

Nuclear Theory - Course 127

NUCLEAR STRUCTURE

The subjects discussed in this course cover many of the topics already discussed in the levels 4, 3 and 2 Nuclear Theory courses. Therefore, it is inevitable that this level 1 Nuclear Theory course will review much of the material already covered. However, the approach to the subject matter will be somewhat different and certainly more detailed. New topics will also be introduced here and there, so that when you have completed this course you should have a good basic knowledge of the physical processes occurring in nuclear reactors.

For the sake of convenience, this course will be divided into two parts. These are:

- (a) Reactor Theory - The Steady State. This section deals with such topics as the critical size of a reactor, neutron balance in a reactor, moderator and reflector properties. All these discussions centre around a reactor in which the chain reaction is just being maintained and where no variations with time are occurring.
- (b) Reactor Theory - Time Dependent Effects. This section could be considered as being a discussion of the effects that arise from a "Disturbance of the Steady State". The type of topics that would be covered in this section would be temperature change effects, effects of changes in reactivity, reactor regulation and protection, build-up of poison and other effects that vary over short or long periods of time.

The first four lessons of this course will deal with those aspects of nuclear physics which are important for an understanding of reactor theory.

Equivalence of Mass and Energy

Einstein showed that mass and energy are equivalent. The relationship between mass and energy changes may be written:

$$\underline{\Delta E = \Delta m c^2}$$

Where ΔE is the energy change expressed in joules, Δm is the accompanying change in mass given in kilograms and c is the velocity of light, equal to 3×10^8 meters per second.

Δm mass in kg.
 c vel. of light
 ΔE joules.

A convenient and very common unit of energy in nuclear physics is the *electron volt* (abbreviated *eV*). It is the energy gained by an electron in being accelerated through a potential difference of 1 volt.

$$\begin{aligned} 1 \text{ eV} &= 1.6021 \times 10^{-19} \text{ joule} \\ 1 \text{ keV} &= 10^3 \text{ eV} \\ 1 \text{ MeV} &= 10^6 \text{ eV} \end{aligned}$$

$$\begin{array}{c} N \\ \times \\ Z \\ A \\ \text{--- mass no.} \\ Z \\ \text{--- atomic no.} \end{array}$$

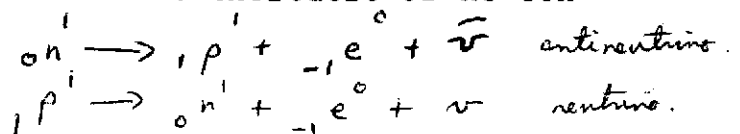
The Nucleus, Nuclear Particles

The atomic nucleus consists of Z protons and N neutrons, where Z and N are the *atomic number* and *neutron number* respectively. The total number of *nucleons* in the nucleus, that is, neutrons and protons, is equal to $Z + N = A$, where A is the *atomic mass number*.

A nuclear species with a given Z and a given A is called a *nuclide*. To distinguish a particular nuclide it is written in the form ${}_Z^A X$, where X is the chemical symbol for the element. Nuclides with the same Z but different A are called *isotopes*. Every element has a number of isotopes - stable and unstable - which range from 3 (hydrogen) to 26 (tin), with an average of about 10 isotopes per element.

The mass of the proton is 1.67252×10^{-27} kg. It carries a positive charge of 1.60210×10^{-19} coulombs (C), equal in magnitude to the negative charge of the electron, and it is a stable particle.

The mass of the neutron is marginally greater than that of the proton, namely 1.67482×10^{-27} kg, and it is electrically neutral. The neutron is not stable unless it is bound in a nucleus. A free neutron decays to a proton with the emission of a β^- particle and an antineutrino, a process which on the average occurs in about 12 minutes. You will see later in this course that the average lifetime of neutrons in a reactor before they are absorbed or leak from the system is no greater than a millisecond. The instability of the neutron is therefore of no consequence in reactor theory.



Nuclear Masses

The masses of atoms are conveniently expressed in *atomic mass units*, or *amu*. The actual mass of a nucleus is measured on a scale, the *physical scale*, such that the mass of the neutral C^{12} atom is precisely 12 amu, and hence $1 \text{ amu} = 1.660438 \times 10^{-27}$ kg.

