Final Exam 2001 - Solution

1a. Because D has a much lower absorption cross section than H. ∴ much better chance of surviving the slowing down process.

1.b

Boron is a good absorber, ∴ safe to assume negligible scattering (no buildup).

$$\therefore \quad \frac{I(x)}{I(0)} = 0.001 = e^{-\Sigma_a x}$$

$$\therefore \quad x = -\frac{\ln(0.001)}{\Sigma_a} = \frac{+6.91}{103}$$

Σ_a ↖ $103\ cm^{-1}$

$$= \underline{\underline{0.0671\ cm}}.$$

2 Validity of Fick's Law

We re-evaluate each assumption in turn:

a *Infinite medium.* This assumption was necessary to allow integration over all space but flux contributions are negligible beyond a few mean free paths due to the factor, $e^{-\Sigma_t r}$. Thus as long as we are at least a few mean free paths from the reactor extremities, all is okay. Corrections can be made at the reactor surfaces as shown later in this chapter.

b *Uniform medium.* A non-uniform medium ($\Sigma_s = \Sigma_s(\mathbf{r})$) requires a re-evaluation of the derivation of Fick's Law. Now the interaction rate, $\Sigma_S\,\phi$, is a function of space due to both ϕ and Σ variations in space. Detailed calculations show, however, that the extra current (ie. scattering) contributions caused by a locally larger Σ_S are <u>exactly</u> cancelled by larger attenuations (($e^{-\Sigma_t r} = e^{-(\Sigma_s + \Sigma_a)r}$) <u>iff</u> (if and only if) $\Sigma_s >> \Sigma_a$ or Σ_s / Σ_t = constant. It should be noted however that $\Sigma_S\,(\mathbf{r})$ can lead to large values of $\frac{\partial \phi}{\partial \mathbf{r}}(\mathbf{r})$ which violates assumption (e).

c *Sources.* As per assumption (a), we can get away with sources as long as they are more than a few mean free paths away.

d *Isotropic scattering.* Anisotropic scattering can be corrected for by detailed considerations of transport theory in which D is re-evaluated:

$$\frac{\Sigma_s}{2}\sqrt{\frac{D}{\Sigma_a}}\ln\left[\frac{\Sigma_t + \sqrt{\Sigma_a/D}}{\Sigma_t - \sqrt{\Sigma_a/D}}\right] = \frac{1 + 3D\Sigma_s\,\bar{\mu}}{1 + 3D\Sigma_s\,\bar{\mu}}$$

Where

$$\bar{\mu} \equiv \cos\theta \quad \text{(average of the scattering angle in the lab system)}$$

$$= \frac{2}{3A}$$

Expanding the equation in D, above:

$$D = \frac{1}{3\Sigma_t\,(1 - \bar{\mu})(1 - 4\Sigma_a/5\Sigma_t + \ldots)}$$

$$= \frac{1}{3\Sigma_t\,(1-\bar{\mu})} \quad \text{for } \Sigma_a/\Sigma_t << 1$$

$\therefore\ D = \frac{\lambda_{tr}}{3}$ as previously defined in the supplemental material at the end of the chapter on *Basic Definitions and Perspectives*

e

Slowly varying flux. Further expansions of φ are necessary to account for large variations in $\phi\,(\mathbf{r})$. It can be shown that 2nd order terms cancel and that third order terms are not important beyond a few mean paths. Therefore, provided $\frac{d^2\phi}{d\mathbf{r}^2}(r)$ is small over a few mean free paths, all is okay. Large variations in ϕ occur when Σ_a is large (compared to Σ_S).

f

Time - in dependent flux. The time it takes a slow neutron to traverse 3 mean free paths (in cm.) is

$$\Delta t \sim \frac{3\lambda_s}{v} \sim \frac{3x1cm}{2x10^5 cm/s} \sim 1.5x10^{-5}s.$$

If is changed at 10%/s (a high rate), then

$$\frac{\Delta\phi}{\phi} = \frac{\Delta\phi/\phi}{\Delta t} x\Delta t \sim 0.1\Delta t = 1.5x10^{-6}.$$

This is a very small fractional change of flux amplitude in the time it takes a neutron to move a significant physical distance.

