Nuclear Theory - Course 127

LOW POWER CONSIDERATIONS

At low power Xenon poison and fission product buildup are of little significance. Thermal effect may also be ignored. Hence considerations of reactor operation at low power are somewhat simpler and more straightforward than those of high power operation.

Change of Neutron Flux and Neutron Power Following a Reactivity Change

Consider a reactor operating in a steady state with $k_e = 1$ but with no source of neutrons present. The reactivity is now changed by δk and then held fixed. This is known as a STEP CHANGE in reactivity. If n is the neutron density in the reactor, δk is the excess of neutrons from one generation to the next, ie, if the neutron density is n in one generation it will be increased by $n\delta k$ by the next generation. Since the generation time is $\mathcal{L}$, the rate of increase of neutrons is $\frac{n.\delta k}{\mathcal{L}}$

$$\text{ie, Rate of increase} = \frac{dn}{dt} = \frac{n.\delta k}{\mathcal{L}}$$

$$\text{Rearranging we get} \quad \frac{dn}{n} = \frac{\delta k}{\mathcal{L}}.dt$$

$$\text{Integrating both sides} \int \frac{dn}{n} = \frac{\delta k}{\mathcal{L}} \int dt$$

$$\text{or } \log_e n = \frac{\delta k}{\mathcal{L}}.t + \text{constant}$$

To determine the constant, let $n = n_o$ when $t = o$, ie, immediately prior to the reactivity change.

$$\text{Then constant} = \log_e n_o$$

$$\text{Therefore} \quad \log_e n = \frac{\delta kt}{\mathcal{L}} + \log_e n_o$$

$$\text{or} \quad \log_e\left(\frac{n}{n_o}\right) = \frac{\delta kt}{\mathcal{L}}$$

$$\text{or} \quad \frac{n}{n_o} = e^{\frac{\delta k}{\mathcal{L}}.t}$$

Hence $n = n_o e^{\frac{\delta k}{\ell} \cdot t}$ ....(1)

This equation gives the neutron density, n, at some time t sec following the δk change in reactivity.

The neutron flux and the power level, in a reactor, are both proportional to the neutron density and, therefore, both will change, with time, in the same way.

Thus $\phi = \phi_o e^{\frac{\delta k}{\ell} \cdot t}$ ....(2)

and $P = P_o e^{\frac{\delta k}{\ell} \cdot t}$ ....(3)

δk may be positive or negative in these equations and so the power will increase exponentially if δk is positive and decrease exponentially if δk is negative. The greater the value of δk the faster the increase or decrease in power will be. The smaller the value of the neutron lifetime, ℓ, the faster the increase or decrease in power will be. Therefore, the value of ℓ is important.

The Reactor Period

The reactor period was defined, in the previous lesson, as being the time required for the power (or flux or neutron density) to change by a factor of e. The significance of this definition is more apparent if equation (3) is examined.

$P = eP_o$ when $t = T$

So $T = \frac{\ell}{\delta k}$ is the reactor period.

The preceding equations now become:

$$P = P_o e^{\frac{t}{T}} \quad ; \quad \phi = \phi_o e^{\frac{t}{T}} \quad ; \quad n = n_o e^{\frac{t}{T}}$$

From these equations, it may be seen that the shorter the reactor period, ie, the smaller the value of T, the faster the power changes will be.

Power Changes With Prompt Neutrons Only

If all the neutrons in a reactor were prompt neutrons, the neutron lifetime, ℓ, would be about 0.001 sec.

Hence, if $\delta k = 0.5$ mk, $T = \frac{0.001}{0.0005} = 2$ sec and, in 1 sec the power would increase by a factor of 1.65, ie, $P = 1.65\ P_o$.

If $\delta k = 2$ mk, $T = \frac{1}{2}$ sec and, in one second, $P = 7.4\ P_o$.

If $\delta k = 5$ mk, $T = \frac{1}{5}$ sec and, in one second, $P = 148\ P_o$.

The above examples indicate how rapid the power increases would be, even for small increases in reactivity. The graphs in Fig. 1 show the variation of power with time for various positive values of δk.

