Final 99 Solutions

1. (a) $$\frac{dN_A}{dt} = -\lambda N_A - \sigma\phi N_A + R$$
$$\equiv -\lambda' N_A + R$$

(b) To solve, let $N_A = X + C$

$$\therefore \frac{d(X+C)}{dt} = -\lambda'(X+C) + R$$

ie $$\frac{dX}{dt} = -\lambda' X - \lambda' C + R$$

choose C to be $= R/\lambda'$ $\Rightarrow \frac{dX}{dt} = -\lambda' X$

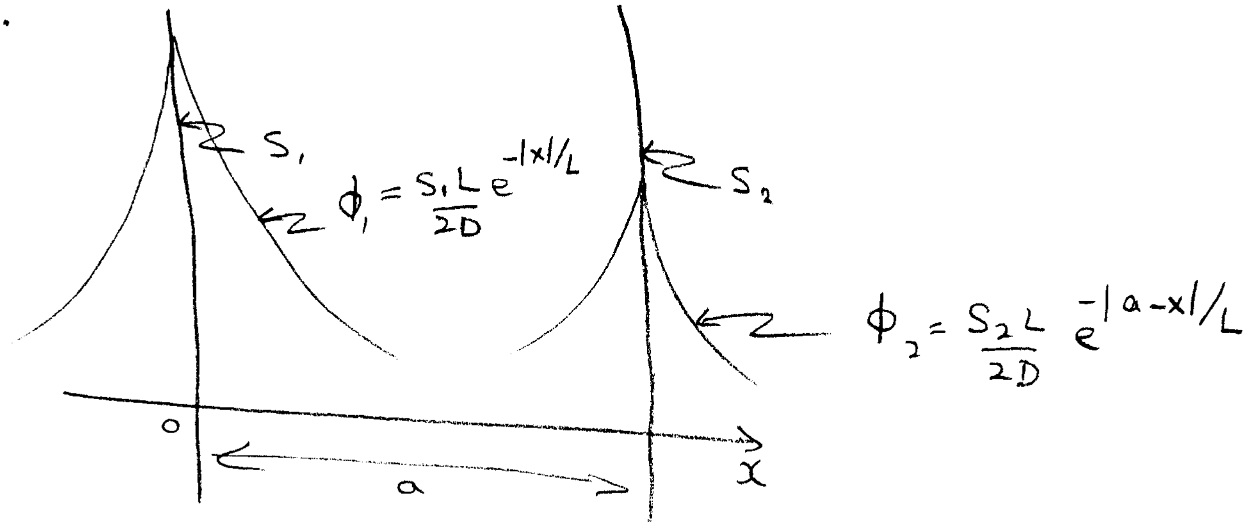
$$\Rightarrow X = X(0)\, e^{-\lambda' t} + \text{const.}$$

$$\therefore N_A(t) - R/\lambda' = \left(N_A(0) - R/\lambda'\right) e^{-\lambda' t} + \cancel{\text{const}}^{=0}$$

$$\therefore N_A(t) = N_A(0)\, e^{-\lambda' t} + \frac{R}{\lambda'}\left(1 - e^{-\lambda' t}\right)$$

$$\therefore N_A(t) = N_A(0)\, e^{-(\lambda+\sigma\phi)t} + \frac{R}{(\lambda+\sigma\phi)}\left(1 - e^{-(\lambda+\sigma\phi)t}\right)$$

2.



(a) $0 = S - \Sigma_a \phi + D\nabla^2 \phi$

$$\Rightarrow \frac{d^2\phi}{dx^2} - \frac{1}{L^2}\phi = \frac{S}{D}$$

$$\Rightarrow \phi = \frac{SL}{2D} e^{-|x|/L} \text{ when } S \text{ is at origin.}$$

$$\therefore \underset{\text{(superposition)}}{\phi = \phi_1 + \phi_2} = \frac{S_1 L}{2D} e^{-|x|/L} + \frac{S_2 L}{2D} e^{-|a-x|/L}$$

(b) $$J \simeq -D\frac{d\phi}{dx} = \frac{S_1}{2} e^{-|x|/L} - \frac{S_2}{2} e^{-|a-x|/L}$$

for $x \in (0, a)$.

