RFSP Model

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Static Two-Energy-Group Neutron Diffusion Equation for Eigenvalue Problems

Two-group equation solved by RFSP:

$$-\vec{\nabla} \cdot D_1(\vec{r})\,\vec{\nabla}\phi_1(\vec{r}) + (\Sigma_{a1}(\vec{r}) + \Sigma_m(\vec{r}))\,\phi_1(\vec{r}) - \frac{\nu\Sigma_f(\vec{r})}{k_{eff}}\phi_2(\vec{r}) = 0$$

(3.1)

$$-\vec{\nabla} \cdot D_2(\vec{r})\,\vec{\nabla}\phi_2(\vec{r}) + \Sigma_{a2}(\vec{r})\,\phi_2(\vec{r}) - \Sigma_m(\vec{r})\,\phi_1(\vec{r}) = 0$$

Static Two-Energy-Group Neutron Diffusion Equation for Eigenvalue Problems (con't)

where, at position $\vec{r}$

$\phi_1(\vec{r})$, $\phi_2(\vec{r})$ = group-1, group-2 neutron flux respectively,

$\Sigma_{a1}(\vec{r})$ = group-1 (fast) absorption cross-section,

$\Sigma_m(\vec{r})$ = moderation (down-scattering) cross-section,

$\nu\Sigma_f(\vec{r})$ = neutron-production cross section,

$\Sigma_{a2}(\vec{r})$ = group-2 (thermal) absorption cross-section,

$D_1(\vec{r})$ = group-1 (fast) diffusion coefficient,

$D_2(\vec{r})$ = group-2 (thermal) diffusion coefficient, and

k_{eff} = reactor multiplication constant (inverse of eigenvalue)

Diffusion coefficients D_g are related to transport cross-sections:

$$D_g(\vec{r}) = \frac{1}{3\Sigma_{tr,g}(\vec{r})} \qquad g = 1,2 \qquad (3.2)$$

Diffusion Equation in Matrix Notation

We can write the diffusion equation in *matrix* notation.

Define the flux vector:

$$\phi(\vec{r}) = \begin{pmatrix} \phi_1(\vec{r}) \\ \phi_2(\vec{r}) \end{pmatrix}$$

The equation can be written as an *eigenvalue* equation:

$$M\phi(\vec{r}) = \frac{F\phi(\vec{r})}{k_{eff}} \equiv \lambda F\phi(\vec{r})$$

where *M* is the scattering, leakage, and absorption matrix:

$$M = \begin{pmatrix} -\vec{\nabla}\cdot D_1\vec{\nabla} + \Sigma_{a1}(\vec{r}) + \Sigma_{1\to 2}(\vec{r}) & 0 \\ -\Sigma_{1\to 2}(\vec{r}) & -\vec{\nabla}\cdot D_2\vec{\nabla} + \Sigma_{a2}(\vec{r}) \end{pmatrix}$$

and *F* is the production matrix:

$$F = \begin{pmatrix} 0 & \nu\Sigma_f(\vec{r}) \\ 0 & 0 \end{pmatrix}$$

and the eigenvalue $\lambda = \frac{1}{k_{eff}}$

Notes to Diffusion Equation in RFSP

- No upscattering
- No explicit fast fission; i.e. $\Sigma_{f1} = 0$ [Σ_{f1} lumped into Σ_{f2}]

  $\Rightarrow$ "1-and-a-half groups" [P.S. True two-group version being developed so can use directly properties from WIMS-AECL]
- Eigenvalue equation: has solution only for certain values of k_{eff} ($\Rightarrow$ *flux harmonics*)
- Largest value of k_{eff} corresponds to the *fundamental (physical steady state*)
- "Gap" between k_{eff} and 1.0 tells us how far the reactor is from criticality (in configuration modelled)
- Related quantity, not in equation: "H" factor - ratio of bundle power to bundle-average flux; H factors translate flux distribution to power distribution
- RFSP solves the *finite-difference* form of the diffusion equation; fluxes are calculated at mid-points of *mesh cells*.

Notes Relating to ^{135}Xe

Nuclear properties are not a function of *configuration* only:

Suppose some configuration is being modelled (i.e., a certain irradiation distribution, certain device positions).

Does not uniquely define properties of core; properties will depend also on *power level*.

One (*but not the only*) reason for this is the *saturating fission products*: most important is ^{135}Xe (what are some others?).

Concentration of ^{135}Xe increases with flux (power), so local properties will depend on flux level. ^{135}Xe is a strong absorber.

This is *implicit* in the diffusion equation. We may wish to show it *explicitly* by writing, for example,

$$\Sigma_{a2}(\vec{r}) = \Sigma_{a2,noXe}(\vec{r}) + \Delta\Sigma_{a2,Xe}(\vec{r})$$

[effect of ^{135}Xe often included wholly in thermal absorption, Σ_{a2}]

Notes Relating to ^{135}Xe (con't)

In the above, the effect of ^{135}Xe will vary with position, depending on local flux (power) - greatest absorption at highest powers
$\Rightarrow$ "distributed" ^{135}Xe.

XE trailer card in *SIMULATE and *TIME-AVER calculates effect of distributed ^{135}Xe according to above equation.

Data Prerequisite to Solve Diffusion Equation

- ***What*** **is in the reactor**
- ***Where*** **it is**
- ***In what state*** **it is (e.g., *irradiation*)**

Geometry

Length and radius of calandria, radius of notch

Reflector thickness

Lattice pitch, bundle length

Mesh lines used in finite-difference model

Distinction between *mesh array* (for flux calculation) and *lattice array* (defining fuel bundles)

Axial and radial extrapolation lengths

This data entered via *DATA GEOMETRY cards.

Lattice Properties

Dave Jenkins has explained earlier the 3-tier scheme for CANDU neutronics calculations: lattice properties, device properties, superposition.

Lattice properties are calculated for each basic lattice cell of reactor (also for reflector).

The lattice cell is quite complex (fuel, coolant, tubes, moderator). This detail can be handled only by transport theory. The *cell code*, e.g. POWDERPUFS-V (PPV) *homogenizes* the properties over the cell volume to permit diffusion calculations.