TABLE 1 - SOME USEFUL NUMBERS

Avogadro's Number	N_o	$6.02252 \times 10^{26} \text{ (kg.mole)}^{-1}$
Electron rest mass	m_e	$9.1091 \times 10^{-31} \text{ kg}$
		$5.48597 \times 10^{-4} \text{ amu}$
		0.511006 MeV
Elementary charge	e	$1.60210 \times 10^{-19} \text{ coul}$
Neutron rest mass	m_n	$1.67482 \times 10^{-27} \text{ kg}$
		1.0086654 amu
		939.550 MeV
Proton rest mass	m_p	$1.67252 \times 10^{-27} \text{ kg}$
		1.0072766 amu
		938.256 MeV
Speed of light	c	$2.997925 \times 10^8 \text{ m.s}^{-1}$

1 MeV	10^6 eV
	$1.60210 \times 10^{-13} \text{ joule}$
1 amu	$1.660438 \times 10^{-27} \text{ kg}$
	931.478 MeV
	$1.49232 \times 10^{-10} \text{ joule}$
1 watt	$1 \text{ joule/sec} \equiv 3.1 \times 10^{10} \text{ fissions/sec}$
1 day	86400 sec
1 year	$3.156 \times 10^7 \text{ sec}$
1 curie	$3.70 \times 10^{10} \text{ dps}$

The atomic mass of a nuclide should be distinguished from the chemical atomic weight which is the average weight of a large number of atoms of a given element. It is not quite the same as the mass of an individual atom unless the element contains a single isotope. Furthermore, you should note that the atomic weight unit on the *chemical scale* is defined as one-sixteenth of the average weight of an oxygen atom in a natural mixture of stable oxygen isotopes (0.204% O^{18} , 0.037% O^{17} and the rest O^{16}). In many calculations this slight distinction (about 3 ppm) is insignificant and the atomic mass, denoted by A , is used rather loosely.

Binding Energy

The mass of the proton is 1.00728 amu, and the mass of the neutron is 1.00867 amu. The actual mass of a nuclide is not equal to the total mass of its individual nucleons, the difference being called the *mass defect*. This mass defect is a consequence of the equivalence of mass and energy and arises from the binding energy of the nuclide. This is the energy required to split the nuclide into its individual component nucleons. From experiment it is found that, except for a few light nuclides, the binding energy per nucleon in the nucleus increases rapidly as the size of the nucleus increases up to about $A = 60$, but for greater values it decreases again gradually. This means that nuclei of intermediate mass are more strongly bound than the light and the heavy nuclei. Thus energy may be released by combining two light nuclei (*fusion*) or by splitting a heavy one into two of intermediate mass (*fission*).

Nuclear Forces

Between two electric charges of the same sign there is a repulsive force which is called a Coulomb force. Since nuclei may contain a large number of positive protons each repelling the others due to Coulomb forces it is clear that there must be other forces present which are attractive. These are short range nuclear forces. They act between all adjacent nucleons, whether n-p, n-n or p-p, and drop off rapidly on separation of the nucleons.

The lighter stable nuclei contain roughly equal numbers of neutrons and protons (eg, ${}_6C^{12}$, ${}_8O^{16}$, ${}_9F^{19}$, ${}_{11}Na^{23}$). As the number of protons in the nucleus increases, the long range Coulomb forces build up more rapidly than the nuclear forces which only have short range. Therefore, in order for heavier nuclei to remain intact more neutrons are required to supply binding forces between all particles to overcome the disruptive Coulomb forces. As a result, the n/p ratio gradually increases from 1 (light nuclei) to about $1\frac{1}{2}$ (eg, ${}_{82}Pb^{208}$, ${}_{90}Th^{232}$, ${}_{92}U^{238}$). For reasons we will not go into here, there is a limit to the number

