Nuclear Theory - Course 227

# NEUTRON MULTIPLICATION FACTOR AND REACTIVITY

In the previous lesson the neutron multiplication factor (k) was defined as:

$$k = \frac{\text{no. of neutrons in one generation}}{\text{no. of neutrons in the preceeding generation}}$$

This definition is only valid if the neutron power is high enough that the effect of source neutrons (photoneutrons and spontaneous fission neutrons) may be ignored and if k itself is not changing. A more precise way to define k is as the product of six factors, each of which represents a possible fate for a member of the neutron population. Thus:

$$k = \varepsilon p \eta f \Lambda_f \Lambda_t$$

Where:

$\varepsilon$ (epsilon) = Fast Fission Factor. The factor by which the fast neutrons population increases due to fast fission.

$$\varepsilon = \frac{\text{No. of neutrons from thermal fission} + \text{No. of neutrons from fast fission}}{\text{No. of neutrons from thermal fission}}$$

A typical value is about 1.03 for natural uranium fuel

p = Resonance Escape Probability. The probability that a neutron will not undergo resonance capture in U-238 while slowing down.

$$p = \frac{\text{No. of neutrons leaving resonance energy range}}{\text{No. of neutrons entering the resonance energy range}}$$

A typical value is about 0.9 for natural uranium fuel.

$\eta$ (eta) = Reproduction Factor. The number of neutrons produced by thermal fission per neutron absorbed by the fuel.

$$\eta = \nu \frac{\Sigma_f^{fuel}}{\Sigma_a^{fuel}} = \nu \frac{\Sigma_f^{fuel}}{\Sigma_f^{fuel} + \Sigma_{n,\gamma}^{fuel}}$$

A typical value is about 1.2 for natural uranium fuel.

f = Thermal Utilization. The fraction of the thermal neutrons absorbed by the fuel compared to neutrons absorbed in the whole reactor.

$$f = \frac{\Sigma_a^{fuel}}{\Sigma_a^{total\ reactor}}$$

A typical value is about 0.95 for a CANDU reactor core.
Note: Fuel must be defined the same way for both $\eta$ & f.

$\Lambda_f$ = Fast Non-leakage Probability. The probability that a fast neutron won't leak out of the reactor. A typical value is about 0.995.

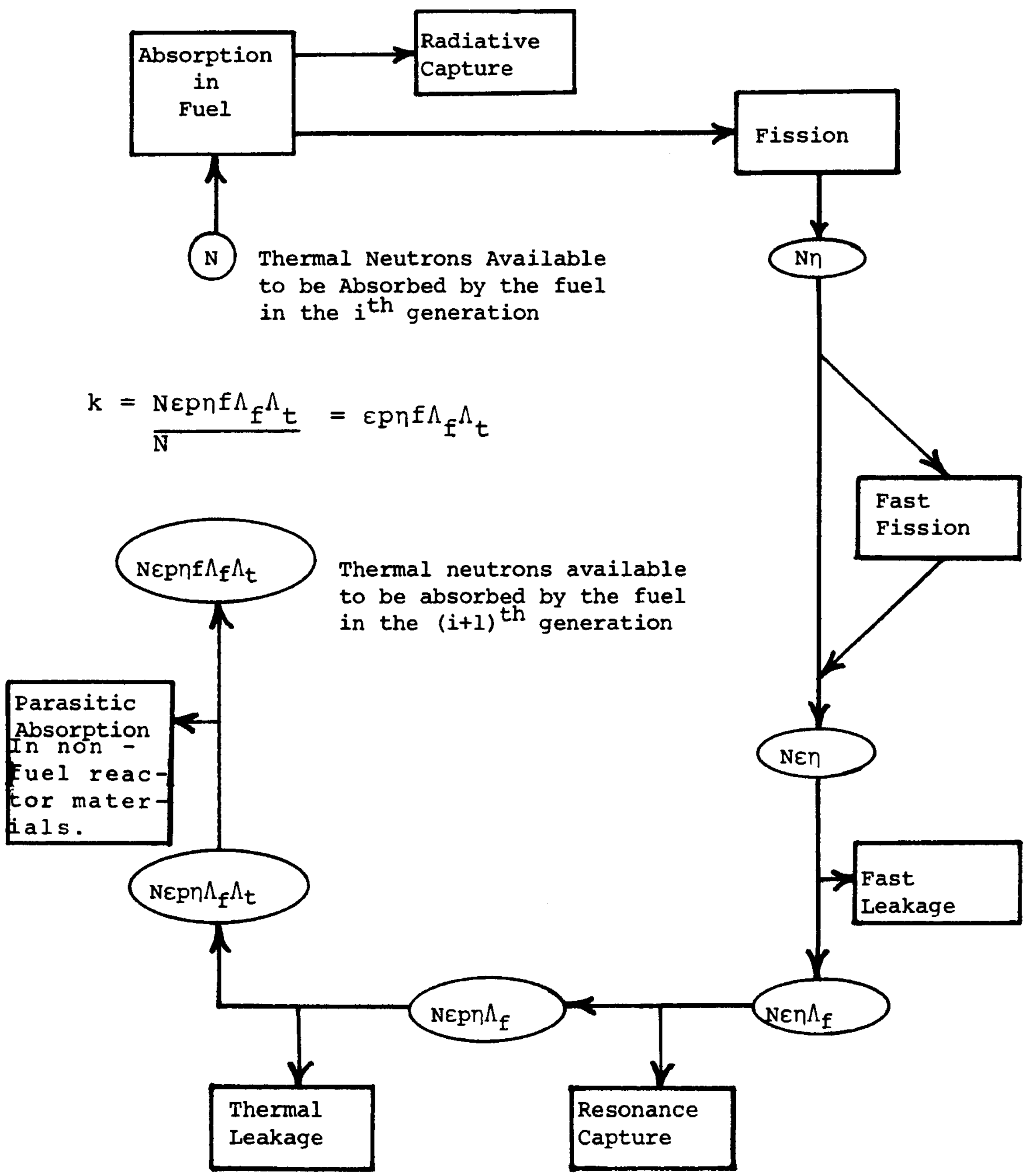
$\Lambda_t$ = Thermal Non-leakage probability. The probability that a thermal neutron won't leak out of the reactor. A typical value is about 0.98.

