Final Exam Dec 1997

1. a) It would take an infinite thickness to attenuate 100% of the beam. [3]

b) (i) $X + n_0^1 = {}_{11}Na^{24} \Rightarrow X = {}_{11}Na^{23}$ [3]

(ii) $X + p_1^0 = {}_{11}Na^{24} \Rightarrow X = {}_{10}Ne^{23}$ (Neon)

(iii) $X + \alpha_2^4 = {}_{11}Na^{24} \Rightarrow X = {}_{9}F^{20}$ (Florine)

c) Best conceivable design:

No leakage ($P_{FNL}, P_{TNL} = 1$)

No resonance absorption ($p = 1$)

No parasitic absorption ($f = 1$)

100% enrichment

$\therefore k = \epsilon \eta f p P_{FNL} P_{TNL} = \epsilon \eta$ [4]

$\eta = \nu (\Sigma_f / \Sigma_a)_{fuel} = \nu \sigma_f^{235} / \sigma_a^{235}$

$\therefore k_{max} = \epsilon \nu \sigma_f^{235} / \sigma_a^{235}$

#2

Soln:

$$\eta = \frac{\sum \nu_j \Sigma_f^j}{\sum \Sigma_a^j} = \frac{\nu \sigma_f^{u235} N^{u235}}{\sigma_a^{u235} N^{u235} + \sigma_a^{u238} N^{u238}}$$

$$= \frac{\nu \sigma_f^5 \gamma}{\sigma_a^5 \gamma + \sigma_a^8 (1-\gamma)}$$

where $\gamma = \frac{N^{u235}}{N^{u235} + N^{u238}} = \frac{N^{u235}}{\text{constant}}$

= enrichment factor

$$\sigma_f^5 = \sigma_f \text{ of } u235 = 577 \text{ b.}$$

$$\sigma_a^5 = \sigma_a \text{ of } u235 = 678 \text{ b.}$$

$$\sigma_a^8 = \sigma_a \text{ of } u238 = 2.73 \text{ b.}$$

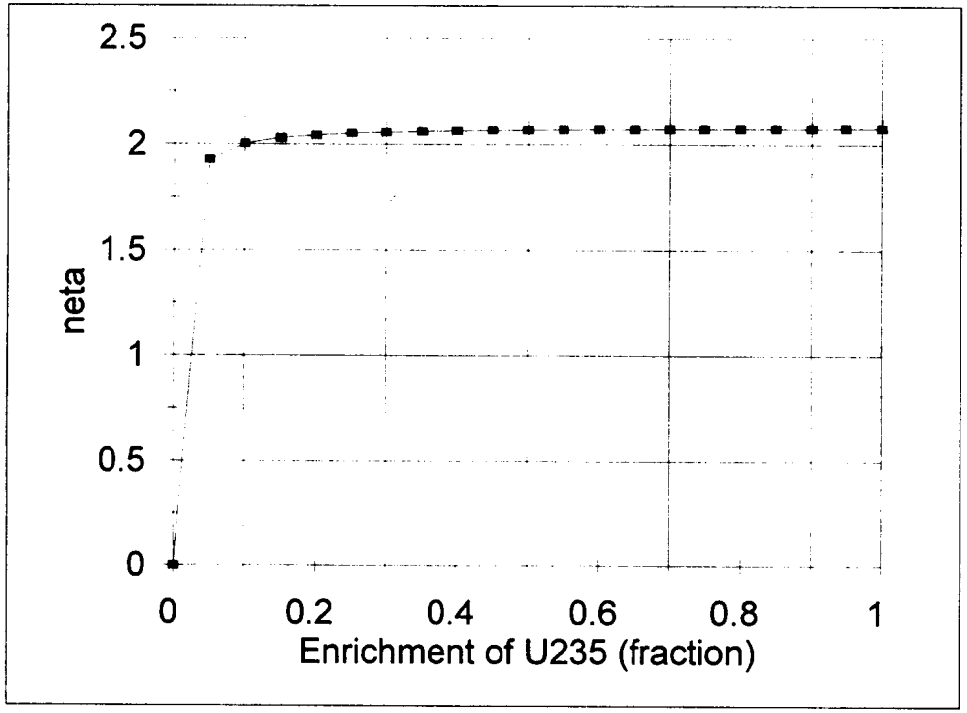
$$\therefore \eta = \frac{2.44 \times 577 \gamma}{678\gamma + 2.73(1-\gamma)}$$