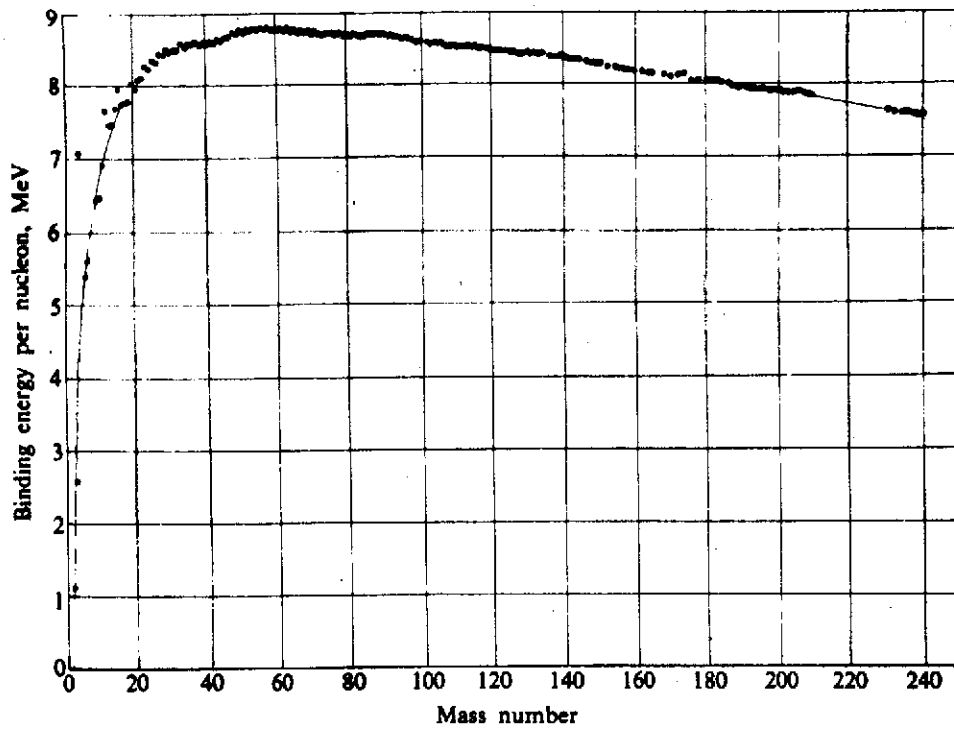


Fig. 1 Binding Energy vs Mass Number

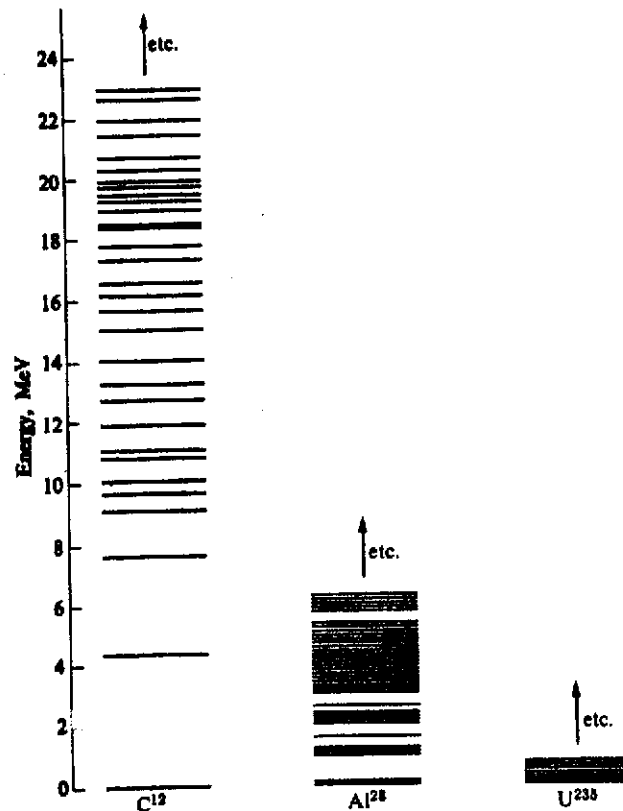


Fig. 2 Nuclear Energy Levels of C^{12} , Al^{28} and U^{238}

of excess neutrons a nucleus can live with, and as a result the heavy nuclei are all unstable and there are no naturally occurring elements having a value of A greater than 238.

Nuclear Energy Levels

A nucleus is said to be in its *ground state* when the nucleons are arranged in such a way that the potential energy is a minimum. If it is not in its ground state it is said to be in an *excited state* and the excess of energy is called *excitation energy*. The potential energy does not take on a continuous range of values, but has discrete values which are termed *energy levels*. For heavy nuclei these energy levels have a minimum separation of about 0.1 MeV, for light nuclei this separation is much greater.

Radioactivity

All the naturally occurring nuclides heavier than lead ($Z = 82$) and a few lighter nuclides are unstable and are *naturally radioactive*. They decay by emitting either an *alpha particle* (helium nucleus) or a *beta particle* (fast electron). In most cases the resulting nucleus, or *daughter*, is produced in an excited state. It then decays to its ground state by the emission of one or more *gamma photons*. Usually, but not always, this occurs instantaneously, ie, within 10^{-14} seconds of the formation of the daughter. A radioactive nuclide, or *radionuclide*, may also decay by capturing one of the inner orbital electrons and this is known as *K capture*. The γ photons after leaving the nucleus may be absorbed by ejecting an electron from an orbit of the same atom; this gives rise to secondary β particles, the process being known as *internal conversion*.

