPER IS COMPLETE. BRING ANY DISCREPANCY TO THE ATTENTION OF YOUR INVIGILATOR.

1. By answering the specific questions given on the following pages, <u>outline</u> a computer program to solve the two group neutron <u>space-time</u> diffusion equations and supporting equations in a 3-dimensional reactor like the McMaster Nuclear Reactor composed of fuel assemblies (containing fuel and water), control assemblies (containing a movable solid control rod and water) and empty assemblies (containing only water). The core is constructed of a 9x9 array of assemblies as shown in the x-y (ie top) view below. Divide the z (vertical) axis into 9 cells. When the control rods are inserted (motion is in the z direction), water is displaced. Assume that the cell properties are homogenized within each cell but that the reactor is heterogeneous overall. Consider flux, delayed precursors, control, poisons and fuel depletion aspects.

North

West

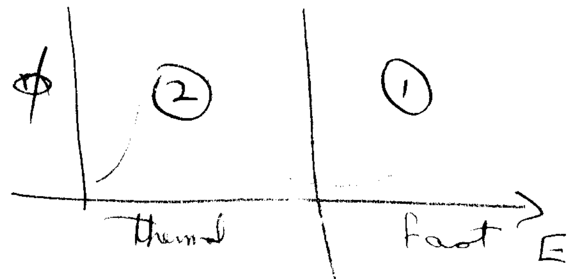
	X AXIS, index i								
Y AXIS index j	Water	Water	Water	Water	Water	Water	Water	Water	Water
	Water	Water	Water	Water	Water	Water	Water	Water	Water
	Water	Water	Fuel	Fuel	Fuel	Fuel	Fuel	Water	Water
	Water	Water	Fuel	Control	Fuel	Control	Fuel	Water	Water
	Water	Water	Fuel	Fuel	Water	Fuel	Fuel	Water	Water
	Water	Water	Fuel	Control	Fuel	Control	Fuel	Water	Water
	Water	Water	Fuel	Fuel	Fuel	Fuel	Fuel	Water	Water
	Water	Water	Water	Water	Water	Water	Water	Water	Water
	Water	Water	Water	Water	Water	Water	Water	Water	Water

East

South

a) (20 marks) For the flux, what are the governing equations, boundary and initial conditions, and finite difference equations?

Choose the two groups as fast & thermal neutron energy groups.



$$\frac{1}{v_1}\frac{\partial \phi_1}{\partial t} = \nabla\cdot D_1 \nabla \phi_1 - \Sigma_{a_1}\phi_1 - \Sigma_{s_1}\phi_1 + \Sigma_{s_{11}}\phi_1 + \Sigma_{s_{21}}\phi_2$$

$$+ \underbrace{\chi_1^c}_{\approx 1} \sum_{\ell=1}^{6} \lambda_\ell C_\ell + (1-\beta)\underbrace{\chi_1^p}_{\approx 1}\left(\nu_1 \Sigma_{f_1}\phi_1 + \nu_2 \Sigma_{f_2}\phi_2\right)$$

can ignore upscatter

can assume all fission neutrons are fast.

can assume all delayed neutrons are born in fast group

$$\frac{1}{v_2}\frac{\partial \phi_2}{\partial t} = \nabla\cdot D_2 \nabla \phi_2 - \Sigma_{a_2}\phi_2 - \Sigma_{s_2}\phi_2 + \Sigma_{s_{12}}\phi_1 + \Sigma_{s_{22}}\phi_2$$

$$+ \chi_2^c \sum \lambda_i C_i + (1-\beta)\underbrace{\chi_2}_{\approx 0}\left(\nu_1 \Sigma_{f_1}\phi_1 + \nu_2 \Sigma_{f_2}\phi_2\right)$$

small compared to

This can be rewritten:

$$\frac{1}{v_1}\frac{\partial \phi_1}{\partial t} = \nabla\cdot D_1 \nabla \phi_1 - \Sigma_{R_1}\phi_1 + \nu_2 \Sigma_{f_2}\phi_2 + \sum_{\ell=1}^{6}\lambda_\ell C_\ell, \quad \phi = \phi(x,y,z,t)$$

$$\frac{1}{v_2}\frac{\partial \phi_2}{\partial t} = \nabla\cdot D_2 \nabla \phi_2 - \Sigma_{R_2}\phi_2 + \Sigma_{s_{12}}\phi_2$$