see plot on following page

Not req'd

******NETA AS A FUNCTION OF ENRICHMENT**********

ENRICHMENT	NETA
0	0
0.05	1.928946
0.1	2.0039
0.15	2.030196
0.2	2.043604
0.25	2.051735
0.3	2.057191
0.35	2.061106
0.4	2.064053
0.45	2.06635
0.5	2.068192
0.55	2.069701
0.6	2.07096
0.65	2.072027
0.7	2.072942
0.75	2.073736
0.8	2.074431
0.85	2.075045
0.9	2.075591
0.95	2.076079
1	2.076519



3.

① $\frac{dN_1}{dt} = -\lambda_1 N_1 \quad \Rightarrow \quad N_1(t) = N_1(0)\, e^{-\lambda_1 t}$

② $\frac{dN_2}{dt} = \lambda_1 N_1 - \lambda_2 N_2$

Since $\lambda_2 >> \lambda_1$, N_1 will not change much in the time that N_2 changes. Thus we can consider $\lambda_1 N_1$ ~ constant in ② and that N_2 quickly comes to a pseudo-equilibrium with N_1, ie:

$$\frac{dN_2}{dt} \sim 0 = \lambda_1 N_1 - \lambda_2 N_2$$

$$\therefore N_2 = \frac{\lambda_1 N_1}{\lambda_2} = \frac{\lambda_1 N_1(0)\, e^{-\lambda_1 t}}{\lambda_2} \qquad [10]$$

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or from $N_2(t) = \frac{\lambda_1 N_1(0)}{(\lambda_2 - \lambda_1)} \left[e^{-\lambda_1 t} - e^{-\lambda_2 t} \right]$

$$\Rightarrow \quad N_2(t) = \frac{\lambda_1 N_1(0)}{\lambda_2} e^{-\lambda_1 t} \quad \text{for } \lambda_2 >> \lambda_1$$

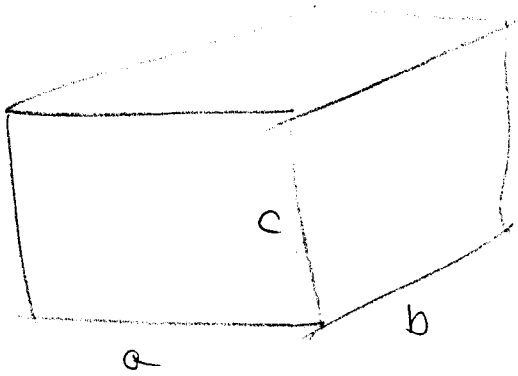
Trial Sol'n

$N_2 = A e^{-\lambda_1 t} + C e^{-\lambda_2 t}$, $N_2(0) = 0$

$\therefore -A\lambda_1 e^{-\lambda_1 t} - C\lambda_2 e^{-\lambda_2 t} = \lambda_1 N_1(0) e^{-\lambda_1 t} - \lambda_2 A e^{-\lambda_1 t} - \lambda_2 C e^{-\lambda_2 t}$

$\therefore A = \frac{\lambda_1 N_1(0)}{\lambda_2 - \lambda_1}$, $C = -A$

4.



a) $-D\nabla^2\phi + (\nu\Sigma_f - \Sigma_a)\phi = 0 \Rightarrow \nabla^2\phi + B^2\phi = 0$

$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$

Try $\phi = A \cos(\pi/a\, x) \cos(\pi/b\, y) \cos(\pi/c\, z)$ [1]

$\Rightarrow B^2 = (\pi/a)^2 + (\pi/b)^2 + (\pi/c)^2 = \frac{\nu\Sigma_f - \Sigma_a}{D}$