Radioactivity is governed by only one fundamental law, namely that the probability of a radionuclide decaying in unit time is constant and independent of external conditions. This constant is called the decay constant and is denoted by λ .

Consider a sample of radioactive material containing only one kind of radionuclide. If there are n atoms of this nuclide present at time t , then the number decaying in the time interval between t and $t + dt$ is $\lambda n dt$. The number of atoms is therefore reduced by dn , where:

$$\begin{aligned} dn &= -\lambda n dt \\ \text{or} \quad n &= n_0 e^{-\lambda t} \end{aligned}$$

where n_0 is the number of atoms at time $t = 0$.

Specific activity
d s⁻¹ μg⁻¹

e.g.

I¹³¹

t_{1/2} = 8 days

131 kgm → 6 × 10²⁶

131 μgm → 6 × 10¹⁷

1 μgm → $\frac{6 \times 10^{17}}{131}$

$$\left(\frac{.69}{8 \times 24 \times 600} \right)$$

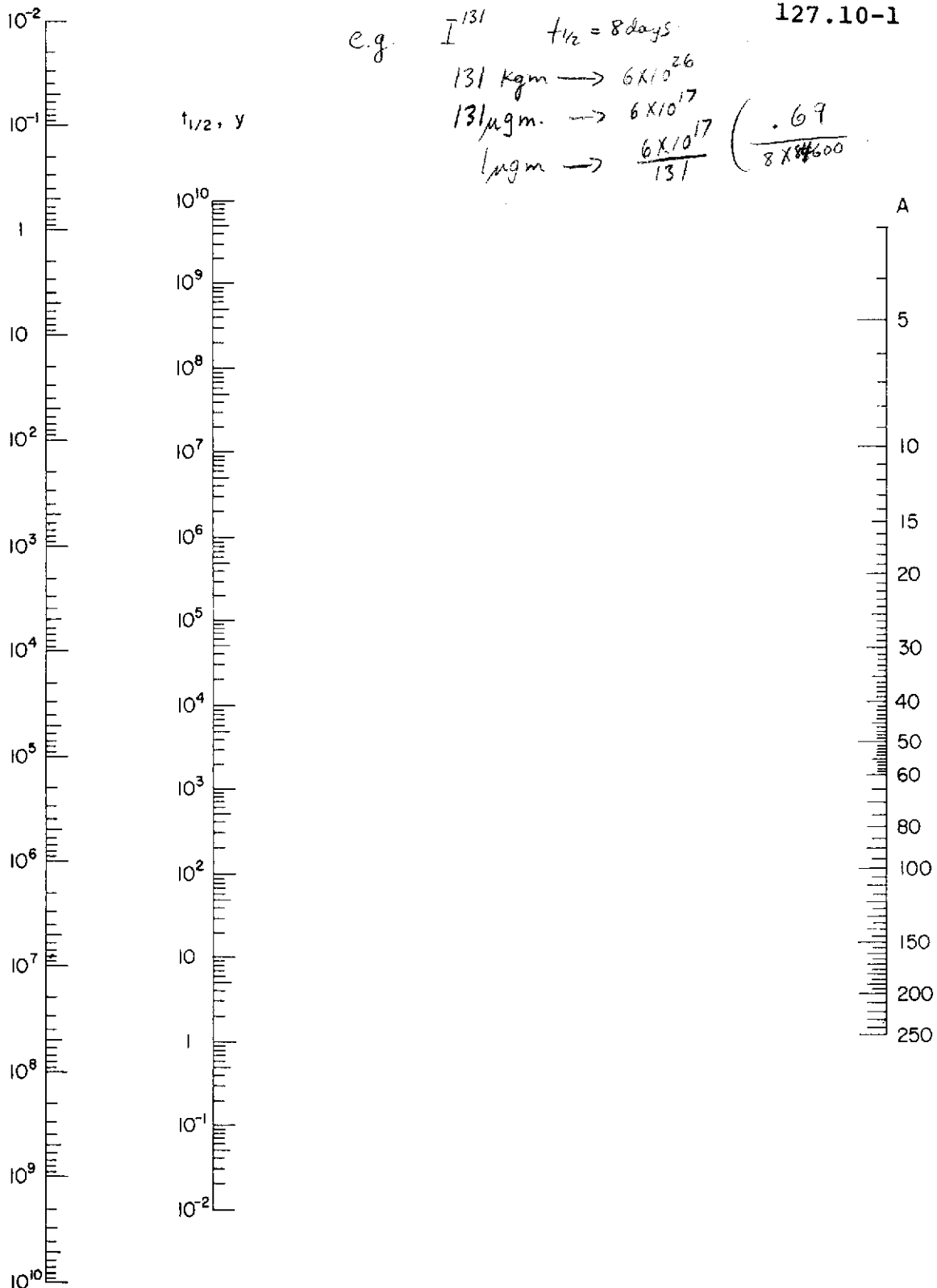


Fig. 3 Nomogram for Calculating Specific Activities

Place a straight edge over the values of the mass number (A) and the half life (in years), and read where it crosses the scale for specific activity in disintegrations per second per microgram (ds⁻¹ μg⁻¹)

The time taken for the number of atoms to be diminished to one half is called the *half-life*, and is denoted by $t_{1/2}$. It is not hard to show that:

$$t_{1/2} = \frac{\ln 2}{\lambda} = \frac{0.693}{\lambda}$$

The *activity* of the sample is just the number of disintegrations per second, $\frac{dn}{dt}$, ie, λn . The unit of activity is the *curie* (abbreviated *Ci*), which corresponds to a decay rate of 3.70×10^{10} disintegrations per second.