3. [10 marks]

a.

$$\cancel{\frac{1}{v_1}\frac{\partial \phi_1}{\partial t}} = \nabla\cdot D_1 \nabla\phi_1 - \Sigma_{a_1}\phi_1 - \Sigma_{s_1}\phi_1 + \Sigma_{s_{11}}\phi_1 + \overset{\text{no upscatter}}{\cancel{\Sigma_{s_{21}}\phi_2}}^{\,0}$$

$$+ \underbrace{\chi_1\left[\overset{\text{small}}{\nu_1\Sigma_{f_1}\phi_1} + \nu_2\Sigma_{f_2}\phi_2\right]}_{②} + \cancel{S_1}\ 0$$

$$\frac{1}{v_2}\frac{\partial \phi_2}{\partial t} = \nabla\cdot D_2\nabla\phi_2 - \Sigma_{a_2}\phi_2 - \Sigma_{s_2}\phi_2 + \overbrace{\Sigma_{s_{12}}\phi_1}^{①} + \Sigma_{s_{22}}\phi_2$$

$$+ \cancel{\chi_2}\left[\nu_1\Sigma_{f_1}\phi_1 + \nu_2\Sigma_{f_2}\phi_2\right] + \cancel{S_2}\ 0$$

0 (no thermal births)

Typically $\Sigma_{s_{12}} \simeq 0$ ∴ Σ_{s_2} + $\Sigma_{s_{22}}$ cancel.

b. Term labelled ① above are the effective sources of thermals (slowing down from fast)

c. Terms labelled ② above are the effective sources of fast neutrons ($\chi_1 = 0$ & the fast fissions, $\nu_1\Sigma_{f_1}\phi_1$ are ~ 3% of the thermal fissions, $\nu_2\Sigma_{f_2}\phi_2$

d. The energy separation is too great for upscatter of any appreciable extent.

e. The poisons affects the Σ_a terms primarily. And the changes in Σ_{a_2} is the more significant since it is the more significant Σ_a.
Xe has a large thermal Σ_a

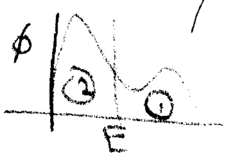
4. [10 marks]

a. $$\frac{1}{v_1}\frac{\partial \phi_1}{\partial t} = \nabla\cdot D_1\nabla\phi_1 - \Sigma_{a_1}\phi_1 - \Sigma_{s_1}\phi_1 + \Sigma_{s_{11}}\phi_1 + \overset{0}{\cancel{\Sigma_{s_{21}}}}\phi_2$$

$$+ \chi_1(1-\beta)\left[\nu_1\Sigma_{f_1}\phi_1 + \nu_2\Sigma_{f_2}\phi_2\right] + \chi_1^c\sum_{i=1}^{6}\lambda_i C_i + S_g^{ext} \quad \text{— (1)}$$

$$\frac{1}{v_2}\frac{\partial \phi_2}{\partial t} = \nabla\cdot D_2\nabla\phi_2 - \Sigma_{a_2}\phi_2 - \Sigma_{s_2}\phi_2 + \Sigma_{s_{12}}\phi_1 + \Sigma_{s_{22}}\phi_2$$

$$+ \overset{0}{\cancel{\chi_2}}(1-\beta)\left[\nu_1\Sigma_{f_1}\phi_1 + \nu_2\Sigma_{f_2}\phi_2\right] + \chi_2^c\sum_{i=1}^{6}\lambda_i C_i + S_g^{ext} \quad \text{— (2)}$$



Assume $\chi_2 \simeq 0$, $\chi_1 = 1$ (no thermal births)

$\Sigma_{s_{21}} = 0$ (no upscatter) $\Rightarrow \Sigma_{s_2} = \Sigma_{s_{22}}$

$S_g^{ext} = 0$ (simplicity)

