ENGINEERING PHYSICS 4D3/6D3

DAY CLASS — Dr. Wm. Garland
DURATION: 3 hours —
McMASTER UNIVERSITY FINAL EXAMINATION — December 13, 2001

Special Instructions:

1. Closed Book. All calculators and up to 6 single sided 8 ½" by 11" crib sheets are permitted.
2. Do all questions.
3. The value of each question is as indicated.
4. Point form is sufficient for discussion type questions.

TOTAL Value: 100 marks

THIS EXAMINATION PAPER INCLUDES 4 PAGES AND 9 QUESTIONS. YOU ARE RESPONSIBLE FOR ENSURING THAT YOUR COPY OF THE PAPER IS COMPLETE. BRING ANY DISCREPANCY TO THE ATTENTION OF YOUR INVIGILATOR.

1. [10 marks]
   a. Explain briefly why D_2O is better moderator than H_2O even though H_2O can slow down a neutron in fewer collisions than D_2O.
   b. Boron is a common material used to shield against thermal neutrons. Estimate the thickness of boron required to attenuate an incident thermal neutron beam to 0.1 % of its intensity. Use $\Sigma_a = 103\ cm^{-1}$.

2. [10 Marks] Fick's Law was developed under a number of assumptions as listed below. Discuss the validity of each in turn.
   a. The medium is infinite;
   b. The medium is uniform;
   c. There are no neutron sources in the medium;
   d. Scattering is isotropic in the laboratory coordinate system;
   e. The neutron flux is a slowly varying function of position;
   f. The neutron flux is not a function of time.

3. [10 marks]
   a. Write out the 2 group neutron diffusion equations in steady state, ignoring the delayed precursors.
   b. Which terms are the effective source terms for the thermal neutrons?
   c. Which terms are the effective source terms for the fast neutrons?
   d. Why can we safely ignore upscatter?
   e. Which terms are the ones most affected by fission product poisons?

4. [10 marks]
   a. Write out the 2 group neutron diffusion equations in transient form, including the delayed precursor equations.
   b. For this case, we can assume χ_g the same as χ_g^C. Why?
   c. Show that the precursors play no part in the steady state solution.
   d. Show how to introduce a controller to keep the flux steady based on modifying the fission source term.
   e. Show how to introduce a controller to keep the flux steady based on modifying the absorption term.

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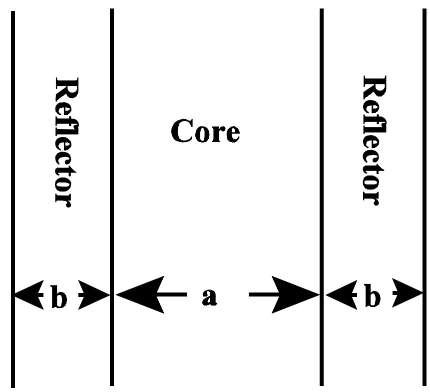
5. [10 marks] Consider an initially pure sample of radioactive material parent, concentration $N_1(t)$, which decays to a daughter product, $N_2(t)$, that is itself radioactive. Determine the long term concentrations of the two isotopes given that the daughter decays much more quickly that the parent. [Hint: This can be solved without much math by considering what N_1 looks like from the point of view of N_2. Alternatively, if you prefer a more mathematical solution, try $N_2(t) = A \exp(-\lambda_1 t) + C \exp(-\lambda_2 t)$ where λ_1 and λ_2 are the decay constants for N_1 and N_2 respectively.]

6. [10 Marks]
   a. For a planar source of neutrons, S neutrons / cm^2 sec, in an infinite absorbing medium, derive the neutron flux distribution (assume one speed).
   b. Why is the distribution in space different from the simple attenuation of a beam in an absorbing media?

7. [15 Marks] For an infinite slab reactor bordered by a reflector on either side as shown:
   a. State the 2 group diffusion equations (steady state) for the core and reflector regions.
   b. State and justify your boundary conditions.
   c. Find the criticality equation. *Hint: With the origin at the centreline, try*



$$\phi_{core} = A_{core}\cos(B_{core}x)$$

$$\phi_{reflector} = A_{reflector}\sinh\left[\frac{a/2 + b - x}{L_{reflector}}\right]$$

$$\text{where } B_{core} = \frac{\nu\Sigma_{f,core} - \Sigma_{a,core}}{D_{core}},\ L_{reflector} = \sqrt{D_{reflector}/\Sigma_{a,reflector}}$$

   d. Sketch the flux distributions that you would expect. Explain any significant features.

