Electrical Equipment - Course PI 30.2

POWER AND POWER FACTOR

OBJECTIVES

On completion of this module the student will be able to:

1. In a few sentences explain the following terms:
   a) active power;
   b) reactive power;
   c) apparent power.

2. Draw a power triangle to represent a specific single phase load.

3. Briefly in writing define "power factor", and show the relationship between the apparent power and active power at:
   pf=1
   pf=0.

4. Given the information as related to the active, reactive, apparent power and power factor, calculate the indicated quantity.

5. State in writing two common methods of improving the power factor.

1.0 INTRODUCTION

This lesson introduces the student to the concept of:

(a) active, reactive and apparent power.

(b) power factor.

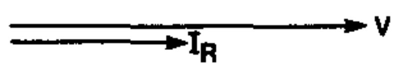
(c) methods of power factor correction.

2.0 POWER

Power is the rate at which energy is consumed in a circuit, resulting in useful work being done by lights, heaters, motors, etc.

2.1 Power in a Resistor

When an alternating current flows through a resistor, energy is dissipated as heat. The rate of energy consumption is termed the active power, P, and is calculated as $P = V \times I_R$ where V is the rms voltage across the resistor and I_R is the rms current through it.



V-I In a Resistor

Figure 1

V and I_R are in-phase with each other, see Figure 1 and the resultant active power, P, has units of watts (or kW or MW).

2.2 Power in an Inductor or Capacitor

Inductors store energy in their magnetic field and capacitors store energy in their electric field. Thus when these elements are fed from an ac source they do not consume energy but simply store it and return it to the system during successive half-cycles. The current flowing from the supply to perform this function is termed reactive current and so we say that inductors and capacitors take reactive power, Q, calculated as:

$$Q = V \times I_L$$

$$\text{or,} \quad Q = V \times I_C$$

where I_L and I_C are the currents in the inductor and capacitor respectively.

(a) Inductor (b) Capacitor

V-I Relationships

Figure 2

Since Q is due to reactive current and is a product of Volts x Amps, the units used are Reactive Volt Amperes. (VAR or kVAR or MVAR).

N.B.: No Active Power is consumed in a pure inductor or capacitor.

3.0 ACTIVE POWER, APPARENT POWER, REACTIVE POWER

3.1 Mechanical Analogy

Consider a horse pulling the loaded mine cart on the tracks in Figure 3. In pulling the cart from point A to point B on the tracks, as shown, the horse must provide power to overcome:

(a) the gravitational force on the loaded cart.

(b) the friction between the cart wheels and the tracks, and bearing friction.

