Nuclear Theory - Course 127

NEUTRON CROSS SECTIONS AND NEUTRON FLUX

When a neutron strikes a nucleus, any of the reactions discussed above may take place, depending on the nucleus and the neutron energy. What determines, then, which reaction will occur? In the case of U-238, for instance, inelastic scattering will not occur unless the neutron energy is greater than 0.1 Mev. To put it another way, there is no chance or probability of inelastic scattering occurring with U-238 unless the neutron energy is greater than 0.1 Mev. We could also say that the chance or probability of U-235 fission occurring is greater with thermal neutrons than with fast neutrons, ie, the probability increases as the neutron energy decreases.

Thus we are always comparing the chances in favour of the various reactions taking place. It is the probability of a particular reaction occurring that is important. Some reactions are more probable with some nuclei than with others or more probable with some neutron energies than with others. Because these reactions are concerned with a neutron striking a target, namely a nucleus, the probability that a particular reaction will occur is measured in terms of a quantity called the *nuclear* or *neutron cross section*.

Neutron Cross Sections and Neutron Flux

To examine the precise measuring of the term "cross section", let us look at what happens when n neutrons per unit volume move with velocity v towards a thin target of surface area S. We will assume that the whole target area is exposed to neutrons, and that all the neutrons travel in the same direction x (see Fig. 1).

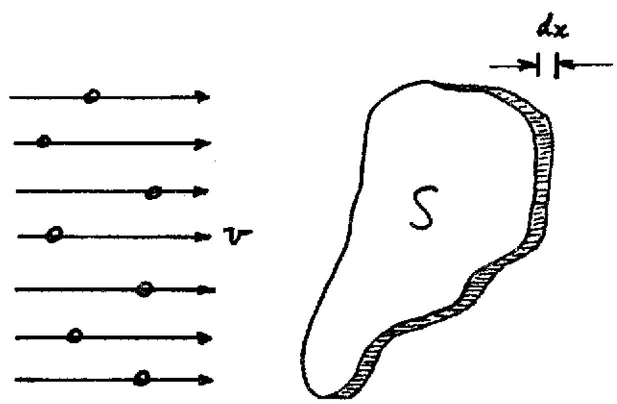

Fig. 1 Neutron Bombardment of a Thin Target

From experiment it is found that the rate R at which a particular reaction occurs is proportional to every one of the following:

(a) $n_x v$, the number of neutrons striking the target in the x direction per unit area and time;

(b) S, the surface area of the target;

(c) dx, the thickness of the target - this is assumed to be sufficiently small for no "shadowing" of the nuclei to occur;

(d) N', a symbol reserved in this course for the number of nuclei per unit volume.

Therefore:

$$R \propto n_x v . N' . Sdx$$

or

$$R = \sigma . n_x v . N' . Sdx$$

σ (sigma) is the constant of proportionality, and could be defined as "the interaction rate per atom in the target per unit nv". It is called the *microscopic cross section*, and a little bit of fooling around with units will show that it has dimensions of area. The usual unit is the *barn* (abbreviated b);

$$1 \text{ b} = 10^{-28} \text{m}^2 = 10^{-24} \text{cm}^2 ;$$

it is the same order of magnitude as the physical diameter of a medium size nucleus.

The reaction rate per unit volume of target material is now seen to be:

$$R = n_x v . \; N' \sigma$$

Since N' and σ are both characteristic of the target material, they are often combined to form the:

$$\textit{macroscopic cross section } \Sigma = N' \sigma$$

We can now go on to consider neutrons arriving from all directions with the same velocity (see Fig. 2).

