Proof of Odds form of Bayes Theorem.

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$$P(A|B) = \frac{P(A)\,P(B|A)}{P(B)}$$

$$O(A|B) = \frac{P(A|B)}{1 - P(A|B)}$$

$$= \frac{P(A)\,P(B|A)}{P(B)} \times \frac{1}{1 - \frac{P(A)\,P(B|A)}{P(B)}}$$

$$= \frac{P(A)P(B|A)}{P(B) - P(A)P(B|A)}$$

$$= \frac{P(A)\;P(B|A)}{P(B|A)P(A) + P(B|\bar{A})P(\bar{A}) - P(A)P(B|A)}$$

$$= P(A)\,\frac{P(B|A)}{P(B|\bar{A})P(\bar{A})}$$

$$= \frac{P(A)}{P(\bar{A})}\,\frac{P(B|A)}{P(B|\bar{A})}$$

$$= O(A)\,\frac{P(B|A)}{P(B|\bar{A})} \qquad \text{QED}$$

Note: Odds: $O = \frac{P}{1-P} \iff P = \frac{O}{1+O}$