Properties depend on:

- **dimensions**
- **type and amount of fuel**
- **type, amount, and purity of coolant and moderator,**
- **temperatures,**
- **fuel irradiation**

First 4 usually don't vary *drastically* over core. But

Fuel irradiation = flux * time [$\omega = \phi \cdot t$] (related to burnup)

Does vary a lot from bundle to bundle - from 0 to exit value.

$\therefore$ Lattice properties often provided as *fuel tables* (vs. irradiation) by cell code.

Device Properties

Device properties given as *incremental cross-sections* over some volume of influence: $\Delta\Sigma$'s calculated by device code (e.g., MULTICELL) and added to Σ's over volume of influence.

Not usually function of device irradiation or fuel irradiation.

Property Smearing

The finite-difference model is a collection of rectangular parallelepipeds, defined by the *mesh lines*. The RFSP user defines the mesh lines in the model.

By definition, the nuclear properties are *uniform* over each parallelepiped (and the flux is calculated at its mid-point).

However, the properties are not *directly* defined over the parallelepipeds. They are defined over:

- lattice cells (for lattice properties)
- defined volume of influence (for reactivity devices)

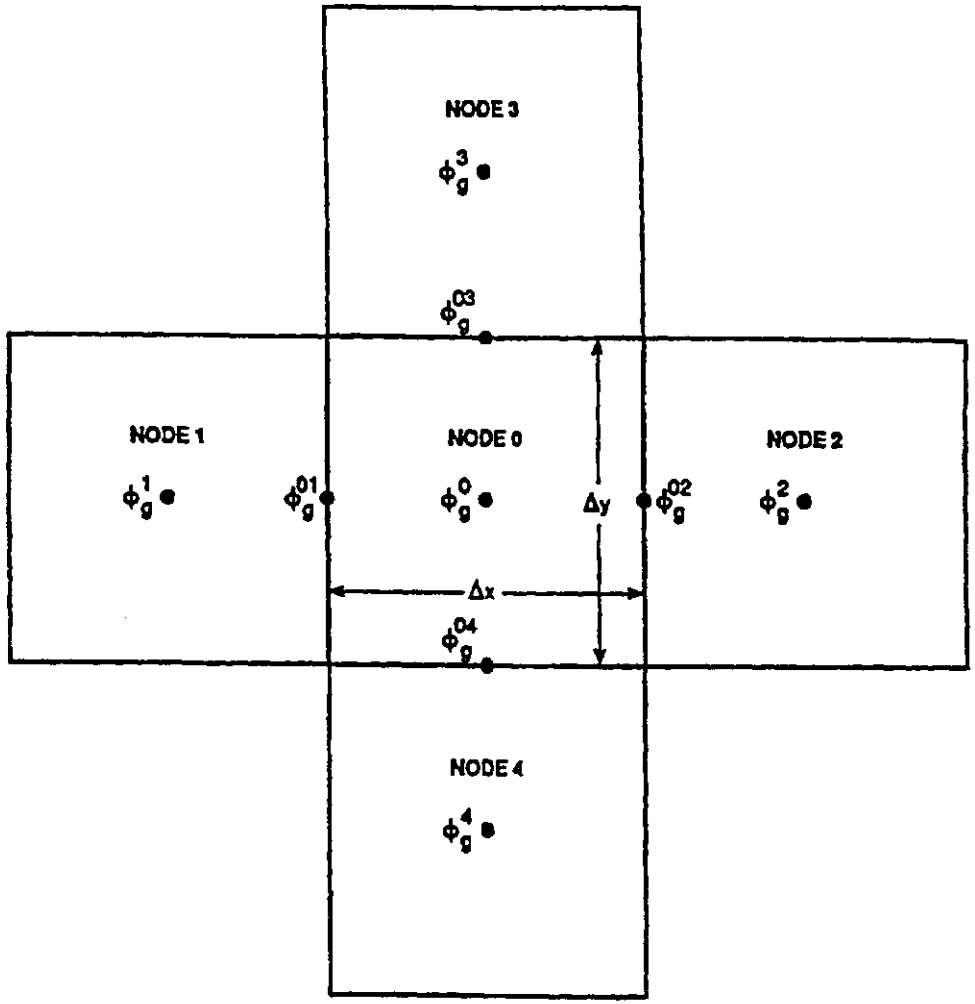
Property Smearing (con't)

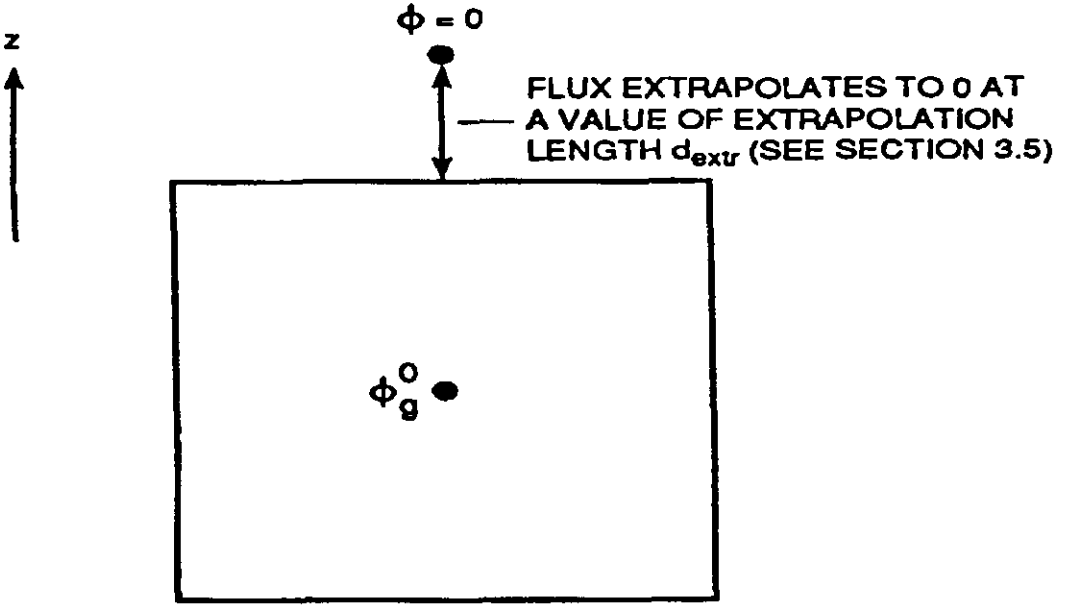
If the boundaries of the lattice cells and/or of the device volumes of influence do not coincide with mesh lines, the properties will be smeared to the *next* (outwardly) mesh line. The smearing involves "diluting" the cross-sections in inverse proportion to volume.