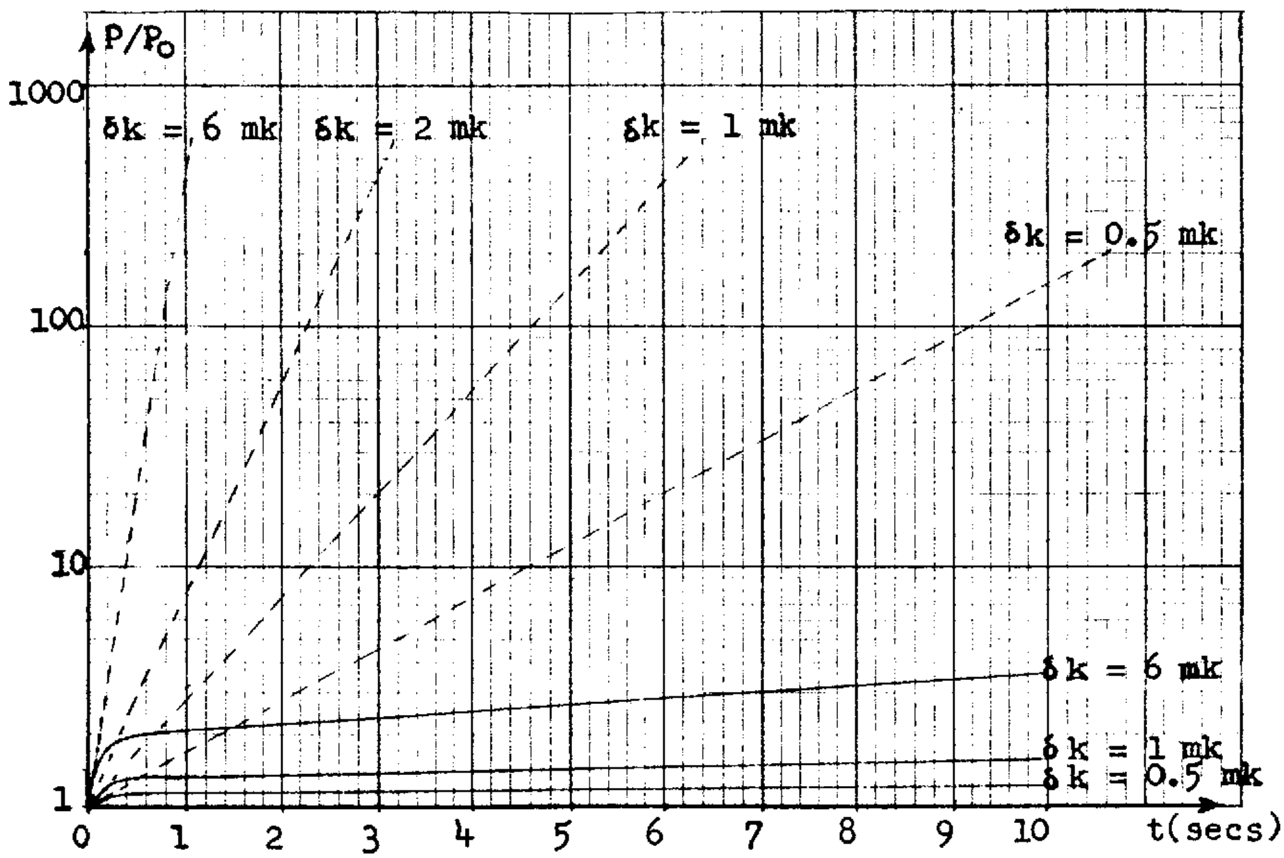


Fig. 1

The increases of power, with prompt neutrons only, are shown as dotted lines. With rapid power increases of the types shown, effective reactor regulation is not possible. Effective protection also becomes difficult since the fastest protective system will take at least 1 second to act. In this period of time, severe damage would have resulted from the high power level reached.

Fig. 2 shows the manner in which the power would decrease, for negative reactivities, if all neutrons were prompt neutrons.