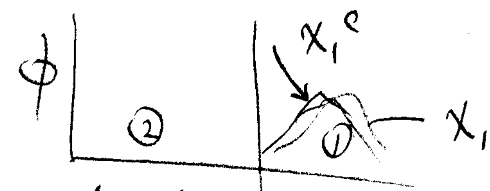
$\nu_1\Sigma_{f_1} \sim 0$ "

$$\frac{\partial C_i}{\partial t} = -\lambda_i C_i + \beta_i\left[\nu_1\Sigma_{f_1}\phi_1 + \nu_2\Sigma_{f_2}\phi_2\right] \quad \text{— (3)}$$

b. $\chi_g \simeq \chi_g^c$ since energy group structure is coarse.

$\chi_2^c = \chi_2 = 0$, $\chi_1 = \chi_1^c = 1$

Energy of precursor neutrons is lower than prompt neutrons but still in group 1.



c. Sub (3) (in steady state) into (1) + (2) to see:

$$\chi_g(1-\beta)[\quad\quad] + \chi_g[\;]\underbrace{\sum \beta_i}_{\beta}$$

since in steady state

(3): $\lambda_i C_i = \beta_i[\quad]$.

Hence β terms cancel.

d. Using the fission term, introduce a fudge factor, k:

$$\frac{1}{v_g}\frac{\partial \phi_g}{\partial t} = \cdots\cdots\cdots \frac{\chi_g(1-\beta)}{k}\left[\nu_1\Sigma_{f_1}\phi_1 + \nu_2\Sigma_{f_2}\phi_2\right]$$

If $\phi_g\uparrow$, raise k, if $\phi_g\downarrow$ lower k. As per class notes, $k^{new} = \dfrac{\int_V (\nu_1\Sigma_1\phi_1 + \nu\Sigma_f\phi_2)^{new}\, dV}{\frac{1}{k^{old}}\int_V (\nu_1\Sigma_{f_1}\phi_1 + \nu\Sigma_{f2}\phi_2)^{old} dV}$

e. Using the absorption term, we envision a control rod that adds absorption when inserted into the reactor.

ie $\Sigma_{a_g} \Rightarrow \Sigma_{a_g}^{0} + \Sigma_{a_g}^{control}(z)$

where z = fraction inserted, $\in (0,1)$.

Devise a controller based on deviation from a flux setpoint:

$$z = a + b(\phi_{measured} - \phi_{setpt.}) + c\frac{\partial \phi_{meas.}}{\partial t}$$

Need to experiment to find a, b, + c. PD controller.

Could also use such a controller to vary k in (d).

5 ① $\frac{dN_1}{dt} = -\lambda_1 N_1 \Rightarrow N_1(t) = N_1(0)e^{-\lambda_1 t}$

② $\frac{dN_2}{dt} = \lambda_1 N_1 - \lambda_2 N_2$

Since $\lambda_2 >> \lambda_1$, N_1 will not change much in the time that N_2 changes. Thus we can consider $\lambda_1 N_1 \sim$ constant in ② and that N_2 quickly comes to a pseudo-equilibrium with N_1, ie:

$$\frac{dN_2}{dt} \sim 0 = \lambda_1 N_1 - \lambda_2 N_2$$

$$\therefore N_2 = \frac{\lambda_1 N_1}{\lambda_2} = \frac{\lambda_1 N_1(0)e^{-\lambda_1 t}}{\lambda_2}$$

[10]

Or from $N_2(t) = \frac{\lambda_1 N_1(0)}{(\lambda_2 - \lambda_1)}\left[e^{-\lambda_1 t} - e^{-\lambda_2 t}\right]$

$\Rightarrow N_2(t) = \frac{\lambda_1 N_1(0)}{\lambda_2} e^{-\lambda_1 t}$ for $\lambda_2 >> \lambda_1$