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8. [15 Marks] From the general set of equations given at the end of this exam:
   a. Set up the **differential** equations that need to be solved (but do not attempt to solve) for the **transient** case of two neutron groups (fast and thermal), no upscatter, no fast fissions, no thermal births. Include the flux, precursor, poison and fuel depletion equations. State your simplifying assumptions and any initial or boundary conditions that you deem relevant.
   b. Set up a simplified and reduced version of the equation set of (a) to model the steady state flux distribution. Do not attempt to solve. Give reasons for your answers. *Hints: What role do the delayed precursors play in the steady state? What role do the poisons play? How is fuel depletion treated in the short term?*
   c. Produce a simplified and reduced version of the equation set of (a) to model fast transients (for times up to a few seconds). *Hint: What can you assume about the precursors, poisons and fuel equations?*
   d. Set up a simplified and reduced version of the equation set of (a) to model a short term reactor transient (for times up to a few minutes). Do not attempt to solve. *Hint: What can you assume about the flux equations in order to speed up the simulation without loss in accuracy? Do you need to track fuel depletion and the poisons?*
   e. Set up a simplified and reduced version of the equation set of (a) to model the xenon and iodine transients as a result of startup from a fresh core. Assume that the ramp up from zero to 100% full power takes places over a short time period compared to the xenon transient time constant. Do not attempt to solve. *Hint: Why do you need to simulate the flux and precursor distributions in space? Do you need to do a transient flux and precursor simulaion? Why or why note? Do you need to track fuel depletion?*

9. [10 Marks] From the general set of equations given at the end of this exam:
   a. Set up the **numerical** difference equation version of the differential equations for the **transient** case of two neutron groups (fast and thermal), no upscatter, no fast fissions, no thermal births. Include the flux, precursor, poison and fuel depletion equations. State your simplifying assumptions that you deem relevant. For simplicity, assume a one-dimensional Cartesian geometry, equal grid spacing, space independent parameters.
   b. How would you handle the initial and boundary conditions? Are the interface conditions necessary? Why or why not?
   c. Outline a solution procedure for a scheme of your choosing. Don't forget to include a controller to control the flux to a desired level.
   d. How could you alter this general numerical code to efficiently simulate the various cases in question 8?

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Additional Information:

The general multigroup neutron diffusion equations with delayed precursors are given by:

$$\frac{1}{v_g}\frac{\partial \phi_g}{\partial t} = \nabla \cdot D_g \nabla \phi_g - \Sigma_{a,g}\phi_g - \Sigma_{s,g}\phi_g + \sum_{g'=1}^{G}\Sigma_{s,g'g}\phi_{g'} + \chi_g(1-\beta)\sum_{g'=1}^{G}\nu_{g'}\Sigma_{f,g'}\phi_{g'} + \chi_g^C\sum_{i=1}^{N}\lambda_i C_i + S_g^{ext}$$

$$\frac{\partial C_i}{\partial t} = -\lambda_i C_i + \beta_i \sum_{g'=1}^{G}\nu_{g'}\Sigma_{f,g'}\phi_{g'}$$

Note that ϕ_g and C_i are functions of $\mathbf{r}$ and t but the notation has been dropped for clarity. The poison equations are:

$$\frac{\partial I(\mathbf{r},t)}{\partial t} = \gamma_I \sum_{g'=1}^{G}\Sigma_{f,g'}\phi_{g'}(\mathbf{r},t) - \lambda_I I(\mathbf{r},t)$$

$$\frac{\partial X(\mathbf{r},t)}{\partial t} = \gamma_X \sum_{g'=1}^{G}\Sigma_{f,g'}\phi_{g'}(\mathbf{r},t) + \lambda_I I(\mathbf{r},t) - \lambda_X X(\mathbf{r},t) - \sum_{g'=1}^{G}\sigma_{a,g'}^{X}\phi_{g'}(\mathbf{r},t)X(\mathbf{r},t)$$

and the fuel depletion equation is:

$$\frac{\partial N_f}{\partial t} = -N_f(\mathbf{r},t)\sum_{g'=1}^{G}\sigma^f{}_{a,g'}\phi_{g'}(\mathbf{r},t)$$

The point kinetic equations are:

$$\frac{dn}{dt} = \left(\frac{\rho - \beta}{\Lambda}\right)n(t) + \sum_{i=1}^{N}\lambda_i C_i$$

$$\frac{dC_i}{dt} = \frac{\beta_i}{\Lambda}n(t) - \lambda_i C_i$$

The associated 'inhour equation" is:

$$\rho = \frac{\omega\ell}{(1+\omega\ell)} + \frac{1}{(1+\omega\ell)}\sum_{i=1}^{N}\frac{\omega\beta_i}{(\omega+\lambda_i)}$$

The "inverse method" equation is:

$$\rho(t) = \beta + \frac{\Lambda}{n(t)}\frac{dn}{dt} - \beta\int_0^\infty \frac{D(\tau)n(t-\tau)}{n(t)}d\tau \quad \text{where } D(\tau) = \sum_i \frac{\lambda_i\beta_i}{\beta}e^{-\lambda_i\tau}$$

Mathematical relationships:

$$\int_0^\infty te^{-\lambda t}dt = 1/\lambda^2$$

...The End...