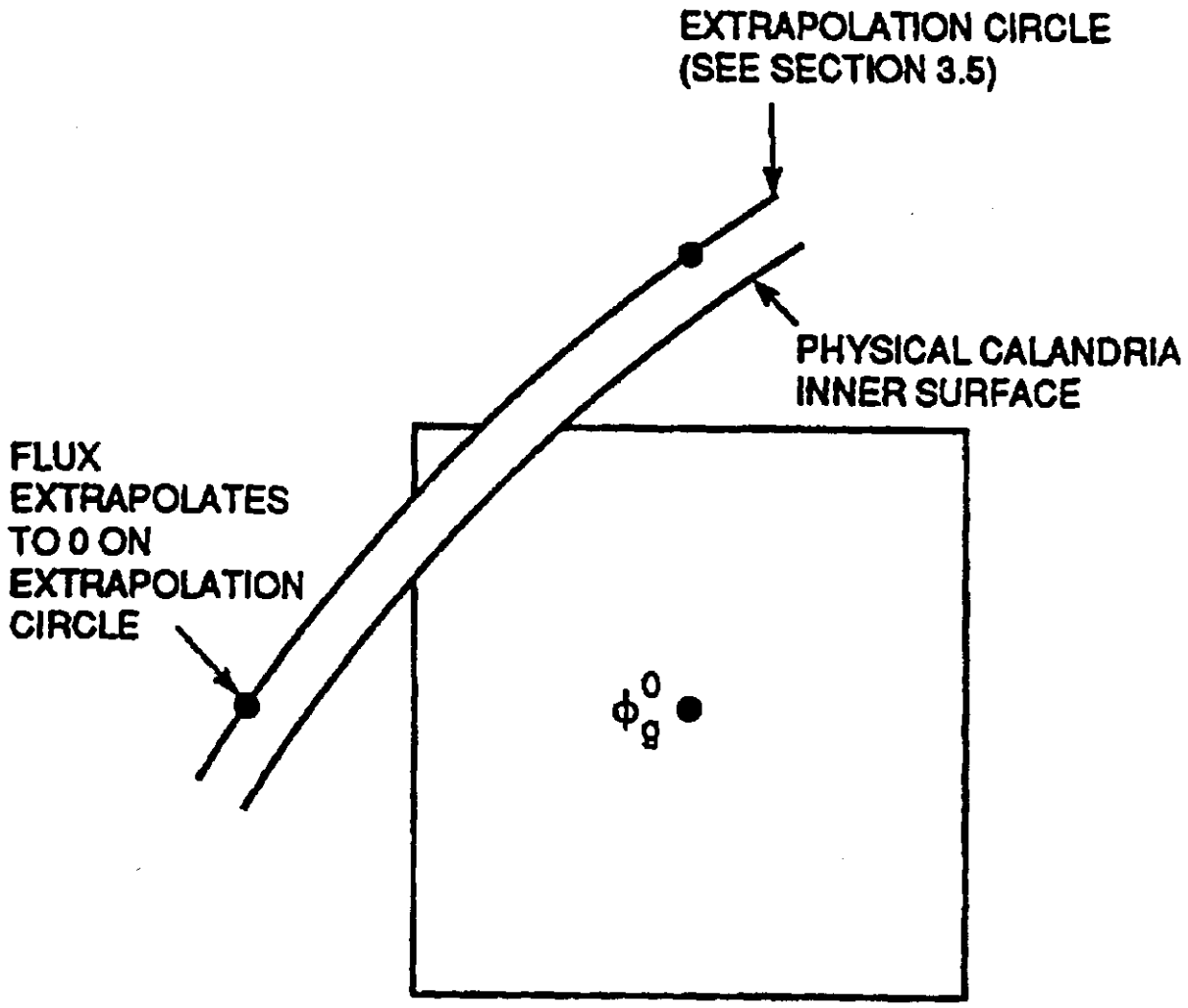
Smearing is not recommended because:

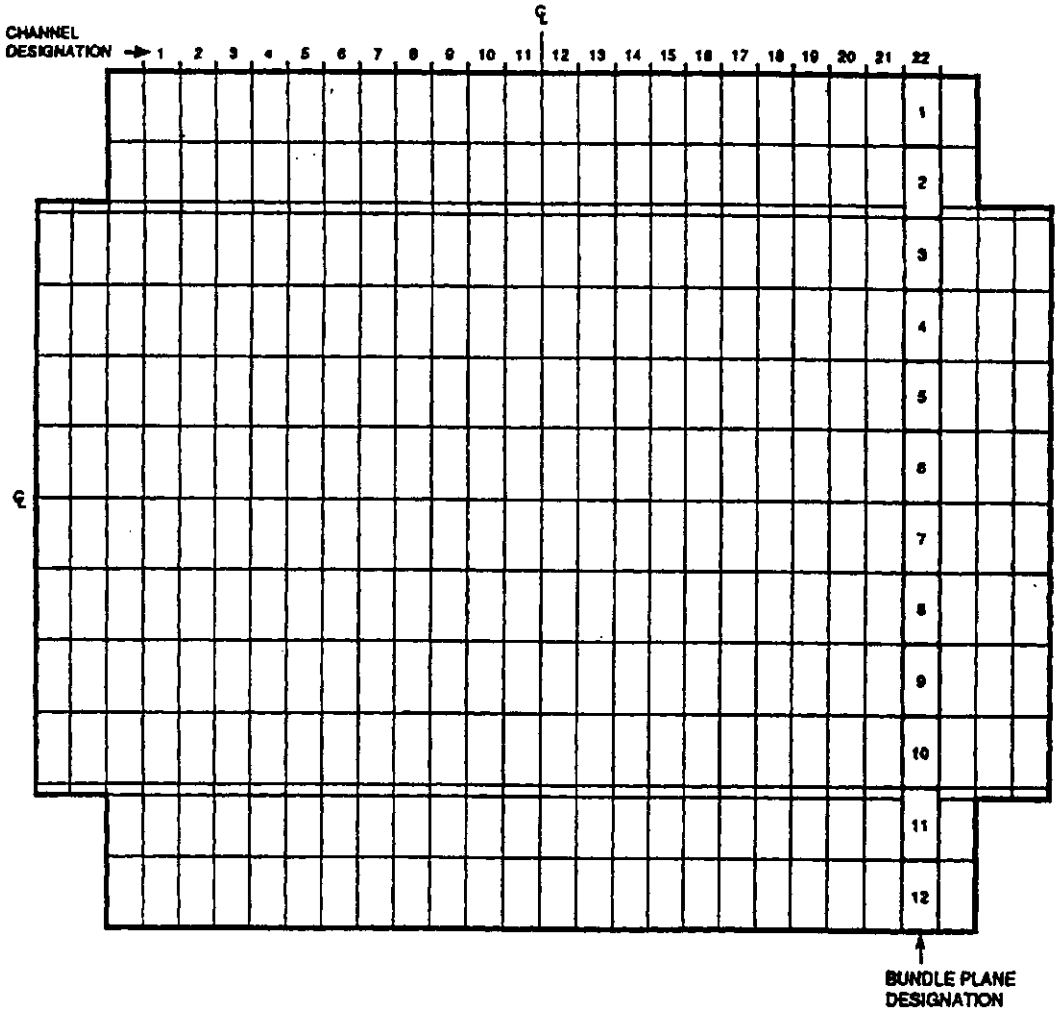
- It denies the volume over which the cell code or device code "wanted" to act especially in terms of absorption, a wider volume of action increases the absorption (even in the presence of dilution); e.g., reactivity worth of shutoff rods is artificially enhanced - not conservative.