Trial Sol'n

$N_2 = Ae^{-\lambda_1 t} + Ce^{-\lambda_2 t}$, $N_2(0) = 0$

$\therefore -A\lambda_1 e^{-\lambda_1 t} - C\lambda_2 e^{-\lambda_2 t} = \lambda_1 N_1(0)e^{-\lambda_1 t} - \lambda_2 Ae^{-\lambda_1 t} - \lambda_2 Ce^{-\lambda_2 t}$

$\therefore A = \frac{\lambda_1 N_1(0)}{\lambda_2 - \lambda_1}$, $C = -A$

#6 - direct from course notes

#6

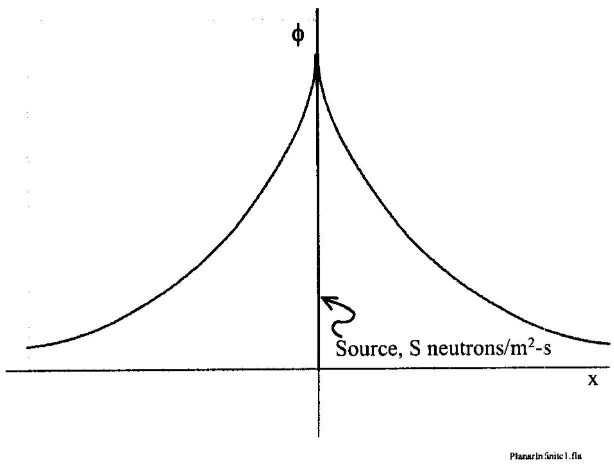
We have previously shown the Steady State Diffusion Equation to be

$$0 = S(\bar{r}) - \Sigma_a(\bar{r})\,\phi(\bar{r}) + D\,\nabla^2\phi(\bar{r})$$

Defining $L^2 = \dfrac{D}{\Sigma_a} \equiv [cm^2]$; $L \equiv$ diffusion length

$$\nabla^2\phi - \frac{1}{L_2}\phi = -\frac{S}{D} \tag{10.1}$$

10.1 Infinite Planar Source



$$\delta(x) = 0,\ x \neq 0$$

$$\int_a^b \delta(x)\,dx = 1,\ a < 0 < b$$
$$= 0 \text{ otherwise}$$

Figure 8 Flux distribution for a planar source

Equation (10.1) reduces to:

$$\frac{1}{\phi}\,\frac{d^2\phi(x)}{dx^2} - \frac{1}{L^2}\phi(x) = -\frac{S\delta(x)}{D} \tag{10.2}$$

and for $x \neq 0$

$$\frac{1}{\phi}\,\frac{d^2\phi(x)}{dx^2} - \frac{\phi(x)}{L^2} = 0 \tag{10.3}$$

Consider the planar source (as shown in figure 9)

$$\lim_{x \to 0} \underbrace{J(x)}_{\text{current from either end}} = \frac{S}{2} \tag{10.4}$$

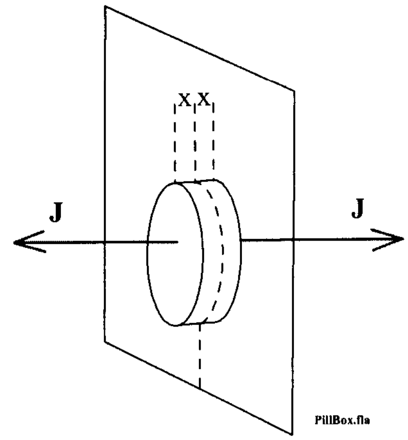


Figure 9 Current "pill box"

The solution to Equation (10.3) has the following form:

$$\phi(x) = A\, e^{-x/L} + C\, e^{x/L} \tag{10.5}$$

For $x > 0$, $C = 0$, otherwise ϕ is non-finite as $x \to \infty$

$$\therefore\ \phi(x) = A\, e^{-x/L},\ x > 0 \tag{10.6}$$

From Fick's Law $J\big|_0 = -D\left.\frac{d\phi}{dx}\right|_0 = +\left.\frac{DA}{L} e^{-x/L}\right|_0 = \frac{DA}{L} = \frac{S}{2}$

$$\therefore\ A = \frac{SL}{2D}$$

$$\therefore\ \phi(x) = \frac{SL}{2D} e^{-x/L} \qquad x > 0 \tag{10.7}$$

Similarly for $x < 0$, giving $\boxed{\phi(x) = \frac{SL}{2D} e^{-|x|/L}}$. Recall that this not valid at or near $x = 0$.