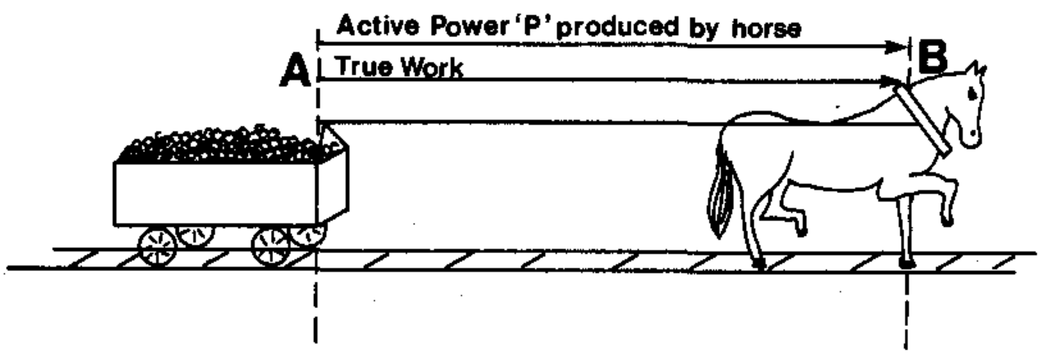
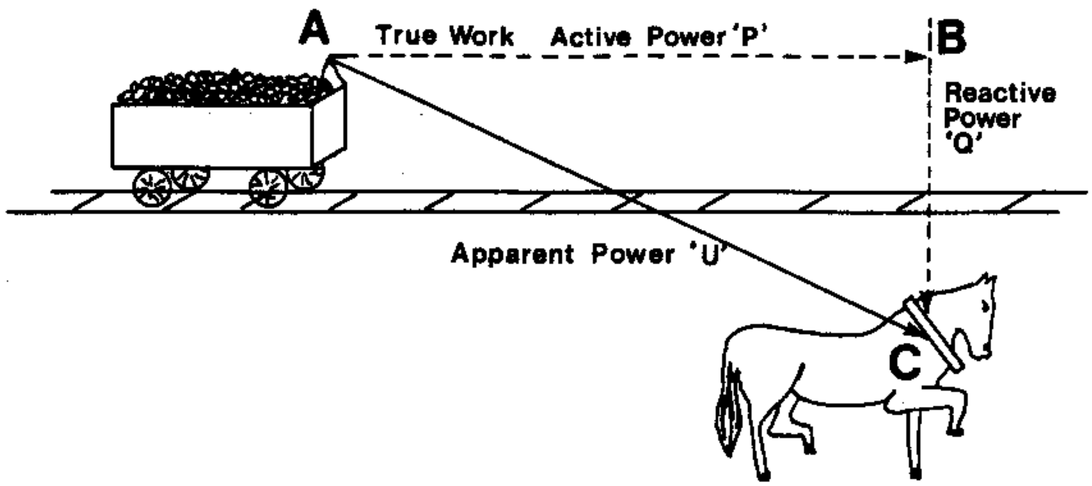


Figure 3

Mechanical Analogy of Active Power P

All the power supplied by the horse results in true work, ie, moving the cart from point A to point B.

Now consider a new untrained horse is brought in which, instead of pulling the cart along the tracks, pulls it at an angle θ as shown in Figure 4. It is the same cart on the same tracks, loaded to the same level, pulled at the same speed and to the same distance.



Horse Pulling the Cart at an Angle

Figure 4

The horse has done a true work equal to the length of the line AB. But it appears that he has done the work equal to the line AC. Line AC is referred to as "apparent work". Apparent work in this case appears to be more than true work.

Where does this difference between the true work and apparent comes from? What is it that the horse is doing differently? To answer this, examine Figure 4 again.

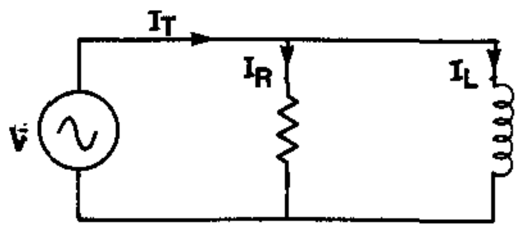
Horse is pulling the cart at an angle θ. The length of the line BC is directly proportional to the angle θ. The amount of work which seem to have been done by the line BC does no good to increase or decrease the true work of moving the cart from A to B. This work is called reactive work because it is in reaction to the angle θ. Reactive work combined vectorially with the true work gives the apparent work.

3.2 Power Triangle

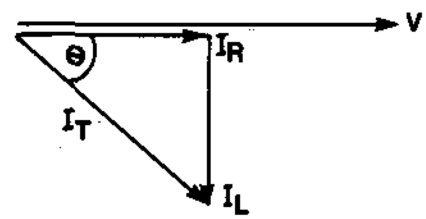
In a circuit containing both R and L or R and C the total current, I_T, will be at some angle θ relative to the voltage, but this can obviously be split into two components, one of which is in-phase with the voltage and one displaced from the voltage by 90°. Comparing this with the case of the horse pulling the mine car, we can see that the component of current which is in-phase with the voltage will produce active power, whilst the reactivecomponent of current, at 90° to the voltage, produces reactive power.