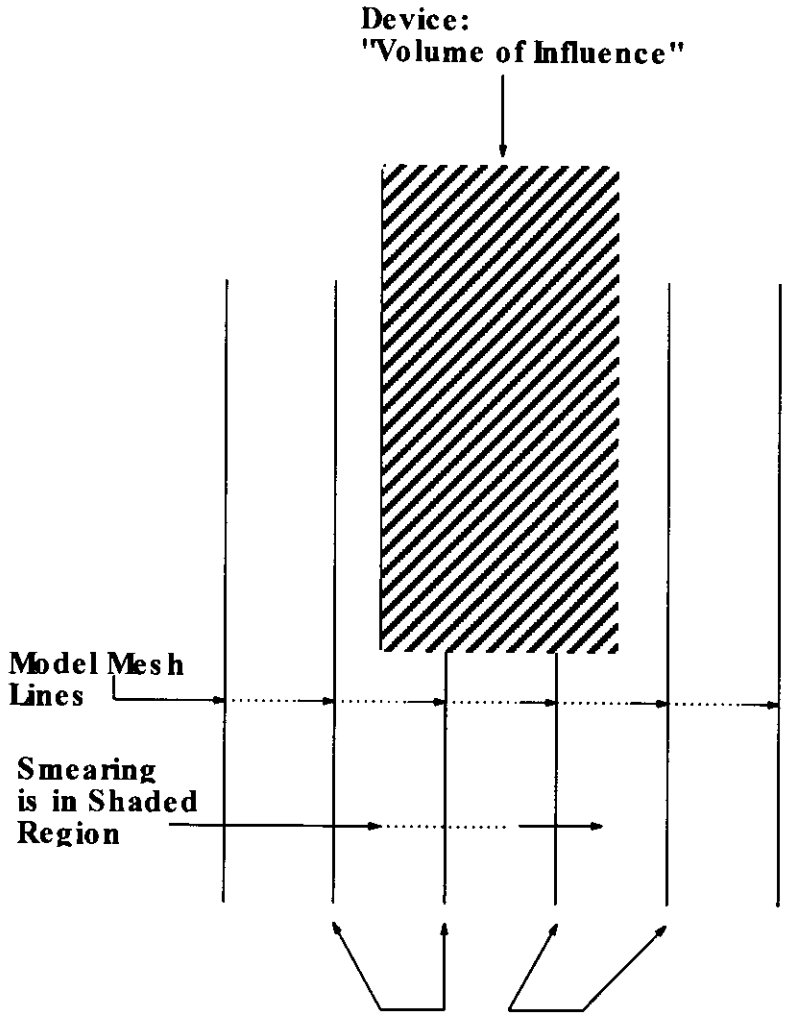
Moral of Story: ensure your model has no smearing


NODE 3
ϕ_g^3
ϕ_g^{03}
NODE 1
NODE 0
NODE 2
ϕ_g^1
ϕ_g^{01}
ϕ_g^0
Δy
ϕ_g^{02}
ϕ_g^2
Δx
ϕ_g^{04}
NODE 4
ϕ_g^4