Power Peaking Factor (PPF) = Peak / Average

$$= \frac{A}{\frac{A}{\text{Volume}} \int_0^{a/2} \cos\left(\frac{\pi x}{a}\right) dx \int_0^{b/2} \cos\left(\frac{\pi y}{b}\right) dy \int_0^{c/2} \cos\left(\frac{\pi z}{c}\right) dz}$$

$\text{Volume} = abc$

$$= \frac{abc/8}{\left(\frac{a}{\pi} \sin\frac{\pi x}{a}\Big|_0^{a/2}\right)\left(\quad y \quad\right)\left(\quad z \quad\right)}$$

$$= \frac{\pi^3}{8} = 3.8758$$ [5]

b)

(i) Xe will build up + decay more in the center of the core than at the edges. The effect is non-linearly related to flux since $Xe \sim \frac{(\gamma_I + \gamma_x)\Sigma_f \phi_0}{\lambda_x + \sigma_a^x \phi_0}$ [1]

This poison will actually reduce the flux peak -> flux flattening

(ii) The central core will burnup more quickly than the edge, causing a flux flattening effect. [1]

(iii) Control rods should be placed near the core center for maximum reactivity effect. This will help lower the flux peaking as well, due to flux depression caused by neutron absorptions by the control rods

(iv) The higher the flux, the higher the measured signal but this may cause early instrument burnout. Put measurement device away from perturbations caused by control rods but in a region where flux is high enough to get a good signal. If the core is large, you may need many measurements to get a good representation of the flux distribution. The large peaking factors show that one cannot assume that a measured flux is representative. [1]

#5 $L^2 = D/\Sigma_a$, $D = \dfrac{1}{3(\Sigma_t - \mu_0 \Sigma_s)}$

For H_2O:

$\mu_0 = 0.676$

$\Sigma_t = 3.45 + 0.022 = 3.47$

$D = \dfrac{1}{3(3.47 - 0.676 \times 3.45} = 0.29296$

$\therefore L^2 = 0.293/0.022 = 13.3165$

$\therefore L = \underline{3.649}$ (H_2O)

For D_2O:

$\mu_0 = 0.884$

$\Sigma_f = 0.449$

$\therefore D = \dfrac{1}{3(0.449 - .884 \times 0.449)} = 6.40$

$\therefore L^2 = 6.4/3.3 \times 10^{-5} = 182,855$

$\therefore L \sim 427.61$ (D_2O)

For 2 reactors, same size but ① = H_2O & ② = D_2O

then $B = B_g^2$ = same and both reactors are critical.

$\therefore K = 1 = \left.\dfrac{\nu \Sigma_f / \Sigma_a}{1 + L^2 B^2}\right|_{①} = \left.\dfrac{\nu \Sigma_f / \Sigma_a}{1 + L^2 B^2}\right|_{②}$

It is constructive to consider two cases –

(A) Small reactor

(B) Large reactor

- Case A small reactor

For small reactors, B^2 is large (greater curvature - ie $B^2 \sim \left(\frac{\pi}{\text{size}}\right)^2$).

Thus, the smaller the critical reactor, the more the denominator $(1 + L^2 B^2)$ is dominated by $L^2 B^2$, to give:

$$K \sim \left.\frac{\nu \Sigma_f / \Sigma_a}{L^2 B^2}\right|_{①} = \left.\frac{\nu \Sigma_f / \Sigma_a}{L^2 B^2}\right|_{②}$$

$$\Rightarrow \left.\frac{\nu \Sigma_f}{D}\right|_{①} = \left.\frac{\nu \Sigma_f}{D}\right|_{②}$$

$D_① = 0.293$, $D_② = 6.40$

∴ a critical small D_2O based reactor has to have about 20x times more $\nu\Sigma_f$ (ie fuel) to be critical. Leakage $(1 + L^2 B^2)$ is much larger in a D_2O reactor for the same size (B^2) because L^2 is so much bigger. This is why you don't see small D_2O reactors.

For H_2O, the leakage is better & more than compensates for the poorer absorption characteristics.