If we take a circuit consisting of R and L in parallel, Figure 5 (a), the vector diagram of currents will be as shown in Figure 5 (b). I_R is in phase with the voltage and I_L lags the voltage by 90°.

$$\bar{I}_T = \bar{I}_R + \bar{I}_L \qquad \text{(Vector Addition)}$$



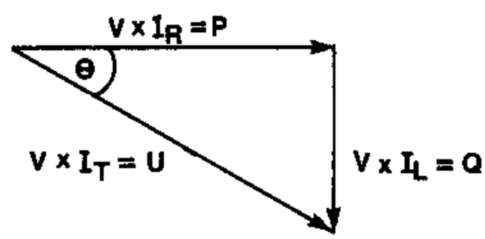
(a) Parallel RL Circuit.



(b) Vectorial Representation.

Figure 5

If we multiply each component of the current triangle by V (a constant), a new triangle is formed, Figure 6.



Power Triangle

Figure 6

This is called the power triangle and the hypotenuse, $U = V \times I_T$, is called the apparent power of the circuit. The units are voltamperes (VA or kVA or MVA).

4.0 POWER FACTOR

4.1 Power Factor (pf) is Defined as Follows:

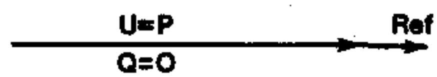
$$pf = \frac{\text{Active Power}}{\text{Apparent Power}}$$

and has a minimum value of zero and a maximum of 1.

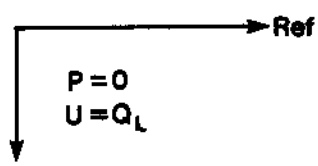
From Figure 6, $pf = \frac{P}{U} = \cos\theta$

Hence the power factor is also equal to the cosine of the phase angle between voltage and current in the circuit. This angle is called the power factor angle. (See Lesson PI 26.34-1; 3.1 and 3.2.)

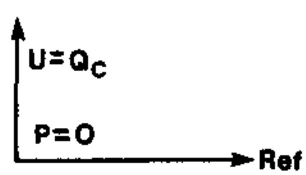
It should now be clear that for a resistor, pf = 1, (θ=0) whereas for an inductor or capacitor, pf = 0 (θ = 90°), see Figure 7.



(a) Resistor-Unity pf. θ = 0°.



(b) Inductor-Zero pf. θ = 90°.



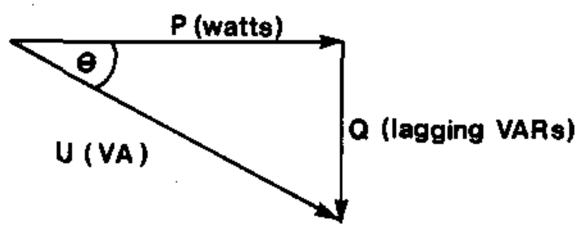
(c) Capacitor-Zero pf. θ = 90°.

Figure 7

4.2 Leading and Lagging Power Factors

Ideally, supply utilities would prefer to supply power to consumers at unity power factor since this would result in active power and apparent power being the same, and the current would be minimum.

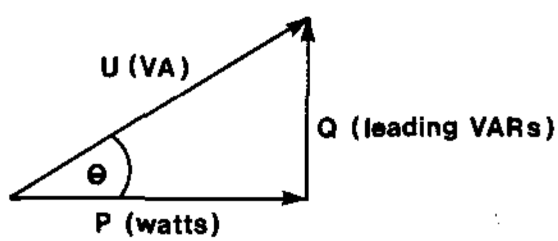
The corresponding I^2R losses in the transmission lines would thus be minimized. However, most industrial loads take inductive reactive VAR's to provide magnetic fields in motors, and since inductive circuits take a lagging current from the supply, the reactive power is designated as lagging VAR's. The power factor under these circumstances is thus said to be a lagging power factor. (See Figure 8.)



Lagging pf Load

Figure 8

The converse is true for a capacitive load where the capacitive reactive VAR's set up electric fields. These are called leading VAR's since capacitors take a leading current from the supply. See Figure 9.

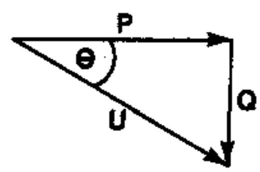


Leading pf Load

Figure 9

4.3 Calculations

Applying the trigonometry to the power triangle, the following relationships can be established.