This solution should make physical sense to you. The flux decays exponentially away from the source as it is absorbed by the medium. This agrees with the beam absorption laws that we have previously derived.

b.) Beam attenuation: $\phi = \frac{\phi_0}{2} e^{-\Sigma_t x}$ cf $\frac{SL}{2D} e^{-\frac{x\sqrt{\Sigma_a}}{\sqrt{D}}}$

does not consider multiple scatters (ie diffusion like process).

7. (15 marks)

a. (4 marks)

$$-D_1^c \nabla^2\phi_1^c + \Sigma_{R_1}^c \phi_1^c = \frac{1}{K}\left[\nu_1 \Sigma_{F_1}^c\phi_1^c + \nu_2 \Sigma_{F_2}^c \phi_2^c\right]$$

$$-D_2^c \nabla^2 \phi_2^c + \Sigma_a^c \phi_2^c = \Sigma_{S12}^c \phi_1^c$$

$$-D_1^R \nabla^2\phi_1^R + \Sigma_{R_1}^R \phi_1^R = 0$$

$$-D_2^R \nabla^2\phi_2^R + \Sigma_a^R \phi_2^R = \Sigma_{S12}^R \phi_1^R$$

b. (4 marks)

$$\phi_i^R(b/2) = 0 \qquad i = 1, 2$$

$$\phi_i^c(a/2) = \phi_i^R(a/2)$$

$$J_i^c(a/2) = J_i^R(a/2)$$

$$(\text{ie } D_i^c \nabla\phi_i^c(a/2) = D_i^R \nabla\phi_i^R(a/2))$$

symmetry or $\nabla\phi_i^c(0) = 0$

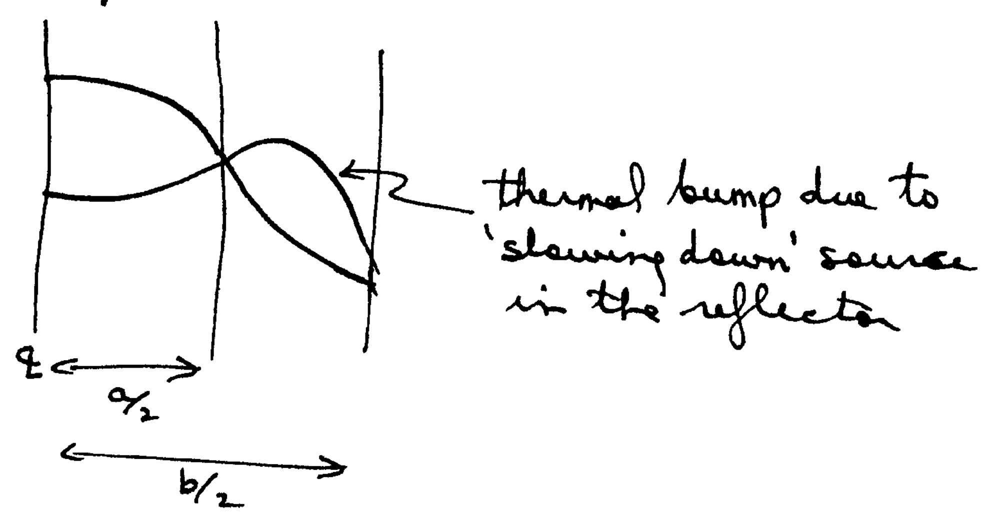
c. (4 marks)

Try solutions of form: $\phi_i^c = A_i \cos\mu_i r + C_i \cosh\lambda_i r$ (or whatever, Bessel?)

& subst. into eqn's in (a). This yields 4 eqn's that should allow the calc. of the buckling coefficients, μ_i + λ_i.