There is more to say about this when multigroup effects are considered (ie moderation effects).

- Case B large reactor

Simply the reverse of Case A

↓

Here B^2 is small & in the extreme, $L^2 B^2 \ll 1$ for both reactors - ie leakage is not a dominant factor. Now Σ_a effects dominate. $\Sigma_{a(1)} \gg \Sigma_{a(2)}$ ∴ you need less fuel to make a large D_2O reactor go critical than a large H_2O one.

Again, this ignores moderation arguements, which will be addressed in multigroup theory.

#6

$$\frac{dn}{dt} = \frac{\rho - \beta}{\Lambda} n(t) + \sum_{i=1}^{6} \lambda_i C_i(t) \quad (1)$$

$$\frac{dC_i}{dt} = \frac{\beta_i}{\Lambda} n(t) - \lambda_i C_i(t) \quad (2)$$

(a) Steady state, $\therefore$ eqn (2) gives $\lambda_i C_i = \frac{\beta_i}{\Lambda} n_1$

+ (1) gives $0 = (\rho - \beta) n_1 + \beta n_1$

$n_1 \neq 0$, $\therefore \rho = 0$, as you might expect since steady state.

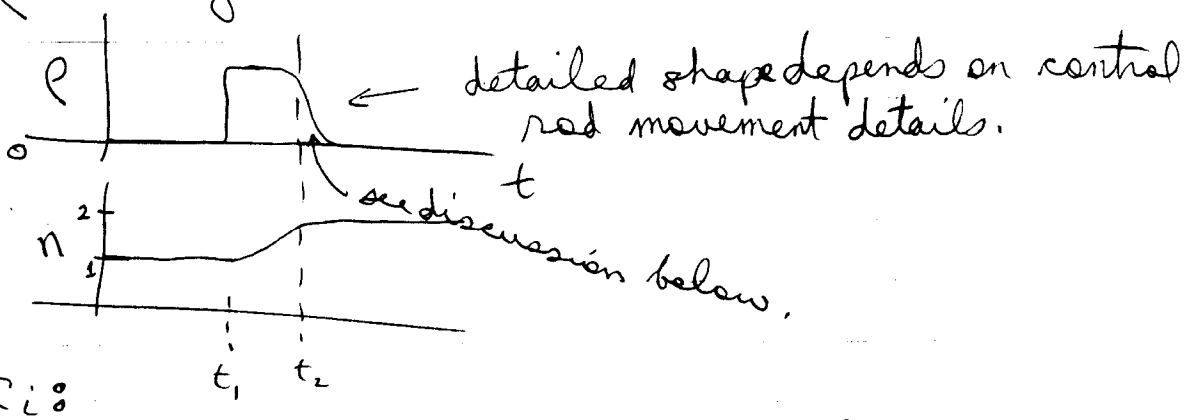
[5] From above, $C_i = \frac{\beta_i n_1}{\lambda_i \Lambda}$

(b) Control rods moves to raise n

$\Rightarrow$ implies $\rho > 0$ for awhile.

$\rho \to 0$ again when at new n

[5]



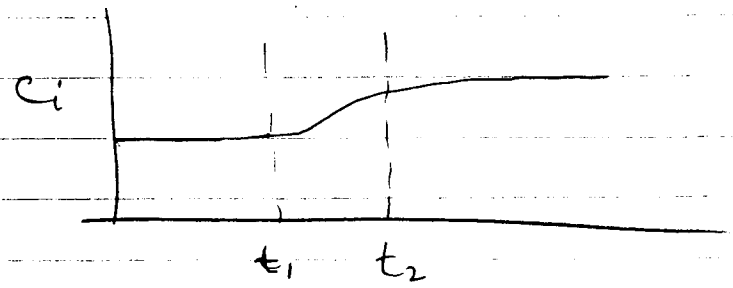
C_i:

n increases since $\rho > 0$ (eqn (1))