Where

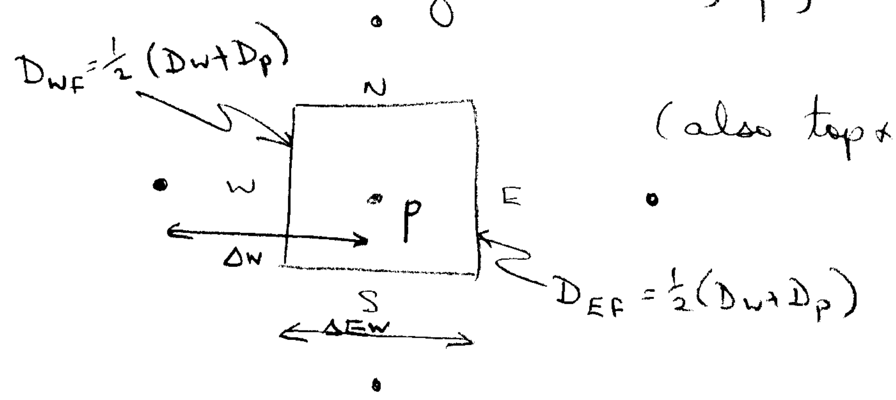
$$\nabla\cdot D\nabla\phi = \frac{\partial}{\partial x} D\frac{\partial \phi}{\partial x} + \frac{\partial}{\partial y} D\frac{\partial \phi}{\partial y} + \frac{\partial}{\partial z} D\frac{\partial \phi}{\partial z}$$

B.C: $\phi = 0$ at extrapolated boundaries.

I.C: ϕ = known at $t=0$.

FDE

As per notes, for a given cell, P,



(also top & bottom neighbours

$$\frac{\partial}{\partial x} D \frac{\partial \phi}{\partial x} = \frac{\dfrac{D_{EF}(\phi_E - \phi_P)}{\Delta E} - D_{WF}\dfrac{(\phi_P - \phi_W)}{\Delta W}}{\Delta_{EW}}$$

~ for north-south & top-bottom.

$$\therefore \frac{1}{v_1} \frac{\phi_{1P}^{t+\Delta t} - \phi_{1P}^{t}}{\Delta t} = \frac{D_{1WF}\,\phi_{1W}^{t+\Delta t}}{\Delta_W \Delta_{EW}} - \left(\frac{D_{1EF}}{\Delta E} + \frac{D_{1WF}}{\Delta_W}\right)\frac{\phi_{1P}^{t+\Delta t}}{\Delta_{EW}} + \frac{D_{1EF}}{\Delta_E}\frac{\phi_{1E}^{t}}{\Delta_{EW}}$$

$$+ \frac{D_{1NF}\,\phi_{1N}^{t+\Delta t}}{\Delta_N \Delta_{NS}} - \left(\frac{D_{1NF}}{\Delta_N} + \frac{D_{1SF}}{\Delta_S}\right)\frac{\phi_{1P}^{t+\Delta t}}{\Delta_{NS}} + \frac{D_{1SF}}{\Delta_S}\frac{\phi_{1S}^{t}}{\Delta_{NS}}$$

$$+ \frac{D_{1TF}\,\phi_{1T}^{t+\Delta t}}{\Delta_T \Delta_{TB}} - \left(\frac{D_{1TF}}{\Delta_T} + \frac{D_{1BF}}{\Delta_B}\right)\frac{\phi_{1P}^{t+\Delta t}}{\Delta_{TP}} + \frac{D_{1BF}}{\Delta_B}\frac{\phi_{1B}^{t}}{\Delta_{TB}}$$

$$- \Sigma_{R1}\,\phi_{1P}^{t+\Delta t} + \nu_2 \Sigma_{f2}\,\phi_{2P}^{t} + \sum_{l=1}^{6} \lambda_l C_{lP}^{t}$$

> Note: Obviously, in a non-fuelled cell, $\Sigma_f = 0$, $C_l = 0$. All coefficients (D, Σ) are locally evaluated.

$$\frac{1}{v_2} \frac{\phi_{2P}^{t+\Delta t} - \phi_{2P}^{t}}{\Delta t} = \Big(\text{ same as above for } \nabla \cdot D \nabla \phi \text{ term but with subscript 2 instead of 1}\Big)$$

$$- \Sigma_{R2}\,\phi_{2P}^{t+\Delta t} + \Sigma_{S12}\,\phi_{1P}^{t+\Delta t}$$

Note the assignment of the superscripts t & $t + \Delta t$. I have assumed that the numerical sweeps will be done west to east, north to south, top to bottom for group 1 first, & then group 2. That means that in the group 1 eqn, ϕ_2 has not yet been updated, whereas in the group 2 eqn, ϕ_1 has been updated. Any point north, west and above the point P will have updated fluxes available. As always, just use the latest fluxes available.

b) (10 marks) For the delayed precursors, what are the governing equations, initial conditions, and finite difference equations?