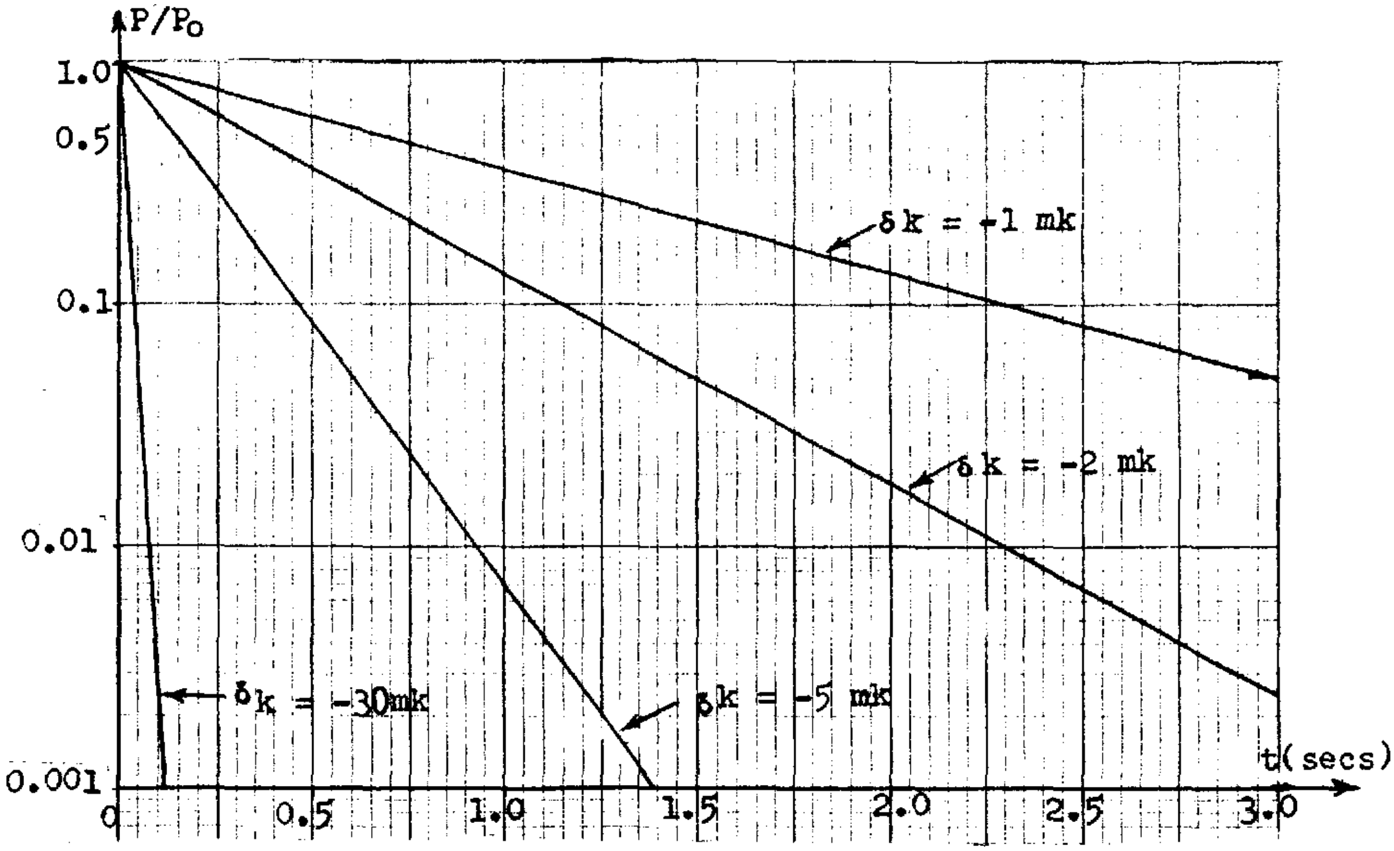


Fig. 2

The decrease in power is quite rapid, even for small negative reactivities. Negative reactivities, well in excess of -30 mk, can normally be introduced by the protective system and, with these large negative reactivities, the power would decrease to one-thousandth of full power in 0.2 sec or less.

Effect of Delayed Neutrons on Power Changes

The following table lists the fission yields, half-life and average life of the six delayed neutron groups:

Yield (%)	Half life (sec)	Average life (sec)	Yield x av. life
0.025	55.60	80.20	2.00
0.166	22.00	31.70	5.26
0.213	4.51	6.51	1.39
0.241	1.52	2.19	0.53
0.085	0.43	0.62	0.05
0.025	0.05	0.07	0.00
99.245	Prompt	0.00	0.00
100.000			9.23

The product of the yield and average life is determined for each group and the sum total of this column found (9.23).