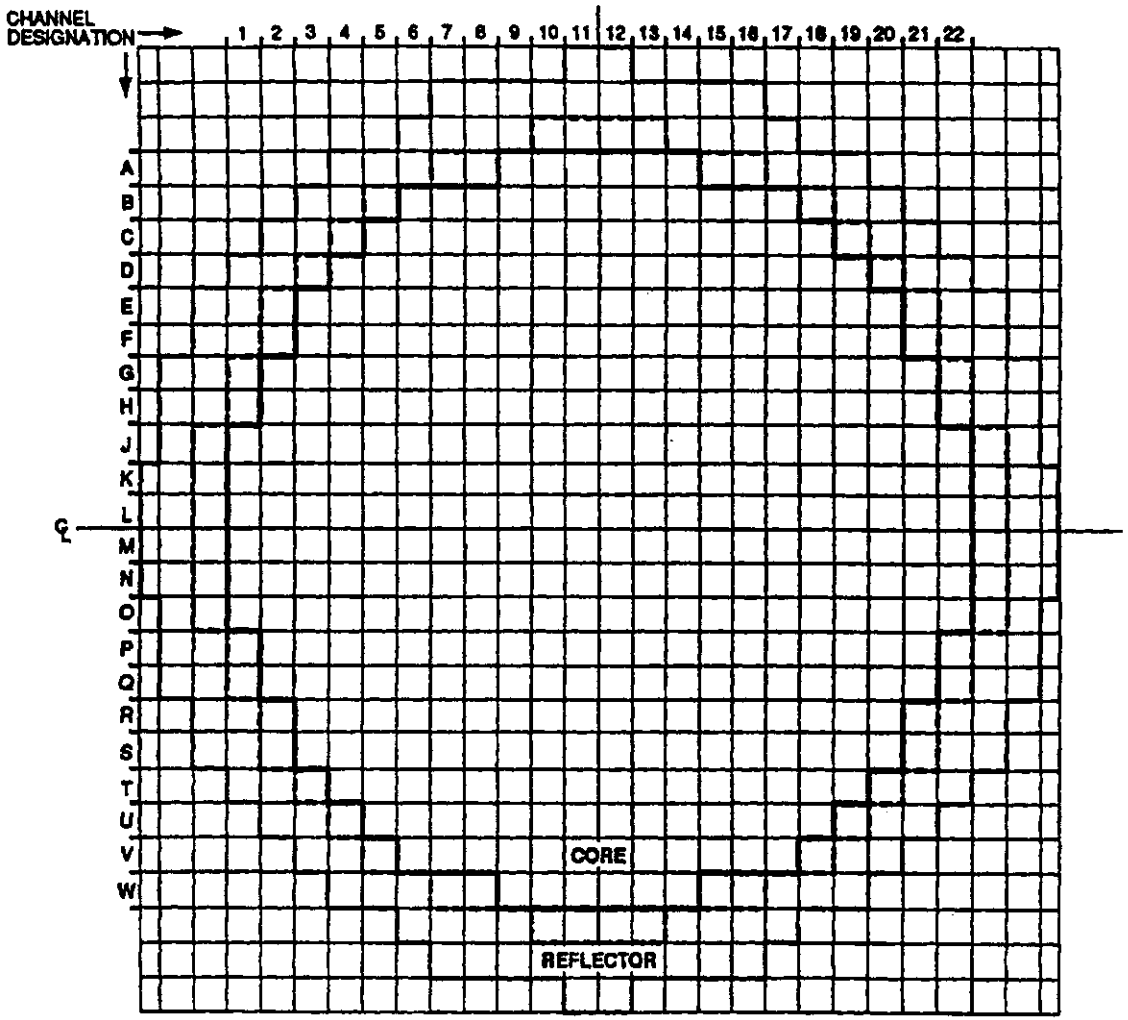

z
ϕ = 0
FLUX EXTRAPOLATES TO 0 AT
A VALUE OF EXTRAPOLATION
LENGTH d_{extr} (SEE SECTION 3.5)
ϕ_g^0

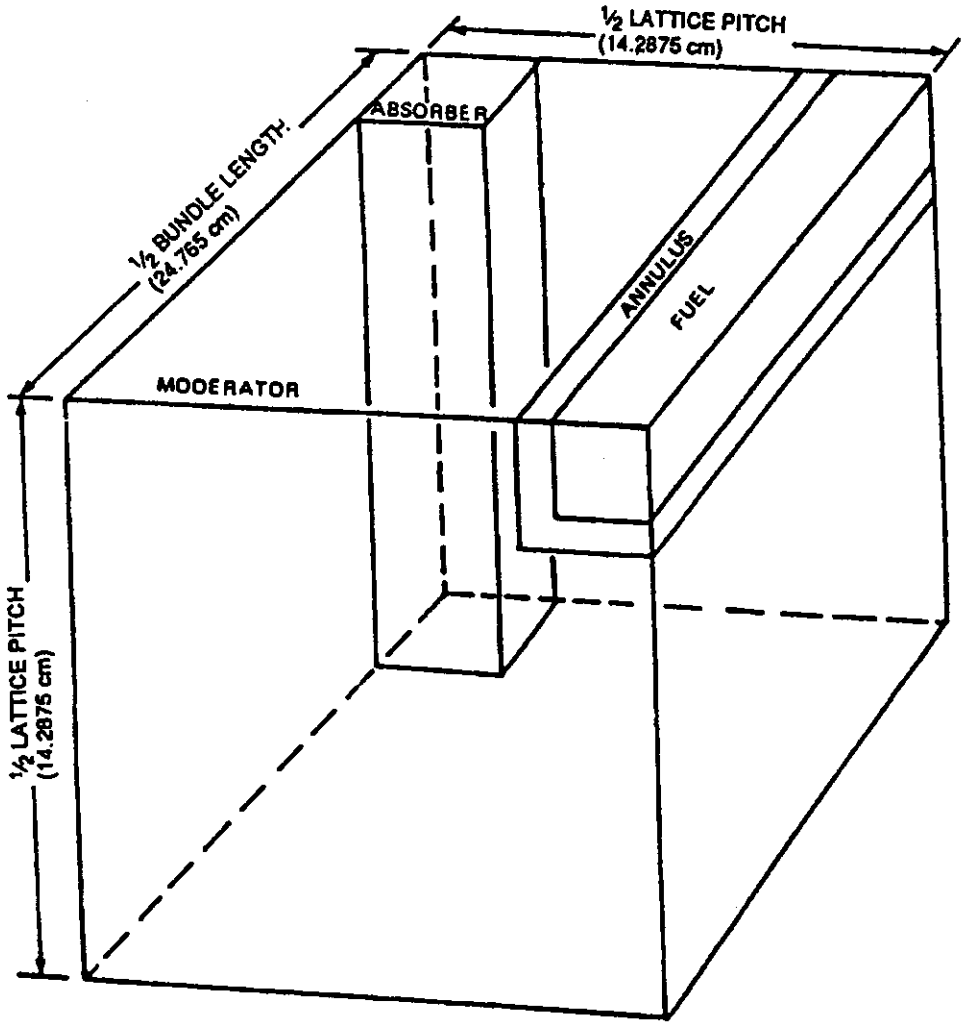

EXTRAPOLATION CIRCLE
(SEE SECTION 3.5)
PHYSICAL CALANDRIA
INNER SURFACE
FLUX
EXTRAPOLATES
TO 0 ON
EXTRAPOLATION
CIRCLE
ϕ_g^0