Next apply the B.C.'s. The condition $\phi_i^R(a/2+b)=0$ should allow you to eliminate one unknown A_i^R or C_i^R for $i=1,2$. Symmetry does the same for the core flux. This leaves 4 unknowns + 4 interface B.C. equations that can be solved (theoretically) for the 4 unknown. These 4 eqn's form the criticality condition.

d) (3 marks)



Thanks to Sinh Nguyen

#8.

a) $\frac{1}{v_1}\frac{\partial\phi_1}{\partial t} = \nabla\cdot D_1\nabla\phi_1 - \Sigma_{R1}\phi_1 + (1-\beta)\nu_2\Sigma_{f2}\phi_2 + \sum_{i=1}^{N}\lambda_i C_i$

1) no upscatter $\Sigma_{s21}=0$

2) no fast fission $\Sigma_{f1}=0$

3) no ~~thermal birth~~ $\chi_2=0 \Rightarrow \chi_1=1$. $\chi_2^c=0 \Rightarrow \chi_1^c=1$.

$\frac{1}{v_2}\frac{\partial\phi_2}{\partial t} = \nabla\cdot D_2\nabla\phi_2 - \Sigma_{a2}\phi_2 + \Sigma_{S12}\phi_1$

$\frac{\partial C_i}{\partial t} = -\lambda_i C_i + \beta_i\nu_2\Sigma_{f2}\phi_2 \quad , i=1,2,\dots,N$

$\frac{\partial I}{\partial t} = \gamma_I\Sigma_{f2}\phi_2 - \lambda_I I$

$\frac{\partial X}{\partial t} = \gamma_X\Sigma_{f2}\phi_2 + \lambda_I I - \lambda_X X + \sigma_a^X\phi_2 X$

$\frac{\partial N_f}{\partial t} = -\sigma_a^F\phi_2 N_f$

for fuel region only

B.C. for flux: $\phi_g(\tilde{r}_s)=0$ vacuum

for others: no need

T.C. $\phi_g(r,0)$ = given

$C_i(r,0)$ = given or $= \beta_i\nu_2\Sigma_{f2}\phi(r,0)/\lambda_i$

$I(r,0) = 0$ or = S.S

$X(r,0) = 0$ or = S.S

$N_f(r,0)$ = known

(*) Discretization (see later)

b) For S.S. flux C_i takes no part discard them

$\nabla\cdot D_1\nabla\phi_1 - \Sigma_{R1}\phi_1 + \nu_2\Sigma_{f2}\phi_2 = 0$

$\nabla\cdot D_2\nabla\phi_2 - \Sigma_{a2}\phi_2 + \Sigma_{S12} = 0$.

I, X, N_f remain.

$C_i \rightarrow$ discard (see question 4c)

X builds up after min $\rightarrow$ hrs will change Σ_{a2}, so calculate it every 5 mins or
and update flux equation.

N_f depletes after hrs $\rightarrow$ days, will affect Σ_f, Σ_a, so calculate it every hour and update flux equation.

c) For fast transient (up to few seconds):

Ignore C_e, I, X, N_f as in a few seconds they don't change considerably

Solve transient equation flux with $\Delta t \sim \mu sec \rightarrow msec$.

d) For short term transient (up to few minutes):

Solve flux and precursors only. I, X, N_f don't change much $\rightarrow$ ignore them.

When flux reaches S.S $\rightarrow$ switch to S.S. version or set $v_g = 1$ and solve precursor equations. We can either update S.S. flux equation or use transient with time step of precursor eq. provided setting $v_g = 1$ or using F-factor to slow down flux transient $\Rightarrow$ no loss of accuracy.

e) For I, Xe-transient:

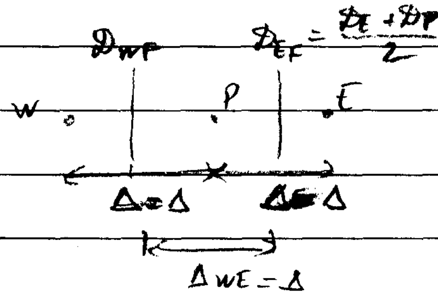
Use S.S. flux version, ignore precursor eq., as flux reaches S.S. in very short time. Use S.S. flux to calculate I & X, and then every 10 min, update the flux eq.