The average life of all neutrons, both prompt and delayed, is this total divided by the total yield of 100%. The average lifetime is then this average life plus the diffusion time.

ie, Average lifetime = $\frac{9.23}{100} + 0.001 = 0.0933$ sec.

Thus, although the delayed neutrons form only 0.755% of all the neutrons which result from fission, they change the average lifetime of all neutrons from 0.001 sec to approximately 0.1 sec, ie, by a factor of 100. Using this new value for ℓ :

when $\delta k = 0.5$ mk, T = 200 sec and, in 1 sec, $P = 1.005\ P_o$

ie, only an increase in power of 0.5% in 1 second.

if $\delta k = 2$ mk, T = 50 sec and, in 1 sec, $P = 1.02\ P_o$

if $\delta k = 6$ mk, T = 16.7 sec and, in 1 sec, $P = 1.06\ P_o$

Therefore, the power increases are much less rapid than with prompt neutrons alone. However, it must be remembered that it would take a tenth to a fifth of a second for the delayed neutrons to become effective. During this initial fraction of a second, the power would increase in the manner described with prompt neutrons alone. The continuous graphs, in Fig. 1, show how the power increases both during this initial period and after the effect of the delayed neutrons are felt. As may be seen, the power increases do not become excessive during the response times of the regulating and protective system. So regulation and protection become practical realities.

Fig. 3 shows how the power decreases for negative reactivities when the effect of the delayed neutrons is considered. Initially the decrease in power is still due, entirely, to the decrease in prompt neutrons and the slopes of the graphs are, initially, equal to those of the corresponding graphs in Fig. 2 (shown in dotted line). After this initial rapid decrease in power, the delayed neutrons become the deciding factor. Each delayed neutron group, in turn, predominates, starting with the shortest half-life group. Eventually, the power decrease is governed entirely by the delayed neutron group having a 55.6 sec half-life.