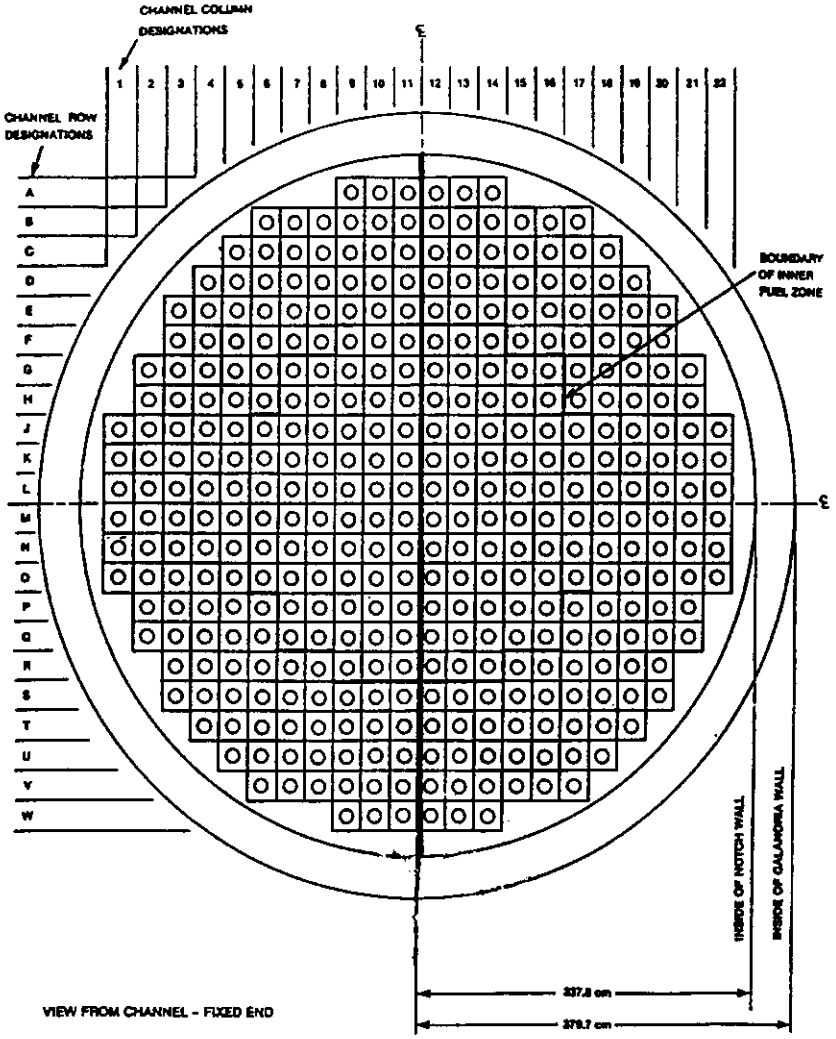

CHANNEL DESIGNATION
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22
1 2 3 4 5 6 7 8 9 10 11 12
BUNDLE PLANE DESIGNATION