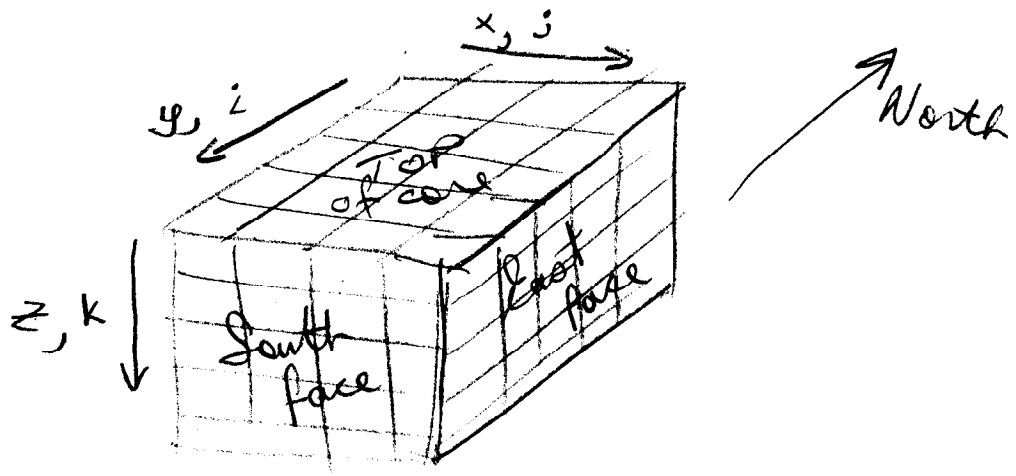
This causes C_i to increase (eqn (2)) but there is a lag in time.

As n_2 is approached, C_i is still below the equilibrium value for n_2 $\therefore \sum \lambda_i C_i$ is smaller than the steady state value at n_2.

∴ need some +ve ρ even when $n = n_2$.



7. a)



Can assume const grid spacing but this is not necessary. Will likely want unequal spacing since reflector areas may not be the same size as the assemblies. Axially uniform properties except for control rods

2 group - thermal & fast. Assume $\chi_1 = 1, \chi_2 = 0$

$$\frac{1}{v_1}\frac{\partial \phi_1}{\partial t} = \nabla \cdot D_1 \nabla \phi_1 \underbrace{- \Sigma_{a_1}\phi_1 - \Sigma_{s_1}\phi_1}_{\equiv -\Sigma_{r_1}\phi_1} + \Sigma_{s_{11}}\phi_1 + \Sigma_{s_{21}}\phi_2 + (1-\beta)\nu_2 \Sigma_{f_2}\phi_2 + \sum_{\ell=1}^{6} \lambda_\ell C_\ell$$

$$\frac{1}{v_2}\frac{\partial \phi_2}{\partial t} = \nabla \cdot D_2 \nabla \phi_2 - \Sigma_{a_2}\phi_2 - \Sigma_{s_2}\phi_2 + \Sigma_{s_{12}}\phi_1 + \Sigma_{s_{22}}\phi_2 + 0$$

($\Sigma_{s_2} = \Sigma_{s_{22}}$ since $\Sigma_{s_{21}} = 0$)

(no fission or precursor sources in thermal group)

$$\frac{\partial C_\ell}{\partial t} = -\lambda_\ell C_\ell + \beta_\ell \nu_2 \Sigma_{f_2}\phi_2 \qquad [6]$$

(13)

b) B.C: $\phi(\tilde{R}, t) = 0$ at extrapolated boundaries (ie at reactor edges) [2]

c) I.C.

$\phi(r, 0) =$ given

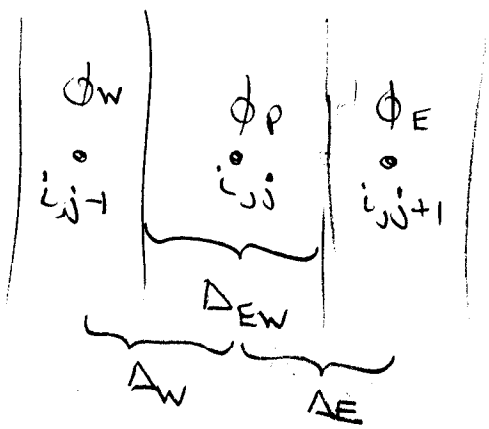
$C_\ell(r, 0) =$ given, eg $= 0$ for fresh core [2]