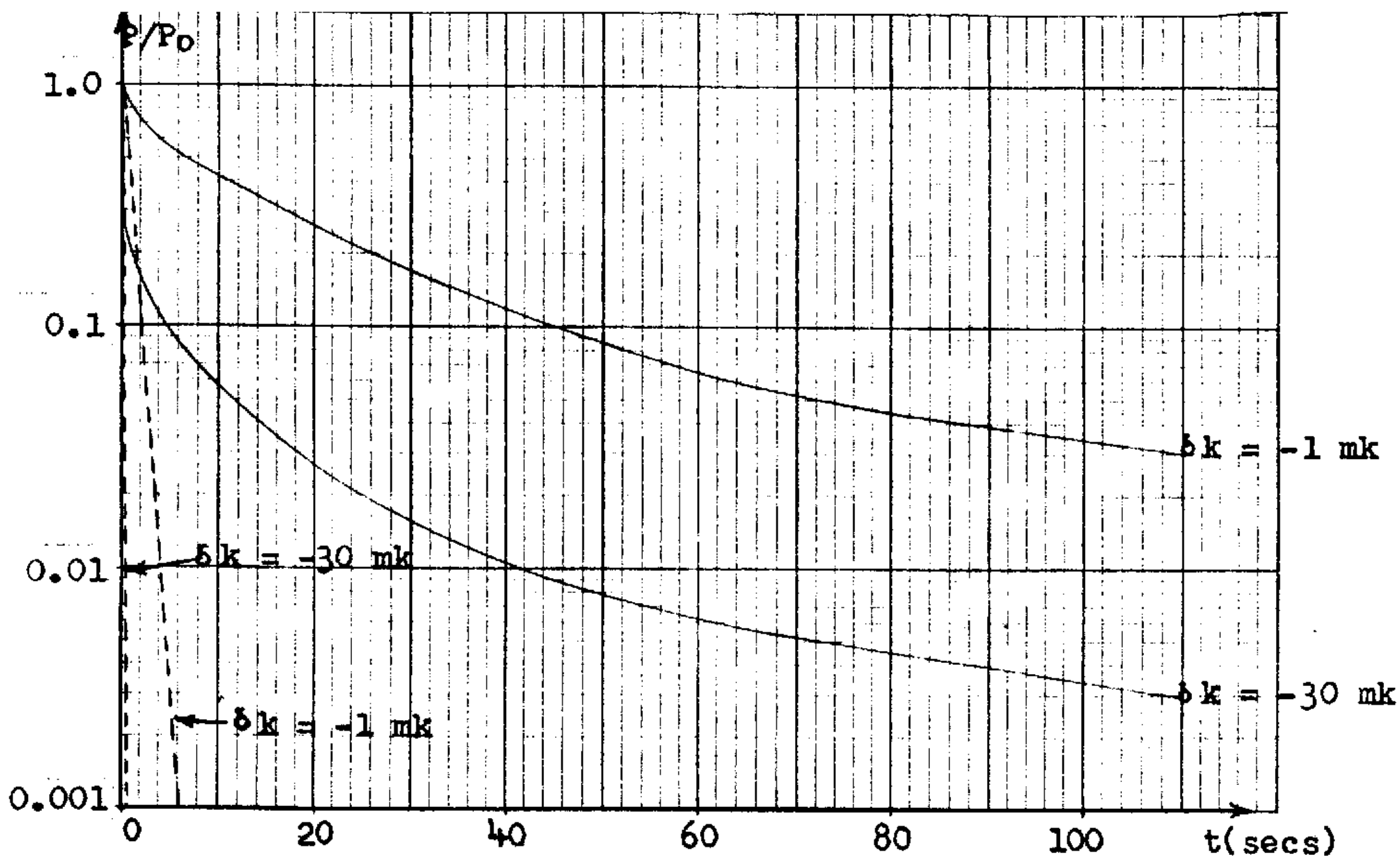


Fig. 3

It is evident that the delayed neutrons cause a considerable decrease in the rate of power reduction. For rapid shutdown, a substantial amount of negative reactivity is required to cause an initial rapid decrease in power, before the delayed neutrons slow down the power reduction. When $\delta k = -30$ mk the power drops to one-tenth of its value in 4 sec but it takes 40 sec for it to drop to 1/100th of its value and 30 min to drop 10 decades, eg, from 100 Megawatts to one-hundredth of a watt.

Prompt Critical

As the value of δk becomes more and more positive, the initial rapid rise in power gets longer and longer and the response of the regulating system has to become progressively faster.

When the reactivity is equal, in value, to the total delayed neutron yield from fission, then the chain reaction can be sustained by prompt neutrons, without the aid of delayed neutrons. The reactor is then said to be PROMPT CRITICAL.

The total delayed neutron yield = 0.755% = 0.00755

Therefore, the reactor is prompt critical when $\delta k = 0.00755$ = 7.55 mk.

By the time the reactivity has reached this value of 7.55 mk the initial rapid rise in power is substantial and the final steady reactor period is less that 2 seconds. The net result is an increase in power by a factor of 3 to 4 during the first second. Such increases are to be avoided and, so, the regulating system is usually designed to avoid δk values even approaching 7.55 mk. The period is normally kept above 10 sec and, preferably around 25 sec or higher.

In a thermal reactor the prompt critical condition is not something suddenly arrived at as the reactivity becomes equal to the delayed neutron yield. As shown in Fig. 4, in the curve for $\ell = 10^{-3}$ sec, there is no sudden decrease in reactor period at prompt critical. Therefore, it is not implied that a thermal reactor with δk = 7.55 mk is much more difficult to control than one in which δk = 7.4 mk, say.