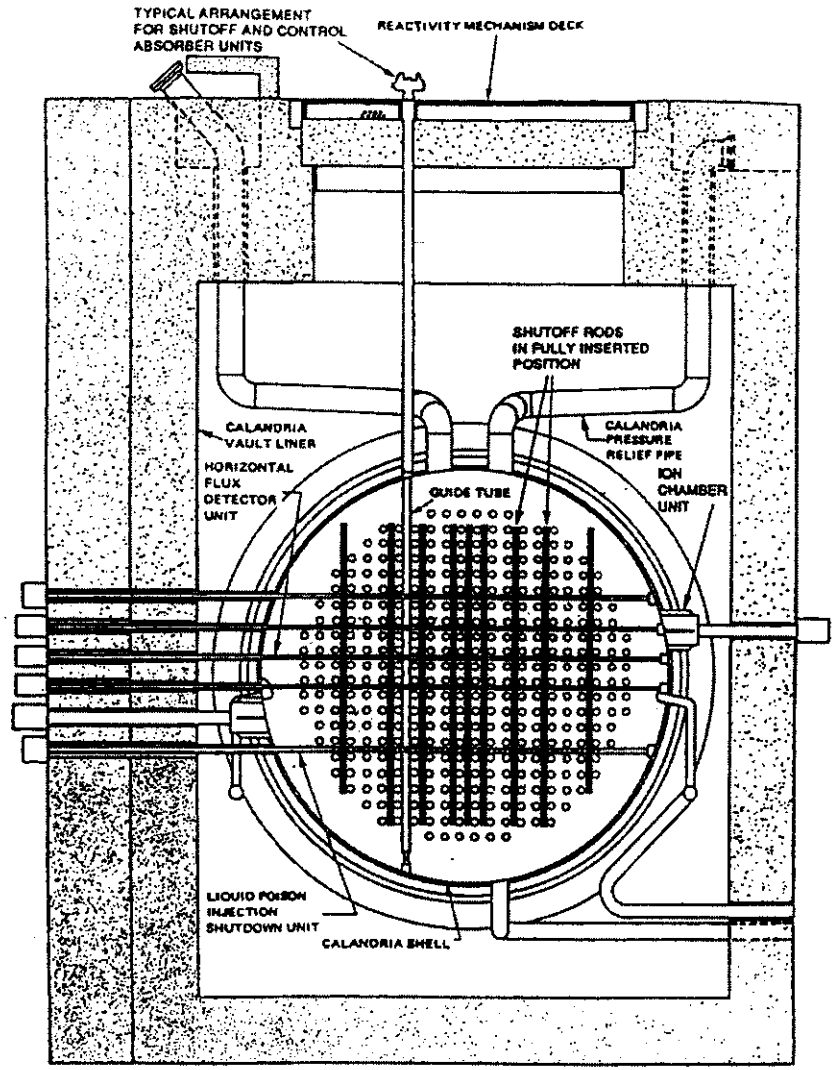


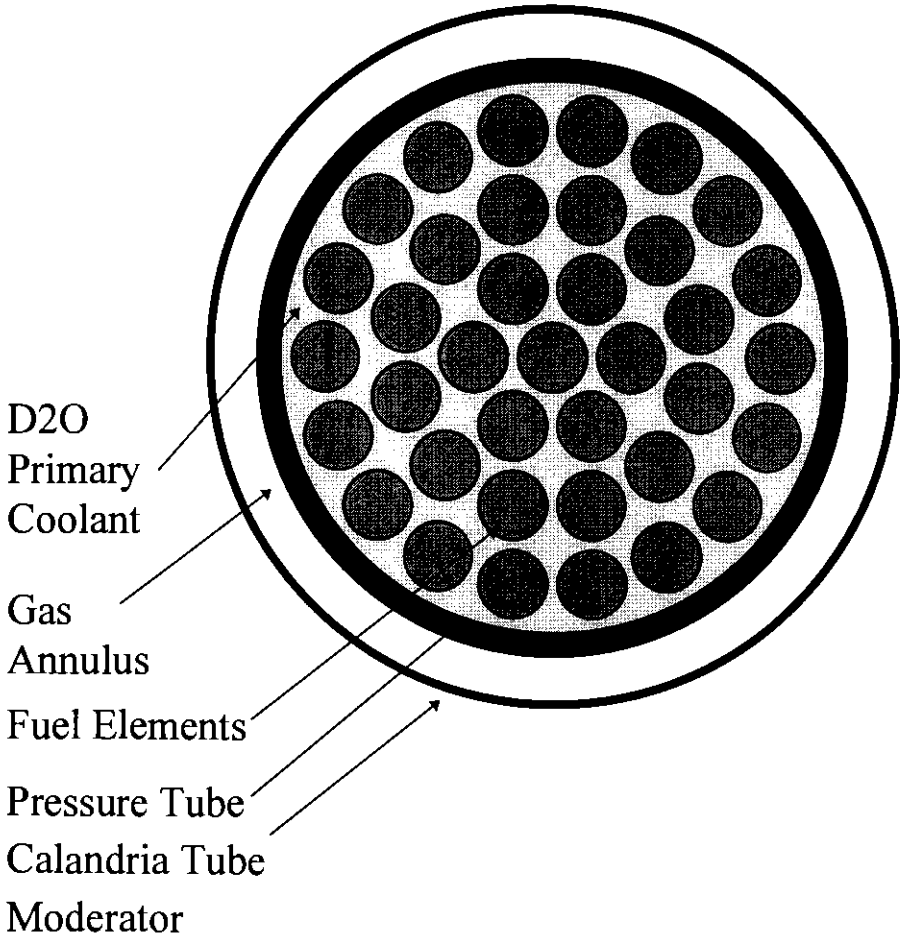
Mesh Lines to Which Device Properties are Smeared

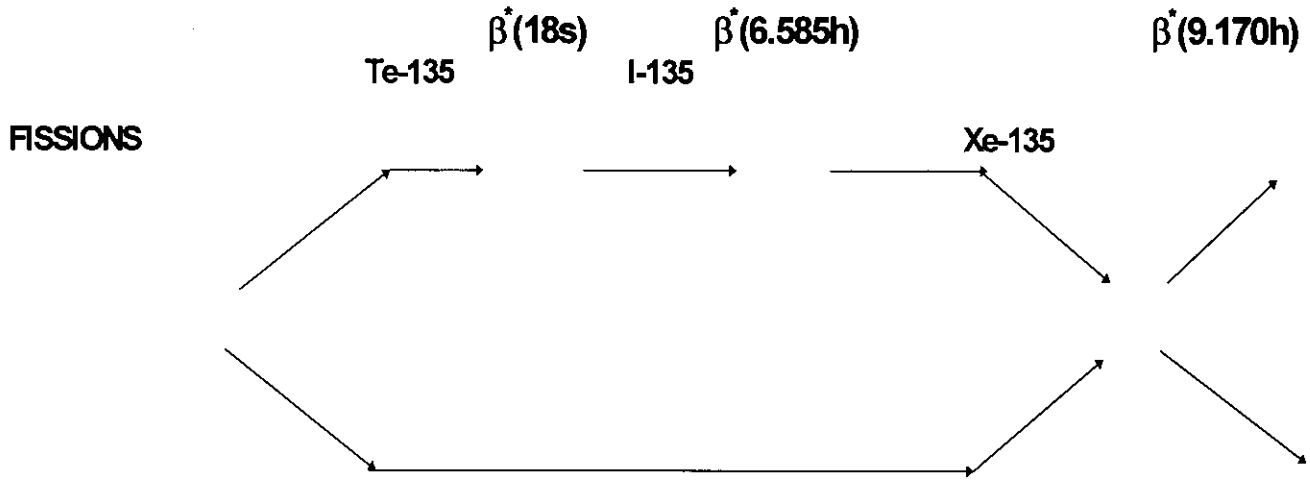

CHANNEL DESIGNATION
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22
A B C D E F G H J K L M N O P Q R S T U V W
CORE
REFLECTOR


½ LATTICE PITCH
(14.2875 cm)
½ BUNDLE LENGTH
(24.765 cm)
ABSORBER
ANNULUS
FUEL
MODERATOR
½ LATTICE PITCH
(14.2875 cm)


CHANNEL COLUMN DESIGNATIONS
CHANNEL ROW DESIGNATIONS
BOUNDARY OF INNER FUEL ZONE
INSIDE OF NOTCH WALL
INSIDE OF CALANDRIA WALL
337.8 cm
379.7 cm
VIEW FROM CHANNEL - FIXED END


TYPICAL ARRANGEMENT FOR SHUTOFF AND CONTROL ABSORBER UNITS
REACTIVITY MECHANISM DECK
SHUTOFF RODS IN FULLY INSERTED POSITION
CALANDRIA VAULT LINER
CALANDRIA PRESSURE RELIEF PIPE
HORIZONTAL FLUX DETECTOR UNIT
GUIDE TUBE
ION CHAMBER UNIT
LIQUID POISON INJECTION SHUTDOWN UNIT
CALANDRIA SHELL


D2O
Primary
Coolant
Gas
Annulus
Fuel Elements
Pressure Tube
Calandria Tube
Moderator


β*(18s)
β*(6.585h)
β*(9.170h)
Te-135
I-135
FISSIONS
Xe-135

Three Irradiation-Distribution Models

To calculate lattice properties, need to know or model the irradiation distribution.

In CANDU modelling, 3 types of irradiation-distribution models have been used:

- **(axially) homogeneous model**
- **time-average model**
- **snapshot model**

We'll treat each in turn.