Power Triangle

Figure 10

$$U^2 = P^2 + Q^2$$

$$U = \sqrt{P^2 + Q^2}$$

$$P = U \cos \theta$$

$$Q = U \sin \theta$$

Example

A transformer is supplying a 10 MW load at 0.85 power factor lagging.

(a) What is the apparent power (VA) supplied by the transformer?

$$P = U \cos \theta$$

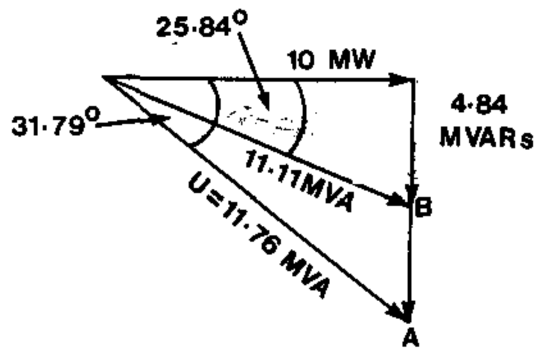
$$\text{Cos } \theta = 0.85$$

$$U = \frac{P}{\cos \theta} = \frac{10 \text{ MW}}{0.85} = 11.76 \text{ MVA}$$

(b) What is the minimum MVA rating of the transformer?

Since the transformer must deliver 11.76 MVA this must be the minimum VA rating of the transformer for it to do the job without overheating. The manufacturer, however, does not make an off-the-shelf transformer of 11.76 MVA. Hence the next higher standard available MVA rating must be used (ie, 15 MVA).

(c)



Power Triangles For Example Part (c)

Figure 11

Existing load conditions are shown at point A in Figure 11.

Original power factor angle = Arc Cos (0.85) = 31.8°.

Original MVAR required = 11.76 sin 31.8° = 6.2 MVAR.

If the existing load power factor is now improved to 0.9.

∴ New power factor angle = Arc cos 0.9 = 25.8°.

New MVAR supplied = 10 tan 25.8° = 4.84 MVAR. New load conditions are shown at point B of Figure 11.

New MVA required = $\frac{10\ MW}{0.9}$ = 11.11 MVA

(d) Comparison of new and original MVA required indicates that we can now place additional load on the transformer, and still stay within the original rating of 11.76 MVA.

Load conditions with the additional load on the transformer are shown at point C in Figure 12, assuming the additional load P, is at unity power factor.

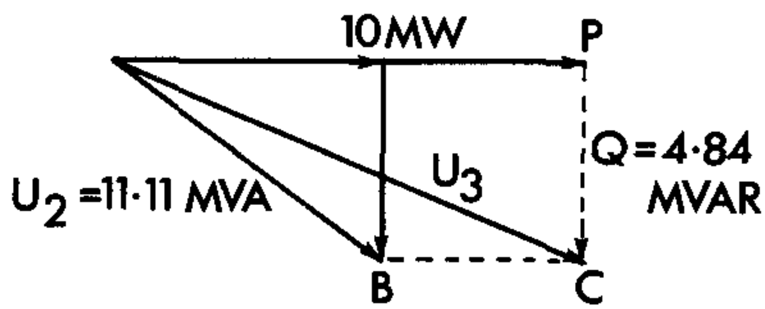


Figure 12

The total active power is now (10 + P)MW and the total reactive power remains constant at 4.84 MVAr.

The apparent power, u_3 = 11.76 MVA, the original rating of the transformer.

$$U_3{}^2 = (10 + P)^2 + Q^2$$

$$\therefore \quad (10 + P) = \sqrt{U_3{}^2 - Q^2}$$

$$= \sqrt{(11.76)^2 - (4.84)^2} = 10.718 \text{ MW}$$

$\therefore$ Additional load, P = 0.718 MW = 718 kW

4.4 Power in Three-Phase Systems

In a balanced 3-phase load, the power in each phase will be identical, so the total power dissipated will be three times the power dissipated per phase.

