Commonly Used Mathematical Notation

1 Logical Statements

Common symbols for logical statement:

$\vee$ **logical disjunction:** "or"

Note:
in mathematics this is always an "inclusive or"
i.e. "on **or** the other **or both**"

$\wedge$ **logical conjunction:** "and"

$\neg$ **logical negation:** "not"

$\rightarrow$ **material implication:** implies; if .. then

Note:
$P \rightarrow Q$ means:
if P is true then Q is also true;
if P is false then nothing is said about Q

can also be expressed as:
if P then Q
P implies Q
Q, if P
P only if Q
P is a sufficient condition for Q
Q is a necessary condition for P

sometimes writen as $\Rightarrow$

$f : X \rightarrow Y$ **function arrow:** function f maps the set X into the set Y

$\circ$ **function composition:** $f \circ g$ function such that $(f \circ g)(x) = f(g(x))$

$\leftrightarrow$ **material equivalence:** if and only if (iff)

Note:

$P \leftrightarrow Q$ means:

means P is true if Q is true and P is false if Q is false

can also be expressed as:

P, if and only if Q
Q, if and only if P
P is a necessary and sufficient condition for Q
Q is a necessary and sufficient condition for P

sometimes writen as $\Leftrightarrow$

$\ll$ is much less than

$\gg$ is much greater than

$\therefore$ therefore

$\forall$ **universal quantification:** for all/any/each

$\exists$ **existential quantification:** there exists

$\exists!$ **uniqueness quantification:** there exists exactly one

$\equiv$ **definition:** is defined as

Note:

sometimes writen as $:=$

2 Set Notation

A set is some collection of objects. The objects contained in a set are known as elements or members. This can be anything from numbers, people, other sets, etc. Some examples of common set notation:

$\{,\}$ **set brackets:** the set of ...

e.g. $\{a, b, c\}$ means the set consisting of a, b, and c

$\{|\}$ **set builder notation:** the set of ... such that ...

i.e. $\{x|P(x)\}$ means the set of all x for which $P(x)$ is true.

e.g. $\{n \in N : n^2 < 20\} = \{0, 1, 2, 3, 4\}$

Note: $\{|\}$ and $\{:\}$ are equivalent notation

$\emptyset$ **empty set**

i.e. a set with no elements. $\{\}$ is equivalent notation

$\in$ **set membership:** is an element of

$\notin$ is not an element of

2.1 Set Operations

Commonly used operations on sets:

$\cup$ **Union**

$A \cup B$ set containing all elements of A and B.
$A \cup B = \{x \mid x \in A \vee x \in B\}$

$\cap$ **Intersect**

$A \cap B$ set containing all those elements that A and B have in common

$A \cap B = \{x \mid x \in A \wedge x \in B\}$

$\setminus$ **Difference** or **Compliment**

$A \setminus B$ set containing all those elements of A that are not in B
$A \setminus B = \{x \mid x \in A \wedge x \notin B\}$

$\subseteq$ **Subset**

$A \subseteq B$ subset: every element of A is also element of B
$A \subset B$ proper subset: $A \subseteq B$ but $A \neq B$.

$\supseteq$ **Superset**

$A \supseteq B$ every element of B is also element of A.
$A \supset B$ $A \supseteq B$ but $A \neq B$.

2.2 Number Sets

Most commonly used sets of numbers:

$\mathbb{P}$ **Prime Numbers**

Set of all numbers only divisible by 1 and itself.
$\mathbb{P} = \{1, 2, 3, 5, 7, 11, 13, 17...\}$

$\mathbb{N}$ **Natural Numbers**

Set of all positive or sometimes all non-negative intigers
$\mathbb{N} = \{1, 2, 3, ...\}$, or sometimes $\mathbb{N} = \{0, 1, 2, 3, ...\}$

$\mathbb{Z}$ **Intigers**

Set of all integers whether positive, negative or zero.
$\mathbb{Z} = \{..., -2, -1, 0, 1, 2, ...\}$.

$\mathbb{Q}$ **Rational Numbers**

Set of all fractions

$\mathbb{R}$ **Real Numbers**

Set of all rational numbers and all irrational numbers
(i.e. numbers which cannot be rewritten as fractions, such as π, e, and $\sqrt{2}$).

Some variations:

$\mathbb{R}^+$ All positive real numbers
$\mathbb{R}^-$ All positive real numbers
$\mathbb{R}^2$ Two dimensional $\mathbb{R}$ space
$\mathbb{R}^n$ N dimensional $\mathbb{R}$ space

$\mathbb{C}$ **Complex Numbers**

Set of all number of the form:

$a + bi$

where:

a and b are real numbers, and
i is the imaginary unit, with the property $i^2 = -1$

Note: $\mathbb{P} \subset \mathbb{N} \subset \mathbb{Z} \subset \mathbb{Q} \subset \mathbb{R} \subset \mathbb{C}$