For a target of unit volume

$$R \text{ (total)} = N' \sigma \; (n_1 v + n_2 v + \ldots n_i v + \ldots)$$
$$= nv . N' \sigma$$

where n is the *neutron density*, which is the number of neutrons per unit volume regardless of their direction of motion. nv is known as the *neutron flux density* (symbol ϕ and often just called

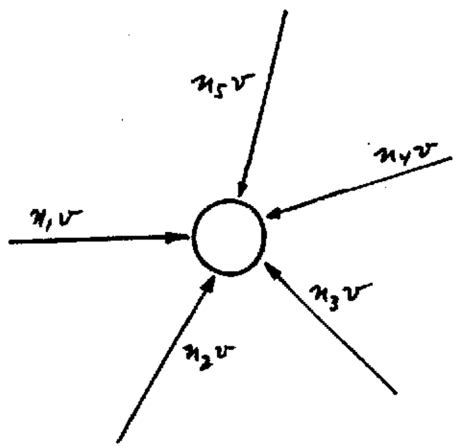

Fig. 2 Isotropic Neutron Bombardment

neutron flux for short). It is usually expressed in units of neutrons.$cm^{-2}s^{-1}$.

The reaction rate for any material exposed to flux ϕ is then:

$R = \phi\Sigma$ per unit volume

Incidentally, it is a common misconception that the neutron flux can be defined as the number of neutrons striking unit area per second. This would be true for a beam, but not for random directions in which case the number hitting unit area would be less (by a factor of 2 actually). *If you insist on a connection with area,* it can be proved that ϕ is the number of neutrons entering an imaginary sphere each second, of total surface area 4 cm^2 and diametral plane area 1 cm^2.

Another point worth mentioning is that when the neutrons have a range of speeds, an appropriate average cross section is usually chosen. For instance, the detailed structure of the thermal neutron distribution can often be ignored (it certainly will be in this course!), if average thermal cross sections are used.

Since different reactions occur with different probabilities, they will have different cross sections. Throughout this course the following nomenclature will be used:-

σ_f = fission cross section

σ_a = absorption cross section

σ_s = elastic scattering cross section

σ_i = inelastic scattering cross section

In those few cases where $\sigma_f \neq 0$, both fission and radiative capture involve a complete absorption of the neutron, and then σ usually includes both reactions, ie, $\sigma_a = \sigma_f + \sigma_{n,\gamma}$.

For your reference, Table 1 on pages 6 and 7 lists the absorption and elastic scattering cross sections for thermal neutrons only (cross sections usually are strongly energy dependent). We can already arrive at some interesting conclusions by taking a look at these.

(a) Water (H_2O) is a better scatterer of neutrons than heavy water (D_2O) or graphite (carbon), but it is also a much heavier absorber than either. This has important implications in choosing a moderator.

(b) Boron and cadmium have very high values of σ_a and therefore are excellent materials when neutron absorption is required, as, for example, in control rods of a reactor.

(c) The capture cross section of zirconium is much smaller than that of iron. This explains the use of zirconium alloys instead of steel for pressure tubes and fuel sheathing in our reactors.

To appreciate the significance of these cross sections, let us look at a typical problem:

Cobalt-60 gamma sources for radiation therapy units are produced by irradiating cobalt pellets in reactors. A typical pellet might be ¼" in diameter and 1" long. Calculate the activity in curies built up in one of these pellets after it has been irradiated for two years in a thermal neutron flux of $5 \times 10^{13}\ n.cm^{-2}s^{-1}$.

All the data required to solve this problem is already given in the Chart of the Nuclides at the end of the first lesson, and in Table 1 of this lesson (page 6):-

Natural cobalt is 100% Co-59; half-life of Co-60 = 5.3 y;
σ_a of Co-59 = 37 b; ρ = 8.8 gcm^{-3}. λ use 5.2

We must first write down the differential equation relating Co-60 production and decay per unit volume, ie,

$$\frac{dc}{dt} = \phi\Sigma_a - c\lambda,$$

where c is the concentration of Co-60, λ its ~~half-life~~ decay constant, and Σ_a the macroscopic absorption cross section of Co-59. Solving this equation yields:

$$c = \frac{\phi\Sigma_a}{\lambda}\,(1 - e^{-\lambda t}).$$

In other words, the cobalt activity per unit volume is:

$$c\lambda = \phi\Sigma_a(1 - e^{-\lambda t})$$

$$= \phi N'\sigma_a(1 - e^{-\lambda t})$$

With the substitution of the values $\phi = 5 \times 10^{13} n.cm^{-2}s^{-1}$, $N' = \frac{N_o}{A}\rho = 9 \times 10^{22}$ atoms cm^{-3}, $\sigma_a = 37 \times 10^{-24} cm^2$, $\lambda = \frac{0.69}{5.2} y^{-1}$ and $t = 2$ y, we get:

$$\underline{c\lambda = 3.9 \times 10^{13}\ cm^{-3}s^{-1}}$$

The activity has to be in units of $cm^{-3}s^{-1}$, because

$$(cm^{-2}s^{-1}) \times (cm^{-3}) \times (cm^2) = cm^3s^{-1}.$$

To find the activity of the whole pellet in curies, we multiply by the volume and divide by $3.7 \times 10^{10}\cancel{s^{-1}}$ $\pi r^2 h$ ie,

$$\text{Activity} = \frac{3.9 \times 10^{13} \times \pi(\cancel{0.25 \times}\ 2.5)^2 \times 2.5}{4 \times 3.7 \times 10^{10}}\ \text{Ci}$$

$$= \underline{810\ \text{Ci}}$$

Actually, the activity will be a bit less than this because of the *self-shielding* of the cobalt pellets. The flux at the centre of the pellet will be less than at the outside, because some neutrons have been removed by absorption. We shall consider this next.

Attenuation of Neutrons

Consider Fig.1 again. After traversing the thickness dx, some neutrons have been removed from the beam. The neutron density will be reduced by an amount dn given by:

$$\frac{dn}{n} = -N'\sigma\ dx;$$

if the target is of thickness x, the neutron density at x is given by:

$$\int_{n_o}^{n_x} \frac{dn}{n} = \int_o^x N'\sigma\ dx$$

or $$\underline{n_x = n_o e^{-N'\sigma x} = n_o e^{-\Sigma x}}$$

TABLE 1 $\times 10^{24}$

Properties of the Elements and Certain Molecules

Element or molecule	Symbol	Atomic number	Atomic or molecular weight*	Nominal density, gm/cm^3	Atoms or molecules per cm^3†	σ_a,‡ barns	σ_s,‡ barns	Σ_a,† cm^{-1}	Σ_s,† cm^{-1}
Actinium	Ac	89	227		N′	800			
Aluminum	Al	13	26.9815	2.699	0.06024	0.235	1.4	0.01416	0.08434
Antimony	Sb	51	121.75	6.62	0.03275	5.5	4.3	0.1801	0.1408
Argon	Ar	18	39.948	Gas		0.63	1.5		
Arsenic	As	33	74.9216	5.73	0.04606	4.5	6	0.2073	0.2764
Barium	Ba	56	137.34	3.5	0.01535	1.2	8	0.01842	0.1228
Beryllium	Be	4	9.0122	1.85	0.1236	0.0095	7.0	0.001174	0.8652
Beryllium oxide	BeO		25.0116	2.96	0.07127	0.0095	6.8	0.0006771	0.4846
Bismuth	Bi	83	208.980	9.80	0.02824	0.034	9	0.0009602	0.2542
Boron	B	5	10.811	2.3	0.1281	759	4	97.23	0.5124
Bromine	Br	35	79.909	3.12	0.02351	6.7	6	0.1575	0.1411
Cadmium	Cd	48	112.40	8.65	0.04635	2450	7	113.6	0.3245