We can track fuel depletion, but do it every hour or so and then update the flux equation.

Thanks to Sinh Nguyen

#9.

a) Use the governing equation given in 8)



Replace $\frac{\partial Y}{\partial t} \approx \frac{Y^{t+\Delta t} - Y^{t}}{\Delta t}$

In 1-D: $\frac{\partial}{\partial x} D \frac{\partial \Phi}{\partial x} = \frac{D}{\Delta^2}\left(\phi_W - 2\phi_P + \phi_E\right)$

$$\frac{1}{v_1} \frac{\phi_{1P}^{t+\Delta t} - \phi_{1P}^{t}}{\Delta t} = \frac{D_1}{\Delta^2}\left(\phi_{1W}^{t+\Delta t} + \phi_{1E}^{t}\right) - \frac{2D}{\Delta^2}\phi_{1P}^{t+\Delta t} + (1-\beta)\nu_2 \Sigma_f \phi_{2P}^{t} + \sum \lambda_i C_i^{t} - \Sigma_{R1}\phi_{1P}^{t+\Delta t}$$

$$\phi_{1P}^{t+\Delta t} = \frac{\phi_{1P}^{t} + v_1 \Delta t \left[\frac{\mathcal{D}_1}{\Delta^2}\left(\phi_{1W}^{t+\Delta t} + \phi_{1E}^{t}\right) + (1-\beta)\nu_2 \Sigma_f \phi_{2P}^{t} + \sum \lambda_i C_i^{t}\right]}{1 + v_1 \Delta t \left(\frac{2D}{\Delta^2} + \Sigma_{R1}\right)}$$

$$\phi_{2P}^{t+\Delta t} = \frac{\phi_{2P}^{t} + v_2 \Delta t \left[\frac{\mathcal{D}_2}{\Delta^2}\left(\phi_{2W}^{t+\Delta t} + \phi_{2E}^{t}\right) + \Sigma_{S12}\phi_1^{t+\Delta t}\right]}{1 + v_2 \Delta t \left(\frac{2D}{\Delta^2} + \Sigma_{a2}\right)}$$