$$J \simeq \frac{S_1}{2} e^{-|x|/L} + \frac{S_2}{2} e^{-|a-x|/L}$$

for $x > a$

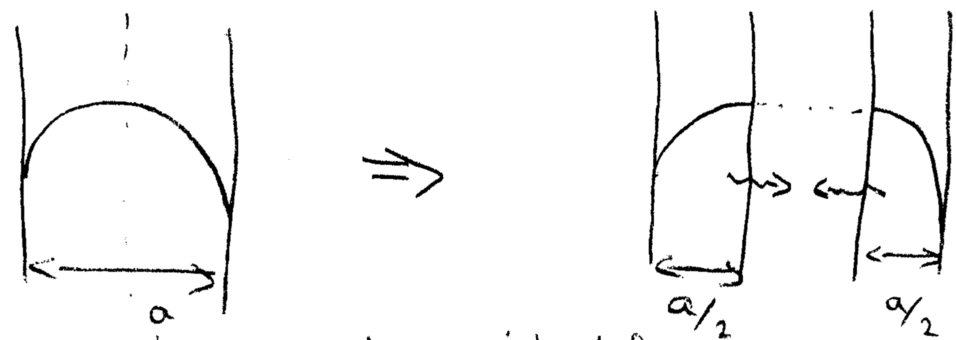
$$J \simeq -\frac{S_1}{2} e^{-|x|/L} - \frac{S_2}{2} e^{-|a-x|/L}$$

for $x < 0$

ie current is the vector sum of sources, reduced by the exponential dropoff due to diffusion.

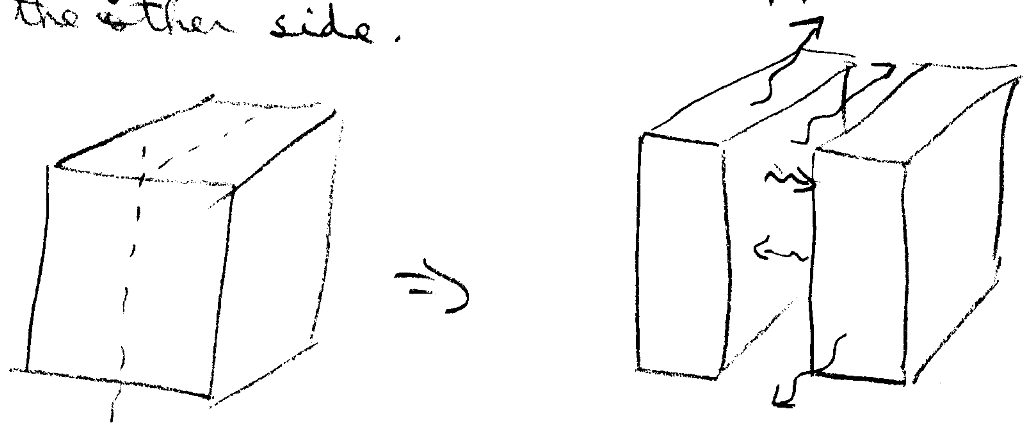
3.

(a)



The reactor remains critical since any neutron leaking from an interior wall appears in the other side.

(b)



The reactor goes subcritical because now more neutrons are non-reentrant.

#4. 2 group, homogeneous.

In steady state:

$$-D_1 \frac{\partial^2 \phi_1}{\partial x^2} \overbrace{+ \Sigma_{a_1}\phi_1 + \Sigma_{s_1}\phi_1 - \Sigma_{s_{11}}\phi_1}^{\equiv + \Sigma_{R1}\phi_1} - \Sigma_{s_{21}}\phi_2$$

$$-\chi_1 (\nu_1 \Sigma_{f_1} \phi_1 + \nu_2 \Sigma_{f_2} \phi_2) = 0$$

$$-D_2 \frac{\partial^2 \phi_2}{\partial x^2} \overbrace{+ \Sigma_{a_2}\phi_2 + \Sigma_{s_2}\phi_2 - \Sigma_{s_{22}}\phi_2}^{\equiv + \Sigma_{R2}\phi_2} - \Sigma_{s_{12}}\phi_1$$

$$-\chi_2 (\nu_1 \Sigma_{f_1} \phi_1 + \nu_2 \Sigma_{f_2} \phi_2) = 0$$

For bare reactor, ϕ_1 & ϕ_2 have same shape (cosine).

