ENGINEERING PHYSICS 3W4

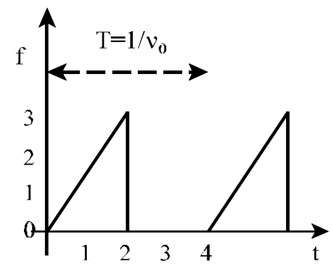
DAY CLASS — Dr. Wm. Garland
DURATION: 30 minutes
McMASTER UNIVERSITY QUIZ #1 — January 27, 2000

Special Instructions: Closed Book. All calculators and up to 3 double sided 8 ½" by 11" crib sheets are permitted.

THIS EXAMINATION PAPER INCLUDES 1 PAGE AND 4 QUESTIONS.

1. [10 marks] For the ramp function as shown, what is the D_0 term of the expansion:

$$f(t) = \sum_{-\infty}^{+\infty} D_n e^{2\pi i n \nu_0 t}, \text{ where } D_n = \frac{1}{T}\int_T f(t) e^{-2\pi i n \nu_0 t} dt, \text{ and } \nu_0 \equiv \frac{1}{T}$$



2. [10 marks] Show that f*g = g*f by a simple change of variables in the definition of the convolution.

3. You are given a simple electronic circuit board, comprised of a few capacitors and resistors, that is supposed to be linear in the range ±20 volts (input) and time-invariant for the life of the product (years). It accepts a voltage input and gives a voltage output. You have a pulse generator and an oscilloscope at hand. You conduct a test by applying a short (say 1 ms) 10 volt pulse in and you measure the voltage out over time. You find that the output voltage immediately jumps to 5 volts and decays exponentially with a time constant about 1 s.
   a. [5 marks] Sketch the system's impulse response function, h(t).
   b. [5 marks] How can you determine if the pulse generated by the generator is sufficiently like a δ function to give the correct response? [Hint: How fast does the circuit respond to the pulse?]

4. What additional test(s) could be performed to show that the circuit in Question 3 is, indeed, linear and time-invariant. Specifically address:
   a. [5 marks] What constitutes a linear response? How linear is linear enough?
   b. [5 marks] What constitutes a time-invariant response? How time-invariant is time-invariant enough?

1.

$$f(t) = \sum_{-\infty}^{\infty} D_n e^{2\pi i n \nu_n t}$$

$$D_n = \frac{1}{T} \int_T f(t) e^{-2\pi i n \nu_n t} dt$$

for $n = 0$, $\quad D_0 = \frac{1}{T} \int_T f(t)\, dt$

$T = 4.$

$$= \frac{1}{T} \int_0^2 \tfrac{3}{2} t \, dt = \frac{1}{4} \cdot \frac{3}{2} \left. \frac{t^2}{2} \right|_0^2$$

$$= \frac{3}{8} \frac{4^2}{2} = \underline{3/4} = D_0$$

You can also see this by inspection: Since D_0 is the D.C. component, the average of $f(t)$ over a cycle is $\frac{\frac{1}{2}(3 \times 2)}{4} = 3/4$.

2. $$f * g = \int_{-\infty}^{\infty} f(\tau) g(t-\tau)\, d\tau$$

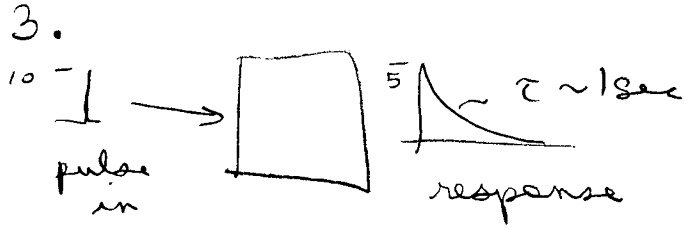
Let $\tau = t - z$

$$\therefore f * g = \int_{\infty}^{-\infty} f(t-z)\, g(z)\, d(-z)$$

$$= \int_{-\infty}^{\infty} g(z)\, f(t-z)\, dz$$

$$= \underline{g * f} \qquad \text{QED}$$

3.



a) $h(t) = \frac{1}{2} e^{-t/\tau}$, $\tau = 1$ second.

b) Try putting in longer + shorter pulses (with the same integrated area as the original pulse). If you get the same response, then you know that the response is not sensitive to the time duration of the pulse. You wouldn't expect any difference in response to a 1 ms pulse compared to a 5 ms pulse when the system time constant is 1 second. But if the τ had been 10 ms, then a 1 ms pulse may not be 'sharp' enough to represent a δ function.

4. (a) To test for linearity try a 1 ms pulse at various amplitudes. If you get a proportional response, ie V volts in, V/2 volts out, then you have linearity. At some voltage, the linearity will break down.

(b) Run the above tests now, run them again later. You should get the same results. You should vary the environment over the range of the expected product environment + duration.

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