$= \dfrac{\beta_\ell \nu_2 \Sigma_{f_2} \phi_2}{\lambda_\ell}$ for equilibrium core

d)

$$\frac{d}{dx}\left(D \frac{d\phi}{dx}\right) = \frac{D_{EF}\phi_E}{\Delta_E \Delta_W} - \left(\frac{D_{EF}}{\Delta_E} + \frac{D_{WF}}{\Delta_W}\right)\frac{\phi_P}{\Delta_{EW}} + \frac{D_{WF}}{\Delta_W}\frac{\phi_W}{\Delta_{EW}} \quad [2]$$

~ for North-South & top-bottom



$$+ \frac{\partial \phi}{\partial t} = \frac{\phi^{t+\Delta t} - \phi^t}{\Delta t}$$

$$\therefore \phi_{1P}^{t+\Delta t} = \phi_{1P}^{t} + \nu \Delta t \left[\frac{D_{EF}\phi_{1E}}{\Delta_E \Delta_W} - (\quad)\frac{\phi_{1P}^{t+\Delta t}}{\Delta_{EW}} + \frac{D_{WF}}{\Delta_W}\frac{\phi_{1W}}{\Delta_{EW}} - \Sigma_{r_1}\phi_{1P}^{t+\Delta t} + (1-\beta)\nu_2 \Sigma_{f_2}\phi_{2P} + \sum_\ell \lambda_\ell C_\ell\right] \quad (1)$$

(evaluate at $t+\Delta t$)

etc.

e) Sol'n algorithm: [4]

In (1), gather up terms in $\phi_{1p}^{t+\Delta t}$ to make it implicit:

$$\phi_{1p}^{t+\Delta t} = \frac{\phi_p^t + v\Delta t\left[(\quad)\phi_w^{t+\Delta t} + (\quad)\phi_E^t + \cdots\right]}{\left[1 + v\Delta t\left(\left(\frac{D_E}{\Delta E} + \frac{D_W}{\Delta W} + \Sigma_r\right)\right)\right]}$$

Sweep through from {west to east / north to south / top to bottom} for group 1,

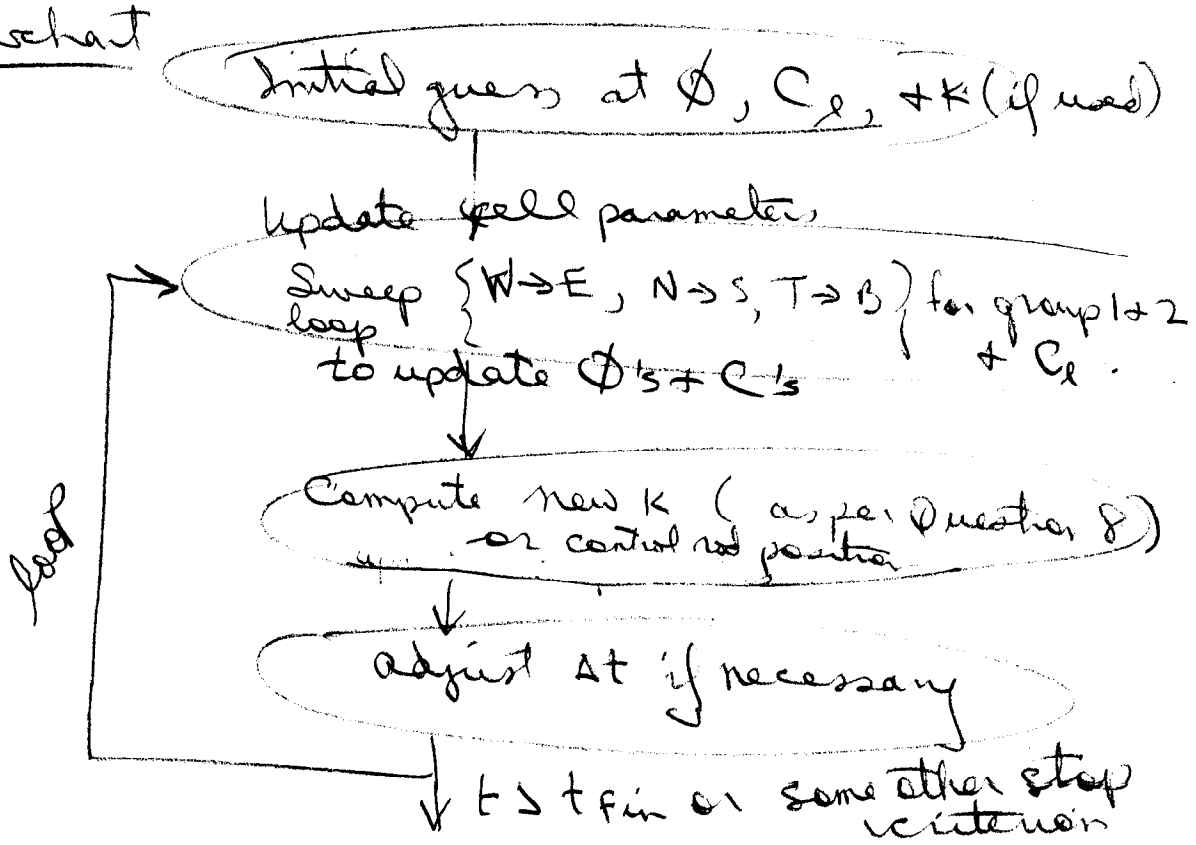
then group 2, then precursors.