Usual B.C: $\phi(\pm a/2) = 0$ (at extrapolated distance)

Let $\phi_1(x) = \phi_1 \psi(x)$, $\phi_2(x) = \phi_2 \psi(x)$

$$+\nabla^2 \psi + B^2 \psi = 0$$

$$\therefore \begin{bmatrix} D_1 B^2 + \Sigma_{R_1} - \chi_1 \nu_1 \Sigma_{f_1} & -\Sigma_{s_{21}} - \chi_1 \nu_2 \Sigma_{f_2} \\ -\Sigma_{s_{12}} - \chi_2 \nu_1 \Sigma_{f_1} & D_2 B^2 + \Sigma_{R_2} - \chi_2 \nu_2 \Sigma_{f_2} \end{bmatrix} \begin{bmatrix} \phi_1 \\ \phi_2 \end{bmatrix} = 0$$

ie $\underline{\underline{A}}\,\phi = 0$

which only has a non-trivial solution if $|\underline{\underline{A}}| = 0$

$$\therefore \boxed{(D_1 B^2 + \Sigma_{R_1} - \chi_1 \nu_1 \Sigma_{f_1})(D_2 B^2 + \Sigma_{R_2} - \chi_2 \nu_2 \Sigma_{f_2}) - (\Sigma_{s_{21}} + \chi_1 \nu_2 \Sigma_{f_2})(\Sigma_{s_{12}} + \chi_2 \nu_1 \Sigma_{f_1}) = 0}$$

criticality condition

Discussion:

For 2 groups, no upscatter is likely (ie $\Sigma_{21} \simeq 0$). Also all fission neutrons are born in the fast group, ie $\chi_2 \simeq 0$, $\chi_1 = 1$. We can also ignore fast fissions usually, ie $\Sigma_{f_1} \simeq 0$ (~3% are fast fissions actually. This is OK given the ± 5% typical errors in measured Σ's.

With these assumptions:

$$(D_1 B^2 + \Sigma_{R_1})(D_2 B^2 + \underbrace{\Sigma_{R_2}}_{= \Sigma_{a_2} \text{ now}}) - \nu_2 \Sigma_{f_2} \Sigma_{S_{12}} = 0$$

is the criticality condition.

#5. (a) If < 4 mk, hard to control. If > 6 mk, too close to prompt critical ($\rho = \beta$).

(b) Calibrate reg. rod by:

- make reactor critical by adjusting the 5 control rods. Reg rod is fully inserted.
- withdraw reg rod slightly (say 1 cm) to initiate a slow power excursion. Wait for fast transients to die out + record long term period ($T = 1/\omega$)
- calc ρ from inhour eqn:

$$\rho = \frac{\omega l}{(1+\omega l)} + \frac{1}{(1+\omega l)} \sum_{i=1}^{N} \frac{\omega \beta_i}{\omega + \lambda_i}$$

to give the worth.

- insert control rods to achieve criticality.
- repeat reg rod withdrawal as above.

(c) Now that the reg rod is calibrated, bring the reactor critical with the reg rod about half inserted.

- ① withdraw one control rod 1 cm or so and note the reg. rod movement required to bring the reactor back to critical. That gives you the mk worth of ~~that~~ 1 cm of the control rod.
- hold that control rod fixed + insert the other 4 control rods slightly to get criticality with the reg rod ~ half inserted.
- repeat ① until rod is withdrawn. Repeat for other control rods.

(d) Yes, they would need to be recalibrated since a rods worth depends on the local flux, ie absorption = $\Sigma_a \phi$. ϕ shape will change over time due to burnup + reconfiguration.

(e) ~~Best~~ locations would be in the high flux areas but spread out so as to minimize flux depressions.

(f) Flux measurements should be away from any control or reg rod because their insertion or removal causes flux distortions.

6.

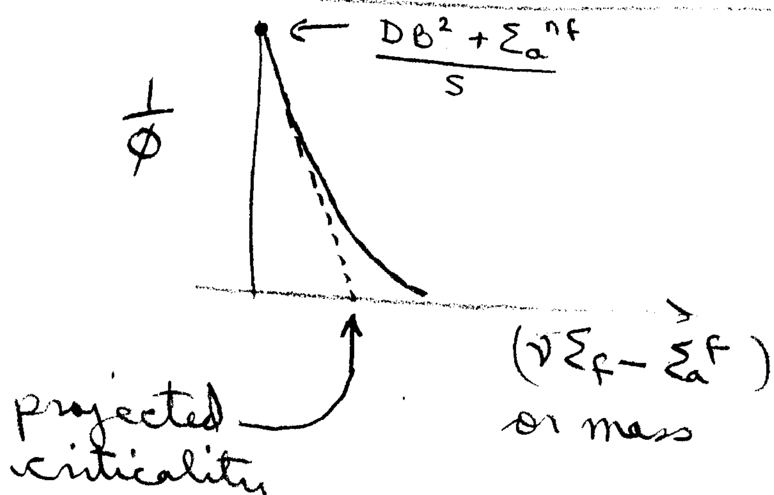
(a) $$\frac{1}{v}\underbrace{\cancel{\frac{\partial \phi}{\partial t}}}_{\text{S.S.}} = D\underbrace{\nabla^2 \phi}_{\simeq -B^2\phi} + (\nu \Sigma_f - \Sigma_a)\phi + \underbrace{\cancel{\sum_i \lambda_i C_i}}_{\text{Can ignore}} + S$$