Calcium	Ca	20	40.08	1.55	0.02329	0.43	3.0	0.01002	0.06987
Carbon (graphite)**	C	6	12.01115	1.60	0.08023	0.0034	4.8	0.0002728	0.3851
Cerium	Ce	58	140.12	6.78	0.02914	0.7	9	0.02040	0.2623
Cesium	Cs	55	132.905	1.9	0.008610	30	20	0.2583	0.1722
Chlorine	Cl	17	35.453	Gas		33	16		
Chromium	Cr	24	51.996	7.19	0.08328	3.1	3	0.2582	0.2498
Cobalt	Co	27	58.9332	8.8	0.08993	37	7	3.327	0.6295
Columbium (see niobium)									
Copper	Cu	29	63.54	8.96	0.08493	3.8	7.2	0.3227	0.6115
Deuterium	D	1	2.01410	Gas		0.0005			
Dysprosium	Dy	66	162.50	8.56	0.03172	940	100	29.82	3.172
Erbium	Er	68	167.26	9.16	0.03203	160	15	5.125	0.4805
Europium	Eu	63	151.96	5.22	0.02069	4300	8	88.97	0.1655
Fluorine	F	9	18.9984	Gas		0.0098	3.9		
Gadolinium	Gd	64	157.25	7.95	0.03045	46,000	4	1401	0.1218
Gallium	Ga	31	69.72	5.91	0.05105	3.0		0.1532	
Germanium	Ge	32	72.59	5.36	0.04447	2.4	3	0.1067	0.1334
Gold	Au	79	196.967	19.32	0.05907	98.8	9.3	5.836	0.5494
Hafnium	Hf	72	178.49	13.36	0.04508	105	8	4.733	0.3606
Heavy water††	D_2O		20.0276	1.105	0.03323	0.0010	13.6	3.323×10^{-5}	0.4519
Helium	He	2	4.0026	Gas		$\leq$0.050	0.8		
Holmium	Ho	67	164.930	8.76	0.03199	65		2.079	
Hydrogen	H	1	1.008665	Gas		0.332			
Illinium (see promethium									
Indium	In	49	114.82	7.31	0.03834	194	2.2	7.438	0.08435
Iodine	I	53	126.9044	4.93	0.02340	6.4	3.6	0.1498	0.08242
Iridium	Ir	77	192.2	22.5	0.07050	460		32.43	
Iron	Fe	26	55.847	7.87	0.08487	2.53	11	0.2147	0.9336
Krypton	Kr	36	83.80	Gas		24	7.2		
Lanthanum	La	57	138.91	6.19	0.02684	8.9	15	0.2389	0.4026
Lead	Pb	82	203.973	11.34	0.03348	0.17	11	0.005692	0.3683
Lithium	Li	3	6.939	0.53	0.04600	71	1.4	3.266	0.0644
Lutetium	Lu	71	174.91	9.74	0.03354	80		2.683	
Magnesium	Mg	12	24.312	1.74	0.04310	0.063	4	0.002715	0.1724
Manganese	Mn	25	54.9380	7.43	0.08145	13.3	2.3	1.083	0.1873
Mercury	Hg	80	200.59	13.55	0.04068	360	20	14.64	0.8136
Molybdenum	Mo	42	95.94	10.2	0.06403	2.6	7	0.1665	0.4482
Neodymium	Nd	60	144.24	6.98	0.02914	50	16	1.457	0.4662
Neon	Ne	10	20.183	Gas		0.032	2.4		
Nickel	Ni	28	58.71	8.90	0.09130	4.6	17.5	0.4200	1.597
Niobium	Nb	41	92.906	8.57	0.05555	1.1	5	0.06111	0.2778
Nitrogen	N	7	14.0067	Gas		1.85	10		