Before we continue with an example, it would probably be best to refresh your memory with respect to *Avogadro's number*, N_0 . This is the number of molecules per kg.mole, or the number of atoms per kg.atom. A kg.atom is A kg where A is the atomic mass of the substance (in amu). $N_0 = 6.0225 \times 10^{26}$ (kg.mole)⁻¹ and this number holds for all substances. For example, 131 kg of I^{131} contain N_0 I^{131} atoms and 18 kg of water also contain N_0 water molecules.

Example: Calculate the activity due to K^{40} in an 80 kg man, assuming that 0.35% of the body weight is potassium. The natural abundance of K^{40} in potassium is 0.0118%, and $t_{1/2} = 1.27 \times 10^9$ years.

$$\begin{aligned} \text{Mass of } K^{40} &= 80 \times 0.35 \times 10^{-2} \times 0.0118 \times 10^{-2} \\ &= 33 \times 10^{-6} \end{aligned}$$

The number of K^{40} atoms, n , is then given by:

$$\begin{aligned} n &= 33 \times 10^{-6} \times \frac{6 \times 10^{26}}{40} \\ &= 5 \times 10^{20} \end{aligned}$$

The activity now becomes:

$$\begin{aligned} \lambda n &= \frac{0.69}{1.27 \times 10^9} \times 5 \times 10^{20} \text{ disintegrations per year} \\ &= 8000 \text{ dps} \\ &= \frac{8000}{3.7 \times 10^4} \mu\text{Ci} \\ &= \underline{0.22 \mu\text{Ci}} \end{aligned}$$

The five pages following this lesson show a *Chart of the Nuclides*. This is a plot of all known nuclides and some of their properties, such as direction of decay, half life and natural abundance. There are charts with considerably more detail, usually much bigger and printed in multi colors, which you can refer to in the stations. The chart reproduced here (by kind permission of Encyclopaedia Britannica) nevertheless gives you most of the information you would normally need to look up. In fact, you will have to refer to it in order to do some of the assignments below.