Use the latest values of ϕ + C_l when possible.

This is equivalent to the SOR technique.

For control, adjust parameters of cell containing the control rod(s) (Σ_{a_2} mainly). Could also change $\nu_2\Sigma_{f_2} \to \frac{\nu_2\Sigma_{f_2}}{k}$ and control k to bring ϕ to the desired value. More on this in question 8.

Flowchart



f) The fast neutron velocities (compared to the delayed precursors) means that the Δt must be small to handle the rapid neutron transients. Yet the slower precursors will dominate the transient response as long as the reactor is below prompt critical. The reactor controller (on k) will keep the flux ~ constant so, unless you want to track the rapid flux transients, you might as well set the neutron velocities to be much slower or even just solve the steady state fluxes & track the precursors in time. The SS. ϕ would have to be recalculated periodically since the C_i's are changing. If you adjust the v_g's, set $v_g \Sigma_g >$ largest λ_i to ensure $\frac{\partial \phi}{\partial t}$ is greater than $\frac{\partial C_i}{\partial t}$. [3]

g) Reactivity control could be made via the k in the $\nu_2 \Sigma_{f2} \phi_2$ term. This effectively adjusts the fissioning rate. This is quite artificial. It is more physically real to model the control rod cells & adjust the Σ_a's in these cells as per Question 8. [1]

8. a) Basically, the control rods are inserted or removed to change the amount of neutron absorption. Since thermal cross sections are higher than fast cross sections, the dominant effect occurs in Σ_{a_2}. [2]

b) Yes, the controller would work. The first term (proportional term) causes a rod insertion if the measured ϕ is > the setpoint. The second term (rate or differential term) causes an insertion if the flux is taking off. Both are correct behaviours. If the flux is at the setpoint and steady, then no rod movement is requested, as desired. [5]

For the SS calculation we adjusted k in the term $\frac{\nu_2 \Sigma_{f2}}{k}$. This is rather artificial but is consistent with the definition of k when you re-arrange the S.S. eqn's.

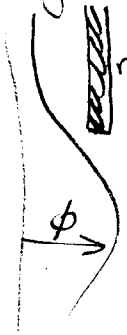
There is no reason why the S.S. could not be achieved by adjusting Σ_{a_2} instead of $\nu_2 \Sigma_{f_2}$. However, since $\nu_2 \Sigma_{f_2}$ has a relatively larger error than Σ_{a_2}, you might as well adjust it. The fudging to achieve SS is just that — a fudge or artificial adjustment of the model to make it critical. The transient, however, is a simulation of an event that is a perturbation of the

steady state. For that, a physically real controller is more appropriate.