For the regions containing fuel:

$$\frac{\partial C_\ell}{\partial t} = -\lambda_\ell C_\ell + \beta_\ell \underbrace{\sum_{g'=1}^{G} \nu_{g'} \Sigma_{f_{g'}} \phi_{g'}}_{\text{all the fission neutrons}}$$

$$\simeq \nu_2 \Sigma_{f_2} \phi_2$$

I.C: $C_\ell(\underline{r}, t=0) = \text{known} = 0$ fresh fuel

$$= \frac{\beta_\ell \nu_2 \Sigma_{f_2} \phi_2(\underline{r}, 0)}{\lambda_\ell}$$ equilibrium fuel.

FDE

$$\frac{C_{\ell_p}^{t+\Delta t} - C_{\ell_p}^{t}}{\Delta t} = -\lambda_\ell C_{\ell_p}^{t+\Delta t} + \beta_\ell \nu_2 \Sigma_{f_2} \phi_{2_p}^{t+\Delta t}$$

where p = cell index (ie given x, y, z location)
(or " i, j, k index)

Do the C_ℓ sweep after the flux sweep since ϕ is changing much more rapidly than C.

c) (10 marks) For the poisons, what are the governing equations, initial conditions, and finite difference equations? For fuelled regions only:

$$\frac{\partial I}{\partial t} = \gamma_I \sum_{g'=1}^{G} \Sigma_{fg'} \phi_{g'} - \lambda_I I$$

$$\simeq \gamma_I \Sigma_{F_2} \phi_2 - \lambda_I I$$ since bulk of fissions are due to thermal flux.

$$\frac{\partial X}{\partial t} = \gamma_X \Sigma_{F_2} \phi_2 + \lambda_I I - \lambda_X X - \Big(\sum_{g'=1}^{G} \sigma_{ag'} \phi_{g'}\Big) X$$

$$\sum_{g'=1}^{G} \sigma_{ag'} \phi_{g'} \simeq \sigma_{a_2} \phi_2$$

IC: $I(\underline{r},0)$ = known (often 0)

$X(\underline{r},0)$ = " " "

FDE

$$\frac{I_p^{t+\Delta t} - I_p^{t}}{\Delta t} = \gamma_I \Sigma_{F_{2p}} \phi_{2p}^{t+\Delta t} - \lambda_I I_p^{t+\Delta t}$$

$$\frac{X_p^{t+\Delta t} - X_p^{t}}{\Delta t} = \gamma_X \Sigma_{F_{2p}} \phi_{2p}^{t+\Delta t} + \lambda_I I_p^{t+\Delta t} - \lambda_X X_p^{t+\Delta t} - \sigma_{a_2} \phi_{2p}^{t+\Delta t}$$

Note the usage of t & $(t+\Delta t)$ subscripts. Do the I eqn after the flux + precursor eqn's & before the X eqn.

d) (10 marks) For the fuel depletion, what are the governing equations, initial conditions, and finite difference equations? For fuelled regions only.

$$\frac{\partial N_f}{\partial t} = -N_f \sum_{g'=1}^{G} \sigma_{a_{g'}}^{f} \phi_{g'} \simeq -N_f \sigma_{a_2}^{f} \phi_2$$

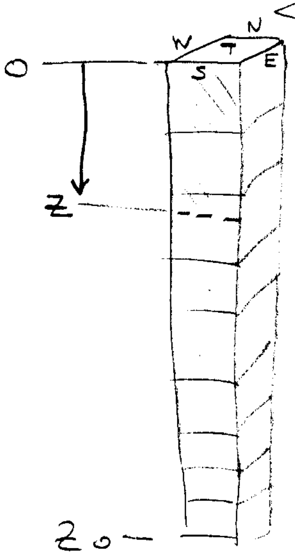
I.C: $N_f(\underline{r}, 0) = \text{known}$ (initial fuel load)

F.D.E: $$\frac{N_{f_p}^{t+\Delta t} - N_{f_p}^{t}}{\Delta t} = -N_{f_p}^{t+\Delta t} \sigma_{a_{2p}}^{f} \phi_{2p}^{t+\Delta t}$$

Do the fuel sweep after the poison calculation since N_f is slowly varying.

e) (10 marks) Devise a simple control mechanism to take the reactor from a low initial flux to full power. Assume all 4 rods move together.