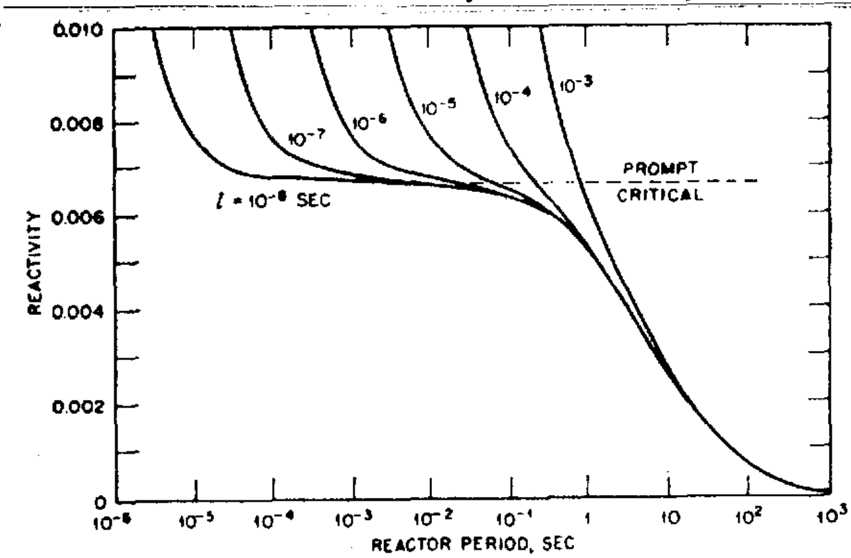


Fig. 4

However, in a fast reactor in which ℓ will have a much smaller value, there is a large decrease in reactor period at prompt critical. Thus the prompt critical condition is far more significant.

Effect of Neutron Sources on Power Changes

The equations at the beginning of the lesson were derived assuming that there was no source of neutrons, in the reactor, other than fission. A source of neutrons may be introduced into the reactor for the initial start-up. Alternatively, if D_2O is used as a moderator, photoneutrons may be produced by the absorption of fission product gamma rays in deuterium, according to the equation:

if γ > 2 meV

$${}_1H^2 + \gamma = {}_1H^1 + {}_0n^1$$

These neutron sources may persist after the reactor has been shut down and the prompt and delayed neutrons have died down.

If the source strength is S neutrons/cc/sec, the equation for the neutron density variation is modified to:

$$n = n_o e^{\frac{t}{T}} - \frac{0.001S}{\delta k} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (4)$$

If P_s is the source strength expressed in watts then the reactor power is given by: doesn't ~~only~~ holds if δk is 0 or very small.

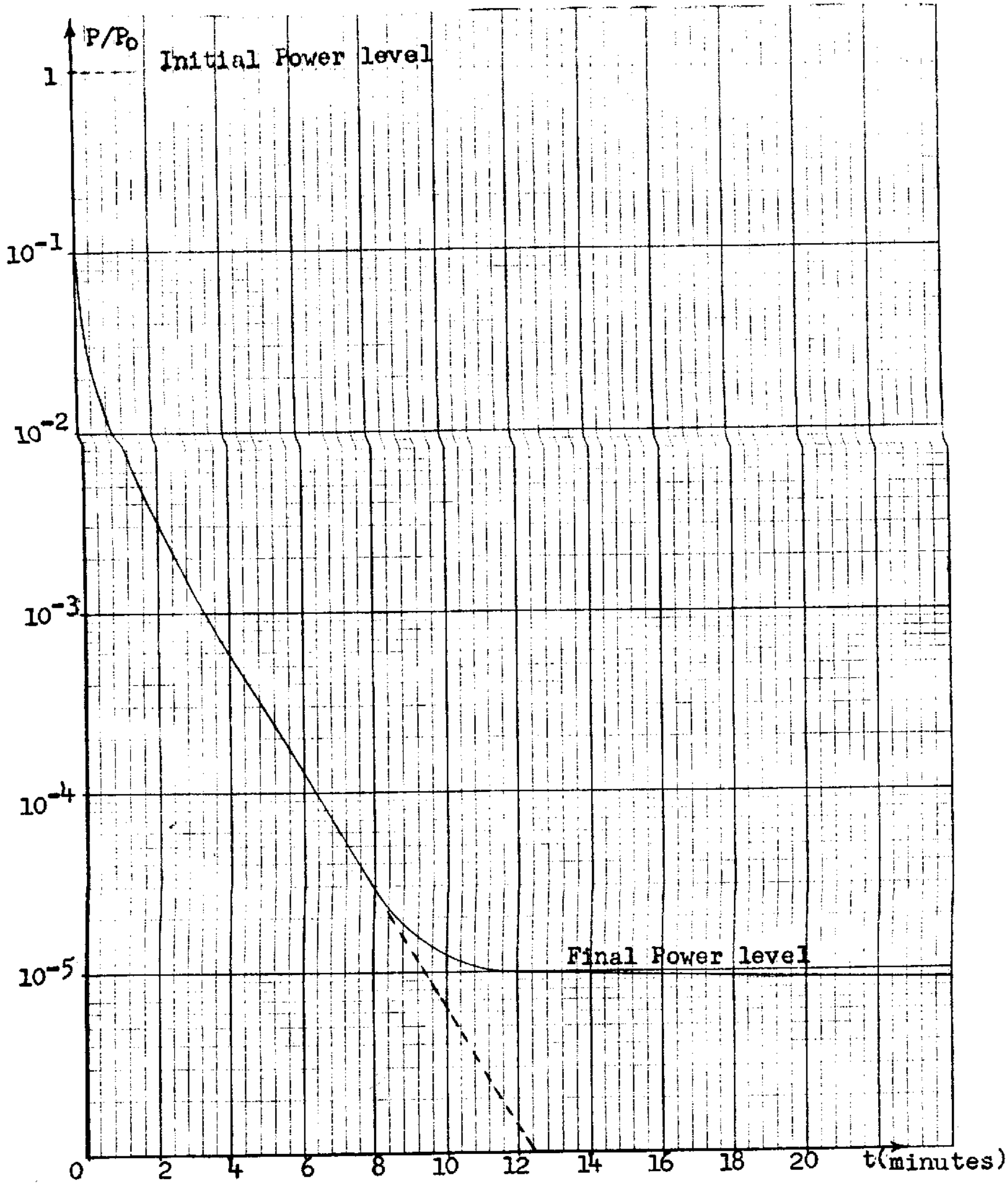
$$P = P_o e^{\frac{t}{T}} - \frac{P_s}{\delta k} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (5)$$