$$C_{iP}^{t+\Delta t} = \frac{C_{iP}^{t} + \cdots}{1 + \lambda_i \Delta t}$$

$I^{t+\Delta t} = \ldots$

$X^{t+\Delta t} = \ldots$

$N_f^{t+\Delta t} = \ldots$

b) I.C. are given at $t=0$ or take the S.S. values for C, I, X at $\phi(0)$

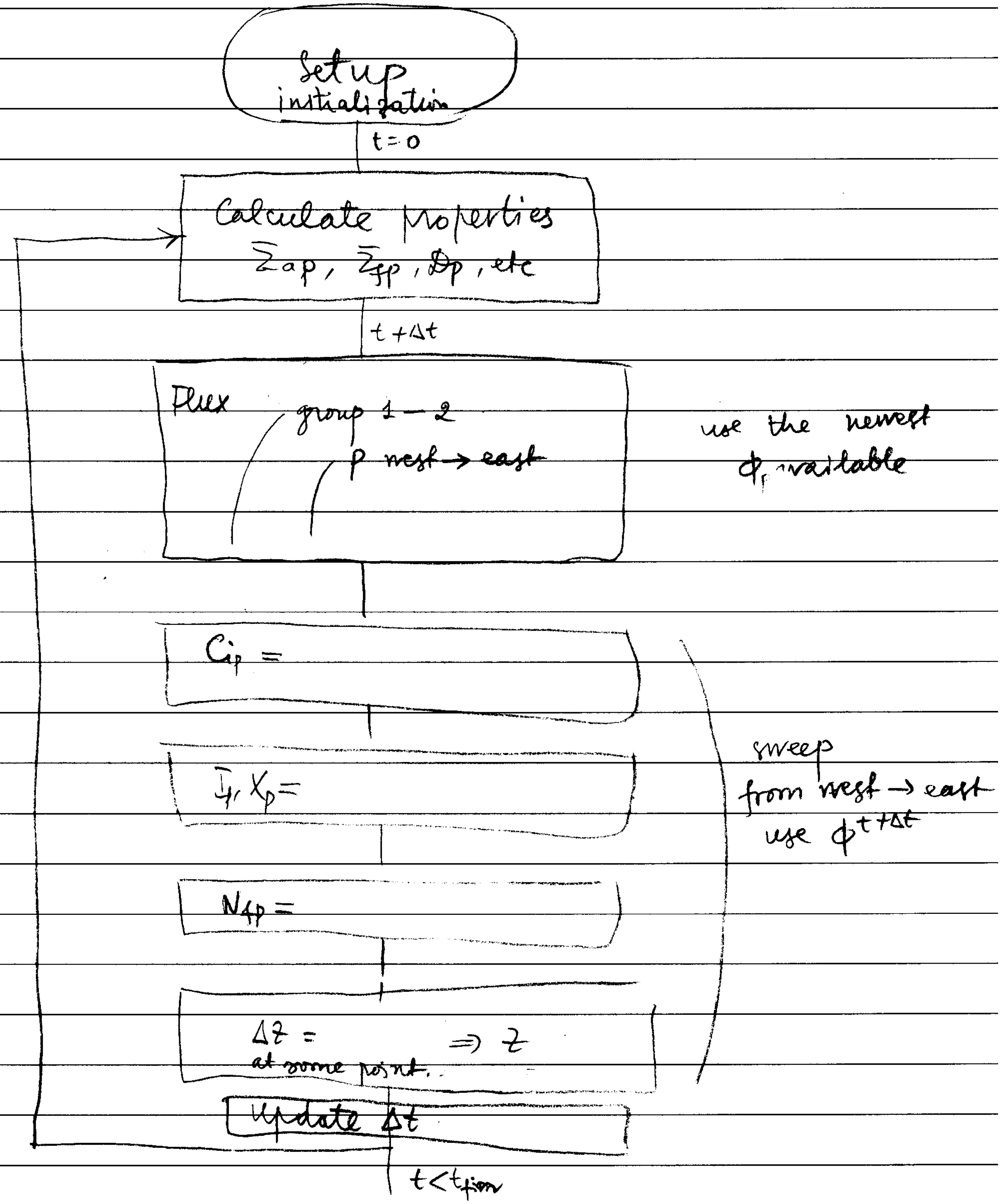
B.C. for flux at edges $\phi_g(\pm a) = 0$.

No need interface B.C. as we can integrate through it but use appropriate $\mathcal{D}, \Sigma$ for that region, i.e. region dependent

c) To control flux, add a controler as in question 4 either k or Σ_{a2}. Preferably, use a control rod to add and remove an absorber $\Rightarrow$ $\Sigma_{a2} = \Sigma_{a2}^{0} + \Sigma_{a2}^{c} \times Z/Z_0$

where $Z = Z + \Delta Z$, $\Delta Z = a_1(\phi_m - \phi_{sp}) + a_2 \frac{\partial \phi_m}{\partial t}$

ϕ_m - measured
ϕ_{sp} - setpoint
a_1, a_2 - +ve constants



d)

Fast transient ; ignore all $C_i \to N_f$, update z

In general take a variable we need to track, for all var with time response < this Δt switch to s.s. version or use F-factor.

For blocks below this, calculate infrequently, say $10\Delta t$ or more and then update properties to recalculate fluxes and so on

Eg. Take I, X. $\Delta t = 10$ min

Solve flux and precursor eq. for steady state only.

Every hour calculate N_f.

It's better to use additional subroutine to calculate Δt for every equations, then we can flexibly simulate various cases we want.