The first four factors, which depend only on the materials of construction, are frequently grouped together and called the multiplication factor for an infinite reactor ($k_\infty$).

$$k_\infty = \varepsilon p \eta f$$

This is normally referred to as the "four-factor formula".

The last two factors are leakage factors which depend on the size and shape (ie, the geometry) of the reactor. Figure 1 shows how each of the factor relates to the neutron life cycle.




Neutron Life Cycle

<u>Figure 1</u>

A thermal neutron which is absorbed by the fuel may be absorbed by fissile material (U-235 or Pu-239) or by non-fissile material (Fission Products, U-238, etc). If it is absorbed by fissile material it may undergo radiative capture or cause fission. If it causes fission, $\nu$ fast neutrons will be produced. The reproduction factor ($\eta$) accounts for all of this. Thus for N thermal neutrons absorbed by the fuel $N\eta$ fast neutrons are produced.

As U-238, Pu-239, and U-235 all have small but finite fission cross-sections for fast neutrons, the fast neutrons can cause additional fissioning to take place. This results in an increase ($\approx$3%) in the fast neutron population. The fast fission factor ($\varepsilon$) accounts for this increase in the fast neutron population. Thus for $N\eta$ fast neutrons from thermal fission we get $\varepsilon N\eta$ fast neutrons from fast and thermal fission.

The two factors $\varepsilon$ and $\eta$ are essentially properties of the fuel and the magnitude of the product $\varepsilon\eta$ fixes the tolerable limits of the other factors which can be regarded as design variables. That is the product $pf\Lambda_f\Lambda_t \geq \frac{1}{\varepsilon\eta}$.

While slowing down the fast neutrons may reach the boundary of the reactor and leak out. To account for this reduction in the population we have the fast non-leakage probability ($\Lambda_f$).

The fast neutrons may also suffer resonance capture while slowing through the resonance energy range. The resonance escape probability (p) accounts for this. Thus for $N\varepsilon\eta$ fast neutrons starting the slowing down process $N\varepsilon\eta p\Lambda_f$ neutrons reach thermal energy.

A certain percentage of the thermal neutron population will diffuse to the boundary and leak out. We use the thermal non-leakage probability ($\Lambda_{th}$) to account for this loss.

The remaining thermal neutrons will either be absorbed by the fuel or by the core material. The thermal utilization factor (f) accounts for this. Thus for $N\varepsilon p\eta f\Lambda_f\Lambda_t$ thermal neutrons, $N\varepsilon p\eta f\Lambda_f\Lambda_t$ are absorbed by the fuel.

From Figure 1 you can see that if we divide the number of neutrons in the $(i + 1)^{th}$ generation by the number in the $i^{th}$ generation we have:

$$k = \frac{N\varepsilon p\eta f\Lambda_f\Lambda_t}{N} = \varepsilon p\eta f\Lambda_f\Lambda_t$$

When k = 1 the reactor is said to be critical. If k is unity and the effects of source neutrons are negligible, neutron power will be constant in a critical reactor. It is important to realize that a reactor may be critical at any power level and that telling someone that a reactor is critical tells them nothing about the reactors power output. By analogy; if I tell you that a car is not accelerating, do you know how fast it is going?

If we want to increase power we must make k greater than one by reducing the losses, with respect to fission, of neutrons. The reactor is then said to be supercritical. Power will continue to increase as long as k is maintained at a value greater than one.

To reduce power we must increase the losses of neutrons thus making k less than one. The reactor then is said to be subcritical and power will decrease until the source neutrons become significant. (This point will be covered in detail in lesson 227.00-9.)

Reactivity

A reactor is critical when k = 1. The factor that determines how subcritical or supercritical a reactor may be, is the amount by which k differs from 1.