ASSIGNMENTS

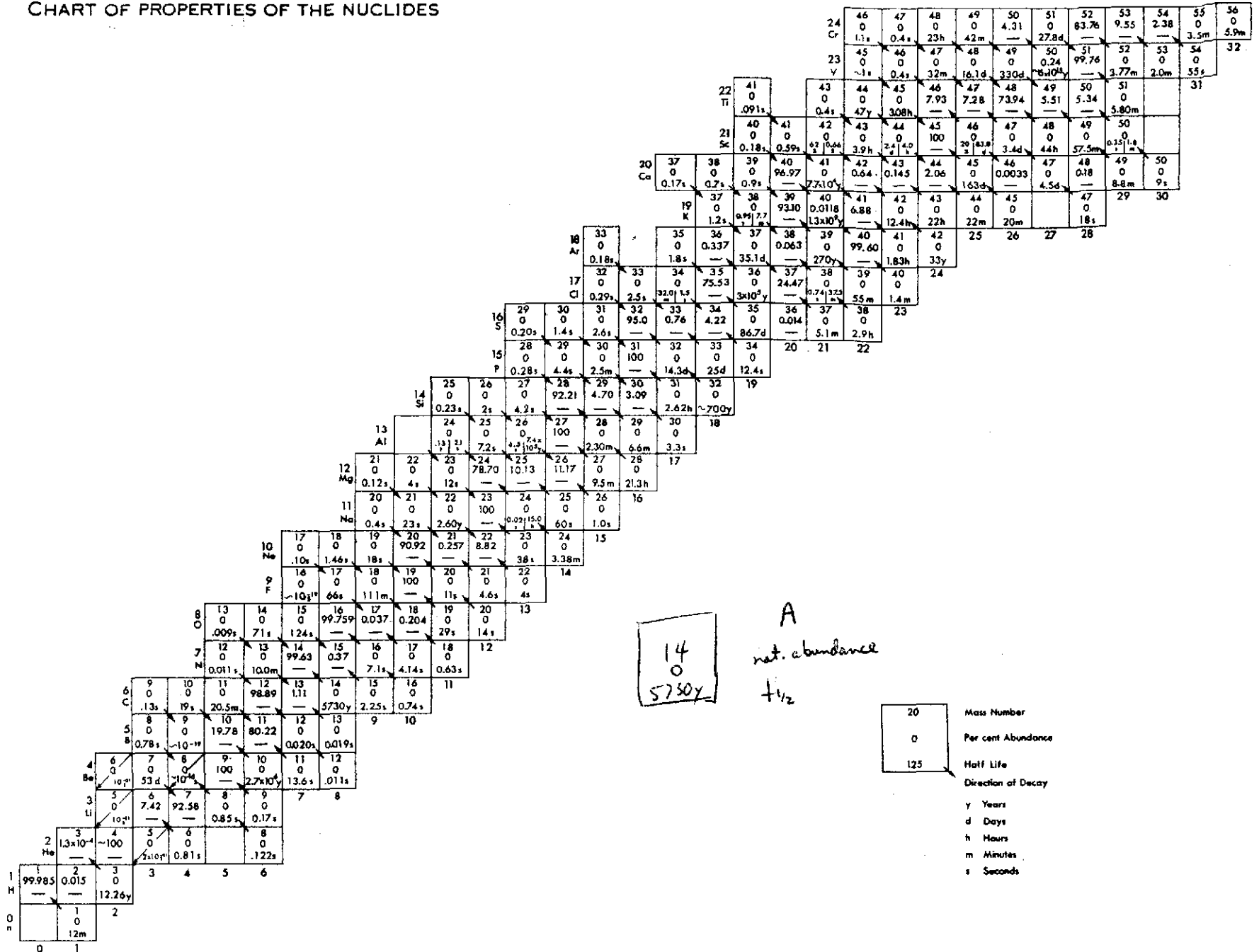
1. Assuming all the mass of an atom to be concentrated in the nucleus, calculate the density of the nucleus in tonnes (1 Mg) per cubic millimeter. The radius of a nucleus of mass number A is believed to be approximately given by $r = 1.2 \times 10^{-15} A^{1/3} \text{ m}$. *7.98 x 10¹⁰ tonnes/mm³ ✓*
2. A carefully purified sample of U^{238} weighs 6.70 mg and undergoes 85 dps. What is the half-life of U^{238} ? *4.36 x 10⁹ yrs. ✓*
3. How much helium at STP will be formed from 1 g U^{238} in one million years? ~~1000000 yrs.~~
~~1000000 yrs.~~
4. The quantity of radiographs taken with x or γ rays is improved as the size of the radiation source is decreased. On this basis, compare the merits of 5 Ci Ra^{226} and 5 Ci Rn^{222} at atmospheric pressure. Consider each source to be spherical.
5. Show that the mean lifetime of radioactive atoms is about 1.4 times their half-life.
6. U^{235} has a half-life of 1.2×10^{17} years for spontaneous fission. Estimate the rate of spontaneous fissioning for 1 gram of U^{235} .
7. As a result of fuel failure, I^{131} activity in the delay tank is 6 Ci. The tank has a volume of 0.85 m^3 and the half-life of I^{131} is 8 days. The permissible release rate through the ventilation exhaust system is $0.05 \mu\text{Ci/s}$. Calculate the total weight of I^{131} in the tank and how long it must be held there before the tank can be exhausted at a rate of $2.4 \times 10^{-4} \text{ m}^3 \cdot \text{s}^{-1}$. *121 days. ✓*

$$\begin{matrix} & A & \text{isobar} \\ & \times & \\ z & & \\ & & \text{isotope} \end{matrix}$$