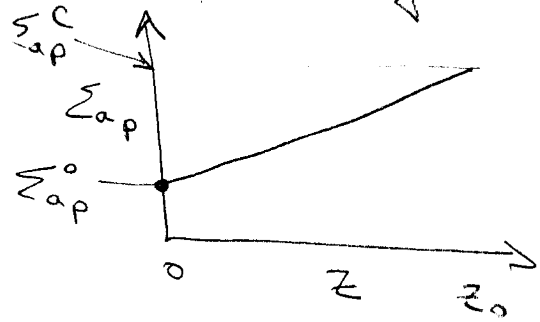
Σ_{a_1} & Σ_{a_2} for the cells containing the control rods is a function of the rod insertion (z):



$\Sigma_{ap} = \Sigma_{ap}^{c}$ if rod completely penetrates the cell

p = point or index of cell in question.

$\Sigma_{ap} = \Sigma_{ap}^{o}$ if rod not in cell at all.



$$\Sigma_{ap} = \Sigma_{ap}^{o} + \left(\Sigma_{ap}^{c} - \Sigma_{ap}^{o}\right)\left(\frac{z}{z_0}\right)$$

Now we need an algorithm to give the desired z for the given reactor situation.

Measure ϕ (say ϕ_2) at point m. Compare that to the desired ϕ set-point (ϕ_{sp})

$$\frac{\Delta z}{\Delta t} = a\left(\phi_m - \underbrace{\phi_{sp}}_{fn(t)}\right) + b\left(\frac{\phi_m^{t+\Delta t} - \phi^{t}}{\Delta t}\right)$$

a & b are +ve constants to be determined by detailed controller design.

Some measure of the rate of change of ϕ

At each Δt, calculate Δz & $z \left(= z^t + \frac{\Delta z}{\Delta t}\cdot\Delta t\right)$

$$\Delta z = \frac{\Delta z}{\Delta t}\cdot\Delta t$$

D:\TEACH\EP4D3\mid2-98.wp8 November 16, 1998

$\phi_{sp}(t) = \phi_{Low} + \left(\text{Rate of desired increase}\right) \times t$, ϕ_{sp} is clipped at $\phi_{\text{Full power}}$.

f) (20 marks) Illustrate the algorithm (using a flow chart and supporting dialogue) to calculate the flux space-time transient and controller action during a startup of the reactor from 1 % full power to 100 % power at a nominal 0.1 % full power a second. Show the overall solution and how the equations interact. Do you need to consider precursors, poisons and fuel depletion? If so, do so. Sketch the flux, precursor, poison and fuel concentration amplitudes, the control rod position and the reactivity over time that you might expect as a solution. **Don't get hung up on details.**

Ignore poisons & fuel depletion. Startup will only take 1000 seconds or so, ∴ poisons + fuel won't change significantly in that time.

Initialize I.C & B.C.
- set controller position + set point
- set flux at low power equivalent
- set cell properties

↓

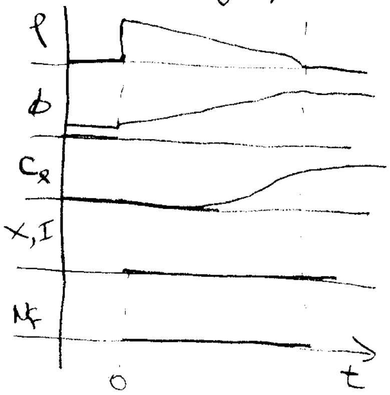
→ Update control cell parameters (Σ, D)

↓

advance Δt

Solve for $\phi^{t+\Delta t}$ for all points
nested sweep
Groups
Top to bottom
North to south
West to east

Don't bother to "converge" ϕ.
Don't normalize ϕ



Sweep C_g eqn's in space using latest fluxes

Update ϕ_{sp}
Calc Δz + z (new rod position)

↓

t > t fin

↓

g) (20 marks) Given that 100 % full power has been reached, illustrate the algorithm (using a flow chart and supporting dialogue) to advance the flux, control rod position, precursors, poisons and fuel solutions in time. Show the overall solution and how the equations interact. **Don't get hung up on details.**

Once you have reached 100% FP, we can keep on calculating ϕ & C but we could, to save on computational effort, use steady state versions. In any case, we now need to add in the poisons & fuel.

Setup

↓

Calc cell properties

↓

Update ϕ & C (either as before or via a steady state version)

↓

Solve for $I^{t+\Delta t}$, $X^{t+\Delta t}$

↓

Solve for $N_F^{t+\Delta t}$

↓

Calc ΔZ & Z

↓ $t > t_{fin}$

You can use the transient version but alter the group velocities to be low; that way you can take bigger Δt's at no loss to the steady state solution in ϕ.

You can save on computation time by noting that N_F is changing very slowly cf I & Xe so that the N_F update can be done infrequently, say every hour. Similarly I & Xe are changing slowly wrt ϕ so do it only every minute or so.