$$\therefore \quad DB^2\phi + (\Sigma_a - \nu\Sigma_f)\,\phi = S$$

(b) $$\therefore \quad \boxed{\phi = \frac{S}{(DB^2 + \underbrace{\Sigma_a}_{= \Sigma_a^f + \Sigma_a^{nf}} - \nu\Sigma_f)} = \frac{S}{DB^2 + \Sigma_a^{nf} + \Sigma_a^f - \nu\Sigma_f}}$$

proportional to mass of fuel

$\boxed{+\Sigma_a^f - \nu\Sigma_f < 0}$



(c) As the mass of fuel increases, criticality will be approached and $\phi \uparrow$. At criticality it is theoretically unlimited.

According to the equations above, $1/\phi$ vs mass should be linear. But non-linear effects such as poison buildup, increase in core size, finite size S, etc, work to reduce ϕ for a given mass, ie $1/\phi$ is higher than that predicted by the linear dropoff – hence ∴ <u>conservative</u>.

#7

$$\rho(t) = \beta + \frac{\Lambda}{n(t)}\frac{dn}{dt} - \beta\int_0^\infty \left[\lambda e^{-\lambda\tau}\frac{n(t-\tau)}{n(t)}\right] d\tau$$

Since $n(t) = n_0 + at$

$$\rho(t) = \beta + \frac{\Lambda a}{(n_0+at)} - \beta\lambda\int_0^\infty e^{-\lambda\tau} \underbrace{\frac{[n_0 + a(t-\tau)]}{(n_0+at)}}_{= \frac{n_0+at}{n_0+at} - \frac{a\tau}{n_0+at}} d\tau$$

$$\therefore \rho(t) = \beta + \frac{\Lambda a}{n(t)} - \beta\lambda\underbrace{\int_0^\infty e^{-\lambda\tau} d\tau}_{= \frac{1}{\lambda}} + \frac{\beta\lambda a}{n(t)}\underbrace{\int_0^\infty \tau e^{-\lambda\tau} d\tau}_{= \frac{1}{\lambda^2}}$$

$$\therefore \rho(t) = \beta + \frac{\Lambda a}{n(t)} - \beta + \frac{\beta a}{\lambda\, n(t)}$$

$$= \frac{a(\Lambda + \beta/\lambda)}{n(t)}$$

#8.

(a) no upscatter $\therefore \Sigma_{21}=0$

no fast fissions $\therefore \Sigma_{f_1}=0$

no neutrons born in thermal region

$\therefore \chi_2 = 0 \Rightarrow \chi_1 = 1$

$\chi_2^c = 0 \Rightarrow \chi_1^c = 0$

Group 1 (fast)

$$\therefore 0 = \nabla\cdot D_1\nabla\phi_1 - \Sigma_{a_1}\phi_1 - \Sigma_{s_1}\phi_1 + \Sigma_{s_{11}}\phi_1 + \cancelto{0}{\Sigma_{s_{21}}\phi_2} \quad (1)$$

$$+ (1-\beta)\,\nu_2\Sigma_{f_2}\phi_2 + \sum_{i=1}^{N}\lambda_i C_i + S_1 \leftarrow \text{ignore.}$$

Group 2 (thermal)

$$0 = \nabla\cdot D_2\nabla\phi_2 - \Sigma_{a_2}\phi_2 - \Sigma_{\cancel{s}_2}\phi_2 + \Sigma_{s_{12}}\phi_1 + \Sigma_{\cancel{s}_{22}}\phi_2 \quad (2)$$

cancel since $\Sigma_{s_{21}}=0$

Precursors

$$0 = -\lambda_i C_i + \beta_i\,\nu_2\Sigma_{f_2}\phi_2 \quad (3)$$

$$+ \quad \beta = \sum_{i=1}^{N}\beta_i$$

Substituting (3) into (1)

$$0 = \nabla\cdot D_1\nabla\phi_1 - \Sigma_{a_1}\phi - \Sigma_{s_1}\phi_1 + \Sigma_{s_{11}}\phi_1$$