J.U. Burnham

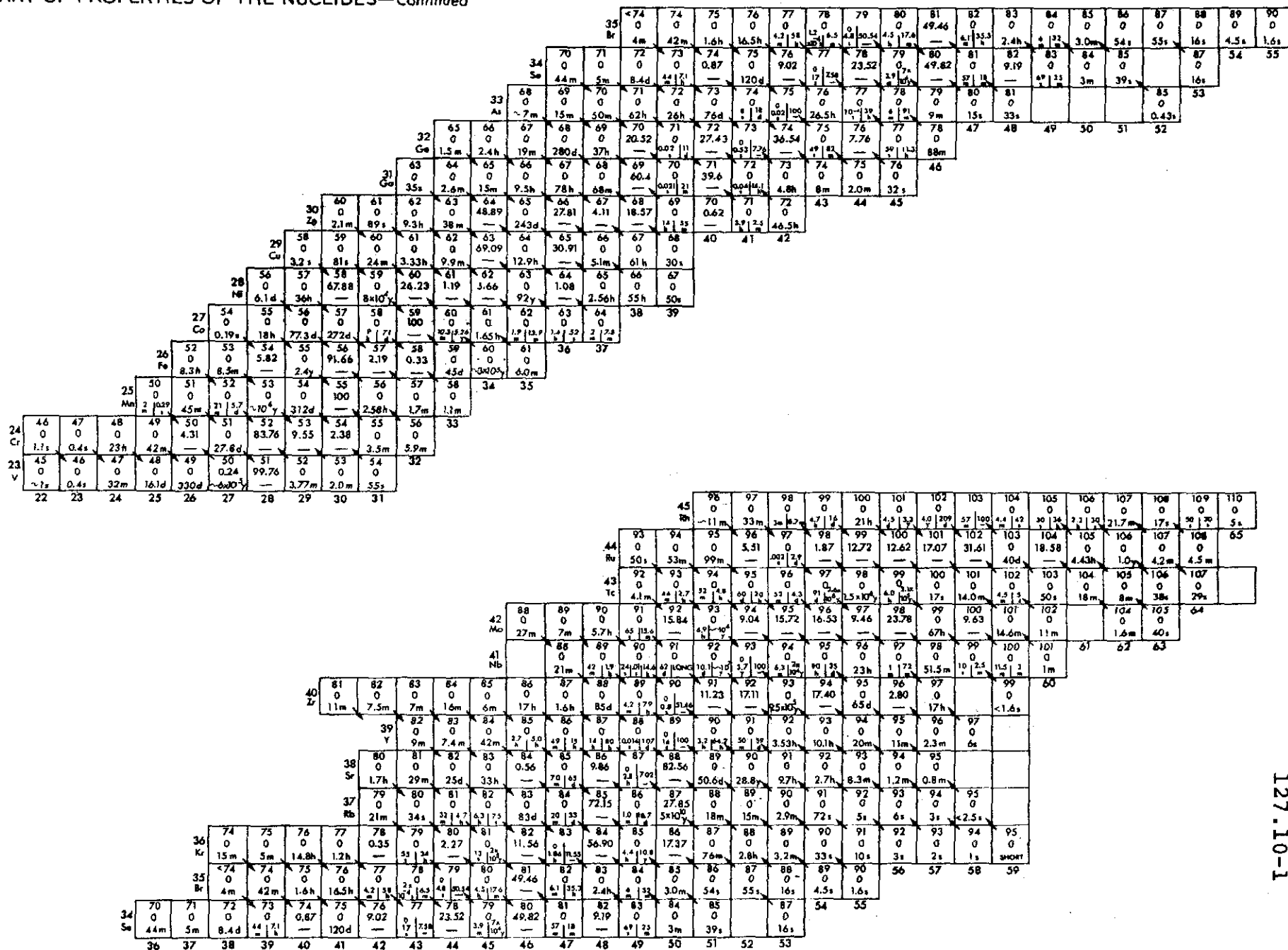
CHART OF PROPERTIES OF THE NUCLIDES

- 10 -



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CHART OF PROPERTIES OF THE NUCLIDES—Continued