If we were comparing the model to a real reactor, we'd first adjust the SS parameters ($\nu_2 \Sigma_{f2}/k$) to match the measured values of criticality with the control rods in the actual steady state positions as measured. Then, we'd let the controller do its thing for transient analysis.

C) Disperse the control rods throughout the core [3]
high thermal flux regions to make them more effective (since they operate via the $\Sigma_{a_2}\phi_2$ term). Several thinner rods are better than one thick, highly absorbing (black) rod since self shielding will reduce the rod effectiveness and since a black rod will cause a deep flux depression. This makes the $\Sigma_{a_2}\phi_2$ term low thereby reducing rod effectiveness. It also makes modelling more error prone since diffusion theory breaks down under these conditions.

Unfortunately the rods will cause axial distortions.

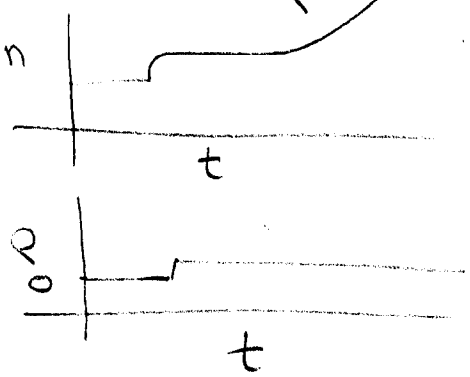


This is unavoidable with solid finite rods.

Flux shape control can be achieved via differential rod movement (ie pull some control rods out + push others in as desired).

d) Movement on the order of msec. is sufficient since the delayed precursors limit the rate of change of the flux to msec → sec range. The precision is the $\rho \ll \beta$ since as $\rho \to \beta$, the reactor period decreases rapidly. At $\rho = \beta$, the reactor is critical on prompt neutrons alone. Prompt neutron lifetime is ~ 50 μsec, far too fast to control. [3]

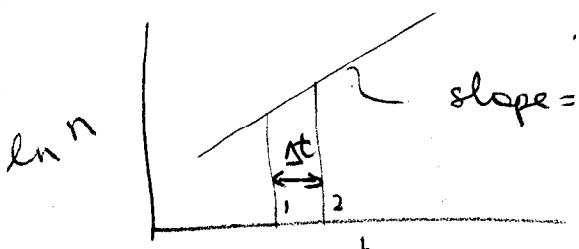
e) Rapid ρ changes cause a "prompt jump" effect of magnitude $\frac{n}{n_0} = \frac{\beta}{\beta - \rho}$. [5]



So a 1mk insertion leads to a $\frac{7mk}{6mk} = 1.16$ jump in power!

This is caused by the prompt neutrons. It is thus advisable to monitor the magnitude of $\frac{dn}{dt}$ (or $\frac{d\phi}{dt}$) and limit the period, τ in $n \sim n_0 e^{+t/\tau}$. This can be achieved by putting a hold on ΔZ_{in} if the rate is too fast.

We have



$$\therefore \frac{1}{\tau} = \text{slope} = \frac{(\ln n_2 - \ln n_1)}{t_2 - t_1} = \frac{\ln n_2 / \ln n_1}{\Delta t}$$

How the algorithm could be:

calc ΔZ_{in} as per controller

calc $\tau_{au} = \frac{\Delta t}{\ln I_2/I_1}$

if $\tau_{au} < \tau_{au\,limit}$ then $\Delta Z_{in} = 0$

f) The flux should be measured in a representative position of the whole core behaviour. For a small core, the flux shape won't change dramatically except near control rods. So stay away from the control rods. The flux measurement needs to be calibrated by thermal power measurements and that calibration can be thrown off by any major core changes. The flux meter needs to be in a high enough flux to get a good reading but ~~too~~ high a flux will lead to early meter failure. [2]