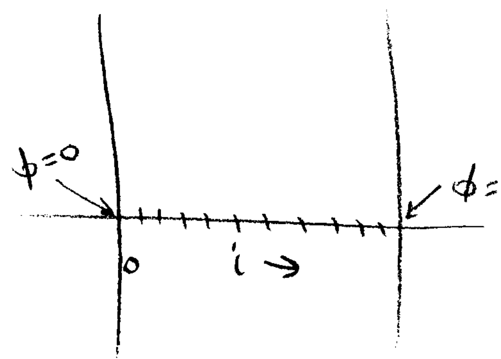
$$+ (1-\cancel{\beta})\,\nu_2\Sigma_{f_2}\phi_2 + \cancel{\beta\nu_2}\Sigma_{f_2}\phi_2$$

$$= \nabla\cdot D_1\nabla\phi_1 \underbrace{- \Sigma_{a_1}\phi - \Sigma_{s_1}\phi_1 + \Sigma_{s_{11}}\phi_1}_{-\Sigma_{R_1}\phi_1} + \nu_2\Sigma_{f_2}\phi_2$$

This makes sense physically since the steady state does not depend on <u>when</u> the neutron is born (prompt or delayed).

b. In a real reactor, poisons build up and fuel is depleted. Once criticality is reached, (where the flux and precursors are in balance), then poison (Xe & Sm) grow in. This adds Σ_a + the control rod must be withdrawn a bit to compensate. This causes local flux and precursor changes (millisec → minute time constant). The poisons grow in over hours / days. The Σ_f term gradually (over days and months) reduces as fuel depletes. The control rod must be withdrawn further. At some point, there will be insufficient fuel and / or too much poison to achieve criticality.

c.



We want to solve:

$$-\frac{D\partial^2 \phi_1}{\partial x^2} + \Sigma_{R_1}\phi_1 = \frac{\nu_2 \Sigma_{f_2} \phi_2}{K}$$

$$-\frac{D_2 \partial^2 \phi_2}{\partial x^2} + \Sigma_{R_2}\phi_2 = \Sigma_{S_{12}}\phi_1$$

We use the usual discretization $\frac{\partial^2 \phi}{\partial x^2} = \frac{\phi_{i+1} - 2\phi_i + \phi_{i-1}}{\Delta x^2}$

The R.H.S. of the flux eqn's act as sources. At any given time we know the core composition ∴ we solve for ϕ_1 + ϕ_2 via iteration as usual, adjusting $K^{new} = K^{old} \times \frac{\text{Source}^{new}}{\text{Source}^{old}}$ as usual

→

We then find updates (over time) to the core composition (+ hence Σ's + D's) by tracking the poisons (Xe + I) + fuel (N_F).
We can do this by solving

$$\frac{\partial I(x,t)}{\partial t} = \cdots$$

$$\frac{\partial X(x,t)}{\partial t} = \cdots$$

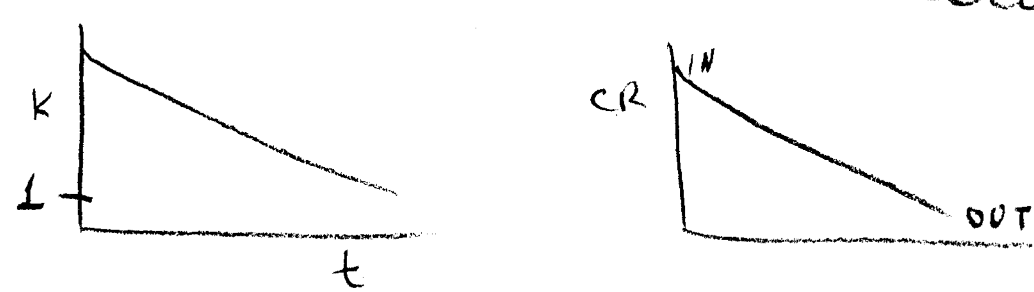
by simple Euler integration with $\Delta t \sim 1$ minute.
Recalc. Σ_a's given the new Xe concentrations.
Then solve for flux again + so on.

Perhaps once per day, calc. N_F from

$$\frac{\partial N_f}{\partial t} = \cdots$$

+ recalc. Σ_F, and so on.

(d) $K_{in}(c)$ will start out > 1 + gradually diminish, ie the core gradually loses reactivity.
So K is $\sim$ to control rod insertion.



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