Consider a star-connected system, with phase voltage V_ϕ, phase current I_ϕ, operating at a power factor given by cos θ.

The apparent power per phase = $V_\phi I_\phi$

For the three phases, total apparent power, U, is thus 3 $V_\phi I_\phi$.

It should be remembered that, in a 3-phase, star-connected system, $I_L = I_\phi$ and $V_L = \sqrt{3}\ V_\phi$ hence

$$U = 3 \frac{V_L}{\sqrt{3}} \quad I_L \quad = \quad \sqrt{3}\ V_L I_L$$

Similarly $P = \sqrt{3} \quad V_L . I_L \cos\theta = u \cos\theta$

and $\qquad Q = \sqrt{3} \quad V_L\ I_L \sin\theta = u \sin\theta$

Identical relationships exist if the load is delta connected.

4.5 Power Factor Correction

On Ontario Hydro power system, the consumer load is inductive in nature due to the many motors and other equipment used by the customer. As a result a large value of reactive power must be supplied by the generator or other equipment. It can be done by one or both of those given below.

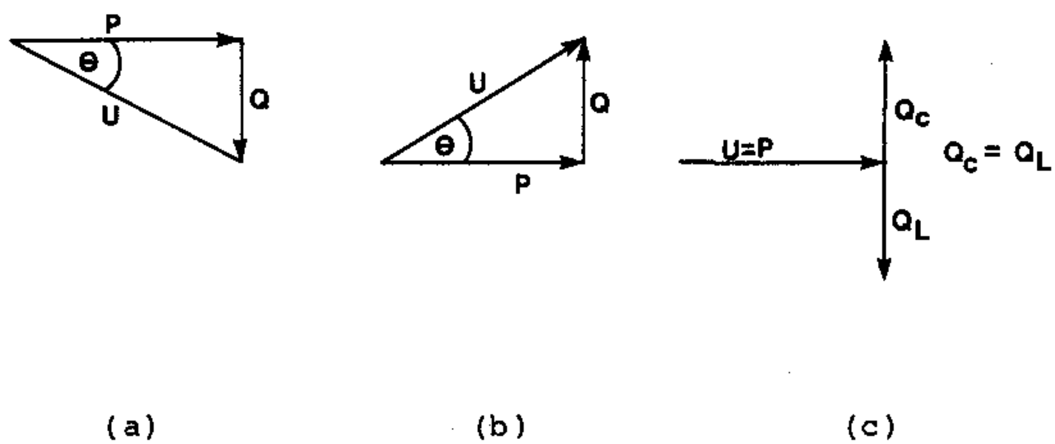
(a) use static capacitors.

(b) use synchronous motors.

In motors and transformers, the use of static capacitors is common.

The principle used in power factor correction is that the inductive load consumes lagging VAR's as shown in Figure 13(a).

Capacitor provides leading VAR's as shown in Figure 13(b). If a capacitor is combined with the inductive load then the lagging VAR's of inductor and the leading VAR's of capacitor will cancel each other as shown in Figure 13(c) and the apparent power will become equal to active power.



Inductive and Capacitive VAR

Figure 13

ASSIGNMENT

1. What is:

   (a) Active Power? (Section 2.1)

   (b) Reactive Power? (Section 2.2)

   (c) Apparent Power? (Section 3.2)

   (d) Draw a power triangle to show P, Q, U and properly label each side. (Section 3.2)

2. Define Power Factor. At pf = 1 and pf = 0 indicate the relationship between apparent power and active power. (Section 4.1)

3. Heat transport pump motor at Bruce NGS-A is 11 000 HP and operates at 0.85 pf and 90% efficiency. What is the minimum VA rating of the transformer feeding the motor? Answer: 10.73 MVA (1 HP = 746 W)

4. In Question 3, what will be the transformer size required to feed this motor if a capacitor is connected in the circuit to improve the power factor to 0.9 lag? Answer: 10.13 MVA.

5. List two common methods used in the Ontario Hydro system for power factor correction. (Section 4.5)

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