$X10^{24}$

Element or molecule	Symbol	Atomic number	Atomic or molecular weight*	Nominal density, gm/cm^3	Atoms or molecules per cm^3†	σ_a,‡ barns	σ_s,‡ barns	Σ_a,† cm^{-1}	Σ_s,† cm^{-1}
Osmium	Os	76	190.2	22.5	0.07124 N	15	11	1.069	0.7836
Oxygen	O	8	15.9994	Gas		<0.0002	4.2		
Palladium	Pd	46	106.4	12.0	0.06792	8	3.6	0.5434	0.2445
Phosphorus (yellow)	P	15	30.9738	1.82	0.03539	0.19	5	0.006724	0.1770
Platinum	Pt	78	195.09	21.45	0.06622	10	10	0.6622	0.6622
Plutonium	Pu	94	239	19.6	0.04939	σ_a = 1015 σ_f = 741	9.6	49.88 36.55	0.4741
Polonium	Po	84	210	9.51	0.02727				
Potassium	K	19	39.102	0.86	0.01325	2.1	1.5	0.02783	0.01988
Praseodymium	Pr	59	140.907	6.78	0.02898	12	4	0.1965	0.1159
Promethium	Pm	61							
Protactinium	Pa	91	231			210			
Radium	Ra	88	226	5.0	0.01332	20		0.2664	
Rhenium	Re	75	186.2	20	0.06596	85	14	5.607	0.9234
Rhodium	Rh	45	102.905	12.41	0.07263	155	5	11.26	0.3632
Rubidium	Rb	37	85.47	1.53	0.01078	0.73	12	0.007869	0.1294
Ruthenium	Ru	44	101.07	12.2	0.07270	2.5	6	0.1818	0.4362
Samarium	Sm	62	150.35	6.93	0.02776	5800	5	161.0	0.1388
Scandium	Sc	21	44.956	2.5	0.03349	23	24	0.7703	0.8038
Selenium	Se	34	78.96	4.81	0.03669	12	11	0.4403	0.4036
Silicon	Si	14	28.086	2.33	0.04996	0.16	1.7	0.1164	0.08493
Silver	Ag	47	107.870	10.49	0.05857	63	6	3.690	0.3514
Sodium	Na	11	22.9898	0.97	0.02541	0.53	4	0.01347	0.1016
Strontium	Sr	38	87.62	2.6	0.01787	1.3	10	0.02323	0.1787
Sulfur (yellow)	S	16	32.064	2.07	0.03888	0.52	1.1	0.2022	0.04277
Tantalum	Ta	73	180.948	16.6	0.05525	21	5	1.160	0.2763
Technetium	Tc	43	99			22			
Tellurium	Te	52	127.60	6.24	0.02945	4.7	5	0.1384	0.1473
Terbium	Tb	65	158.924	8.33	0.03157	46		1.452	
Thallium	Tl	81	204.37	11.85	0.03492	3.3	14	0.1152	0.4889
Thorium	Th	90	232.038	11.71	0.03039	7.4	12.6	0.2249	0.3829
Thulium	Tm	69	168.934	9.35	0.03314	125	7	4.143	0.2320
Tin	Sn	50	118.69	7.298	0.03703	0.63	4	0.02333	0.1481
Titanium	Ti	22	47.90	4.51	0.05670	6.1	4	0.3459	0.2268
Tungsten	W	74	183.85	19.2	0.06289	19	5	1.195	0.3145
Uranium	U	92	238.03	19.1	0.04833	σ_a = 7.6 σ_f = 4.2	8.3	0.3673 0.2030	0.4011
Vanadium	V	23	50.942	6.1	0.07212	4.9	5	0.3534	0.3606
Water	H_2O		18.0167	1.0	0.03343	0.664	103	0.02220	3.443
Xenon	Xe	54	131.30	Gas		24	4.3		
Ytterbium	Yb	70	173.04	7.01	0.02440	37	12	0.9208	0.2928
Yttrium	Y	39	88.905	5.51	0.03733	1.3	3	0.04853	0.1120
Zinc	Zn	30	65.37	7.133	0.06572	1.10	3.6	0.07229	0.2366
Zirconium	Zr	40	91.22	6.5	0.04291	0.18	8	0.007724	0.3433

N.B.!

* Based on C^{12} = 12.00000 amu.

† Four-digit accuracy for computational purposes only; last digit(s) usually is not meaningful. $(\times 10^{24})$

‡ Cross sections at 0.0253 eV or 2200 m/sec. The scattering cross sections, except for those of H_2O and D_2O, are measured values in a thermal neutron spectrum and are assumed to be 0.0253 eV values because σ_s is usually constant at thermal energies. The errors in σ_s tend to be large, and the tabulated values of σ_s should be used with caution. (From BNL-325, 2nd ed., 1958 plus supplements 1 and 2, 1960, 1964, and 1965.)

** The value of σ_a given in the table is for pure graphite. Commercial reactor-grade graphite contains varying amounts of contaminants and σ_a is somewhat larger, say, about 0.0048 barns, so that $\Sigma_a \approx 0.0003851$ cm^{-1}.

†† The value of σ_a given in the table is for pure D_2O. Commercially available heavy water contains small amounts of ordinary water and σ_a in this case is somewhat larger.

Table and data reprinted from Lamarsh: "Introduction to Nuclear Reactor Theory" by permission of Addison-Wesley Publishing Co. Inc.