50	108	109	110	111	112	113	114	115	116	117	118	119	120	121	122	123	124	125	126	127	128	129	130	131	132
Sn	9m	18m	4.0h	35m	—	20y 11h	0.66	0.35	14.30	4.71	24.03	32.83	—	—	—	—	5.94	7.71	—	—	—	—	—	—	—
49	106	107	108	109	110	111	112	113	114	115	116	117	118	119	120	121	122	123	124	125	126	127	128	129	
In	5.3m	32m	40.58	12.43	1.91	0.0	12.39	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	
48	101	102	103	104	105	106	107	108	109	110	111	112	113	114	115	116	117	118	119	120	121	122	123	124	
Cl	15m	30m	10m	57m	55m	—	6.5h	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	
47	100	101	102	103	104	105	106	107	108	109	110	111	112	113	114	115	116	117	118	119	120	121	122	123	
Ag	9m	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
46	98	99	100	101	102	103	104	105	106	107	108	109	110	111	112	113	114	115	116	117	118	119	120	121	
Nd	17m	22m	3.7d	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	
45	96	97	98	99	100	101	102	103	104	105	106	107	108	109	110	111	112	113	114	115	116	117	118	119	
Th	1.1m	33m	2.9	4.7	2.1	2.1	4.2	2.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	
44	93	94	95	96	97	98	99	100	101	102	103	104	105	106	107	108	109	110	111	112	113	114	115	116	
Pa	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
	50m	53m	99m	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	
	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	

61	140	141	142	143	144	145	146	147	148	149	150	151	152	153	154
Pm	6m	9m	34m	265d	363d	18y	1600d	2.6y	5.1	5.3	2.7h	28h	6m	5.5m	2.5m
60	137	138	139	140	141	142	143	144	145	146	147	148	149	150	151
Nd	55m	5h	5.2h	2.7h	2.7h	2.7h	2.7h	2.7h	2.7h	2.7h	2.7h	2.7h	2.7h	2.7h	2.7h
59	134	135	136	137	138	139	140	141	142	143	144	145	146	147	148
Pr	17m	13m	13.5m	70m	2.0h	4.5h	3.4m	19.2h	13.7d	0.3m	5.9h	24m	12m	2m	2.3m
58	129	130	131	132	133	134	135	136	137	138	139	140	141	142	143
Ca	13m	30m	30m	4.2h	6.3h	7.2h	18h	0.19y	0.250	0.19	0.19	0.19	0.19	0.19	0.19
57	124	125	126	127	128	129	130	131	132	133	134	135	136	137	138
La	7m	<1m	1m	3.5m	4.6m	10m	9m	61m	4.8h	4h	6.5m	19.4h	9.5m	6x10y	11x10y
56	123	124	125	126	127	128	129	130	131	132	133	134	135	136	137
Ba	2m	0	0	0	0	0	0	0	0	0	0	0	0	0	0
55	123	124	125	126	127	128	129	130	131	132	133	134	135	136	137
Ce	8m	45m	1.6m	6.2h	3m	32h	30m	9.7d	6.6d	—	—	—	—	—	—
54	118	119	120	121	122	123	124	125	126	127	128	129	130	131	132
Zr	6m	6m	41m	39m	19h	2.0h	—	—	—	—	—	—	—	—	—
53	117	118	119	120	121	122	123	124	125	126	127	128	129	130	131
Hf	7m	14m	20m	1.4h	1.4h	3.5m	13h	4.2d	6.0d	12.8d	—	—	—	—	—
52	107	108	109	110	111	112	113	114	115	116	117	118	119	120	121
Te	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	2.2h	5.3h	—	—	—	—	—	—	—	—	—	—	—	—	—
51	112	113	114	115	116	117	118	119	120	121	122	123	124	125	126
Sb	0.9m	7m	3.4m	31m	—	—	—	—	—	—	—	—	—	—	—
50	108	109	110	111	112	113	114	115	116	117	118	119	120	121	122
Sn	9m	18m	4.0h	35m	—	—	—	—	—	—	—	—	—	—	—
49	106	107	108	109	110	111	112	113	114	115	116	117	118	119	120
In	5.3m	32m	40.58	12.43	1.91	0.0	12.39	—	—	—	—	—	—	—	—