"Homogeneous" Model

This model is based on *continuous refuelling* approximation, and on *bi-directional* feature of CANDU refuelling.

Consider two neighbouring channels (refuelled in opposite directions); fuel moves at speed *r*.

Fuel irradiation in each channel at position $x = x_0$ is integral over time of fuel flux as fuel travels through the core to $x = x_0$:

$$\omega_1(x_0) = \int_0^{t_1(x_0)} \hat{\phi}(x(t))\,dt \qquad (4.1)$$

$$\omega_2(x_0) = \int_0^{t_2(x_0)} \phi(x(t))\,dt \qquad (4.2)$$

"Homogeneous" Model (con't)

where $t_1(x_0)$ and $t_2(x_0)$ are the times to reach position x_0:

$$t_1(x_0) = \frac{x_0}{r} \qquad (4.3)$$

$$t_2(x_0) = \frac{L - x_0}{r} \qquad (4.4)$$

Use Equations (4.3), (4.4) to rewrite ω_1 and ω_2 as integrals over x:

$$\omega_1(x_0) = \int_0^{x_0} \hat{\phi}(x)\frac{dx}{r} = \frac{1}{r}\int_0^{x_0} \hat{\phi}(x)\,dx \qquad (4.5)$$

$$\omega_2(x_0) = \int_L^{x_0} \hat{\phi}(x)\left(-\frac{dx}{r}\right) = \frac{1}{r}\int_{x_0}^{L} \hat{\phi}(x)\,dx \qquad (4.6)$$

"Homogeneous" Model (con't)

The average fuel irradiation at $x = x_0$ over the two channels is thus:

$$\omega_{avge}(x_0) = \frac{1}{2}\left[\omega_1(x_0) + \omega_2(x_0)\right]$$

$$= \frac{1}{2r}\left[\int_0^{x_0} \hat{\phi}(x)\,dx + \int_{x_0}^{L} \hat{\phi}(x)\,dx\right]$$

$$= \frac{1}{2r}\int_0^{L} \hat{\phi}(x)\,dx \qquad (4.7)$$

average irradiation is uniform along the two channels, and is one half of fuel exit irradiation (ω_{exit}) at channel outlet:

$$\omega_{exit} = \omega_1(L) = \omega_2(0) = \frac{1}{r}\int_0^{L} \hat{\phi}(x)\,dx \qquad (4.8)$$

"Homogeneous" Model (con't)

Therefore appropriate within "homogeneous"-model assumptions to use uniform basic-lattice properties along length of the two channels.

Homogeneous-model cross-sections labelled $\Sigma_i(\text{hom})$, with i = various categories of cross-section. "Correct" value of $\omega_i(\text{hom})$ to use along the channels is the "average" value which preserves total reaction rate for fuel as it travels through core. Thus:

$$\Sigma_i(\text{hom}) = \frac{\text{Reaction rate averaged over time}}{\text{Flux averaged over time}}$$

$$= \frac{\frac{1}{T}\int_0^T \Sigma_i(\omega(t))\,\hat{\phi}(x(t))\,dt}{\frac{1}{T}\int_0^T \hat{\phi}\,(x(t))\,dt} \qquad (4.9)$$

"Homogeneous" Model (con't)

where T = transit time of fuel through core:

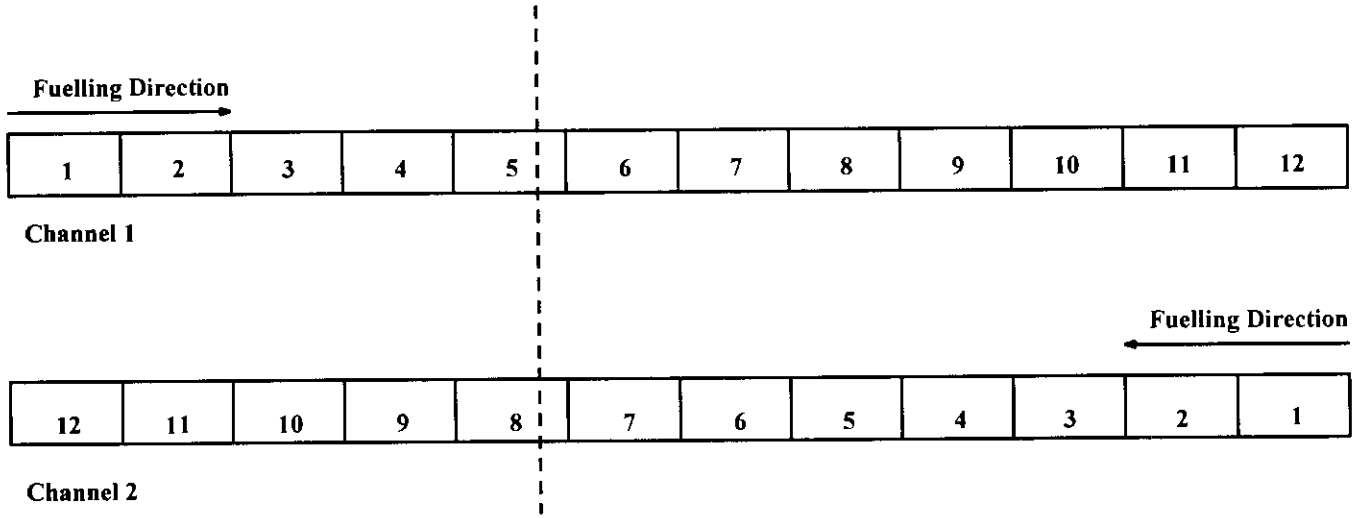
$$T = \frac{L}{r}$$

Integrals over time can be replaced by integrals over irradiation ($d\omega = \hat{\phi}\, dt$):

$$\Sigma_i(\text{hom}) = \frac{1}{\omega_{exit}} \int_0^{\omega_{exit}} \Sigma_i(\omega)\, d\omega \qquad (4.10)$$

ω_{exit} is specific to channel pair. Most often, a 2-region model was used, with inner-core and outer-core values of ω_{exit}.

Eq.(4.10) is the model for averaging cross-sections over time.


Fuelling Direction
1
2
3
4
5
6
7
8
9
10
11
12
Channel 1
Fuelling Direction
12
11
10
9
8
7
6
5
4
3
2
1
Channel 2