This shows that penetration through a distance $x = 1/\Sigma$ reduces the neutron density by a factor of e. It can be shown that this distance $1/\Sigma$ is the average distance a neutron will travel before interacting. This result does not only apply to a beam, but is quite general. The distance $1/\Sigma$ is called the *mean free path*, and is given the symbol λ. Before applying this to a problem on mean free paths in fuel, let us list the thermal neutron cross sections of fuel atoms in Table 2 (the values of ν are given for the sake of completeness). We shall make extensive use of this data later in the course.

TABLE 2

<u>Thermal Neutron Cross Sections of Fuel Atoms (in Barns)</u>

(taken from Atomic Energy Review (IAEA), 1969, Vol 7, No 4, p 3)

	σ_f	$\sigma_{n,\gamma}$	σ_a	σ_s	ν
U-233	530.6	47.0	577.6	10.7	2.487
U-235	580.2	98.3	678.5	17.6	2.430
U-238	0	2.71	2.71	~10	0
nat.U	4.18	3.40	7.58	~10	
Pu-239	741.6	271.3	1012.9	8.5	2.890
Pu-241	1007.3	368.1	1375.4	12.0	2.934

Example: *Calculate the absorption mean free path of thermal neutrons in natural uranium.*

$$\lambda_a = \frac{1}{\Sigma_a} = \frac{1}{\Sigma_f + \Sigma_{n,\gamma}} = \frac{1}{N'(\sigma_f + \sigma_{n,\gamma})}$$

Using the data given in tables 1 and 2, we see that:

$$\lambda_a = \frac{1}{0.048 \times 10^{24} \times 7.58 \times 10^{-24}} \text{ cm}$$

$$= \underline{2.08 \text{ cm}}$$

Incidentally, this rather small value of λ_a helps to explain why the neutron flux at the centre of a fuel bundle is significantly smaller than at its perimeter, giving rise to a so-called *flux depression*.

ASSIGNMENT

1. Prove that the mean free path $\lambda = 1/\Sigma$ for any reaction.

2. U-238 has a very high absorption (σ_a = 8000b) for neutrons of 6.5 eV energy. What is the probability of such neutrons surviving capture in traversing natural uranium of 0.1 mm thickness?

3. Calculate the number of fission neutrons emitted per thermal neutron absorbed in natural uranium and uranium enriched in U-235 to 2% and 10%.

4. A useful expression relating the total thermal power P generated in a reactor to the average neutron flux $\bar{\phi}$ and the quantity of natural UO_2 fuel M is given by:

$$P = \frac{\bar{\phi}.M}{3 \times 10^{12}}$$

where P is in MW, ϕ in $n.cm^{-2}s^{-1}$ and M in Mg. The density of UO_2 is 10.7 $g.cm^{-3}$. Derive this expression.

5. The neutron detectors used in Pickering start up were He-3 proportional counters. They are about 12" long and 2" in diameter, and are filled with He-3 gas at 10 atomospheres. Calculate the expected count rate per unit neutron flux assuming that each neutron reacting in the counter volume will be registered. Also explain why the actual count rate should be less than this, even if the above assumption were valid.

He-3(n,p)H-3 reaction cross-section = 5400 b,
N_o = 0.6 x 10^{24} atoms per 22400 cm^3 at standard temperature and pressure.

6. Part of the ~~The predominant~~ activity in the primary coolant during reactor operation is due to O^{19}. Show that the specific activity ($dis.s^{-1}cm^{-3}$) of N^{16} in the coolant as it leaves the core is given by:

$$A = \frac{\Sigma\phi(1-e^{-\lambda t})}{1-e^{-\lambda T}}$$

$O^{18} + n \rightarrow O^{19} + \gamma$

where t is the core transit time and T the total circuit time.

$\sigma_a \, O^{18}$ = .21 mb.

Calculate this activity for the Douglas Point reactor, for which $\phi = 3 \times 10^{13}$ $n.cm^{-2}s^{-1}$, $t = 0.8s$, $T = 12.7s$ and D_2O density = 0.842 g cm^{-3} at operating temperature.

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