A quantity called reactivity, is used to describe changes in k which are called reactivity changes. Reactivity is defined as:

$$\frac{k - 1}{k}$$

For values of k close to 1 (eg, 0.98 to 1.02) which easily encompasses our normal operating range.

Reactivity may be approximated as

$$\Delta k = k-1$$

This is the accepted meaning of reactivity in Hydro.

The reactivity changes that are made for normal reactor control are always quite small, and they are measured in a unit called the milli-k or mk. (This is not strictly a unit but is a fraction, 1 mk is the same as 0.1%, ie, 0.001).

For Example:

$$\begin{aligned} k &= 1.002 \\ \Delta k &= k - 1 \\ &= 1.002 - 1 \\ &= 0.002 \text{ or } 2 \text{ mk} \end{aligned}$$

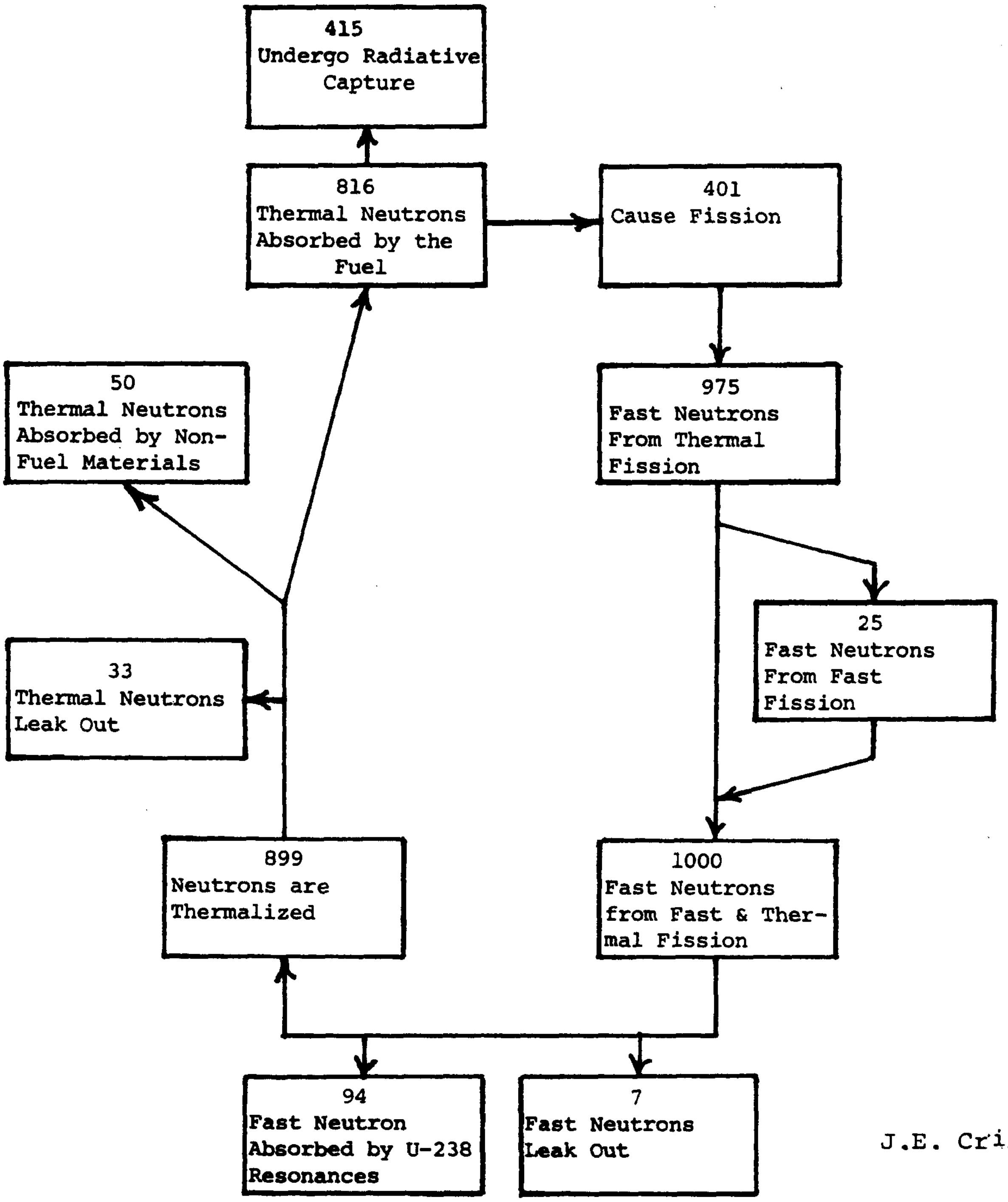
A typical CANDU reactivity control system such as the liquid control zone at Bruce and Pickering have a range of about 6 mk.

## ASSIGNMENT

1. Put your text and your notes away. Now, write the six factor formula, define each of the terms, and sketch the neutron life cycle with the terms used correctly.

2. Calculate the exact value of reactivity for k = 0.95.

3. Calculate each of the six factors for the neutron life cycle shown below.






J.E. Crist