From both these equations, it may be seen that, when δk is positive, the exponential term grows so fast that the source term is insignificant, ie, P_s is usually so small that it does not contribute to the total power produced. It should be noted that this equation does not apply for very small values of δk.

However, if δk is negative, the exponential term decreases and, when the delayed neutrons have died down sufficiently, the source term becomes significant. The power will eventually level out at $P = \frac{-P_s}{\delta k}$, as shown in Fig. 5, instead of continuing to follow the delayed neutron decay, as shown by the dotted line.

When P_s = 30 watts and δk = -30 mk; $P = \frac{30}{.03}$ = 1000 watts and if P_o = 100 Mwatts; $\frac{P}{P_o} = 10^{-5}$

One big advantage of a photoneutron source is that it reduces the range of power which the neutron power instruments have to measure. With a photoneutron source, the range to be covered is around 6 decades, whereas otherwise it would be several decades more. There are logarithmic instruments that will cover 6 decades on one scale and linear instruments that will cover 8 decades fairly adequately with range switching but this is the limit of neutron power measurements.



P/P0
Initial Power level
1
10-1
10-2
10-3
10-4
10-5
Final Power level
0
2
4
6
8
10
12
14
16
18
20
t(minutes)

Fig. 5

ASSIGNMENT

1. If all the neutrons, in a reactor, were prompt neutrons with a lifetime of 0.001 sec:

   (a) Calculate the reactor period and the power increase that occurs in 1 sec if δk = 6 mk.

   (b) What conclusions can be drawn, regarding reactor regulations and protection, with such power increases?

2. Assuming that, with delayed neutrons, the average neutron lifetime is increased from 0.001 sec to 0.1 sec:

   (a) Calculate the reactor period and the power increase that occurs in 1 sec if δk = 6 mk.

   (b) Comment on the significance of the answers when compared with the values obtained in 1(b).

3. Why is a larger amount of negative reactivity required, for a rapid shutdown, because all the neutrons are not prompt neutrons?

4. Explain what is meant by "Prompt Critical" and why this condition, or anything approaching it, should be avoided.

5. How does a neutron source in a reactor affect:

   (a) the power production?

   (b) the neutron power decrease after shutdown?

   (c) the neutron power instrumentation required for normal operation?

A. Williams