Mathematics - Course 421

GRAPHS

I Uses of Graphs

Graphs are used to

(1) display the relationship between 2 or more variables
(2) summarize data pictorially for easy assimilation.

II Rectangular Co-ordinate System

A *rectangular co-ordinate system* is set up by drawing two mutually perpendicular lines *(axes)* which intersect at the *origin, 0*. The vertical axis is usually called the *y-axis;* its upward branch is labelled "*y*" to indicate that *y* increases vertically upwards. The horizontal axis is usually called the *x-axis;* its rightward branch is labelled "*x*" to indicate that *x* increases horizontally rightward. The axes divide the *xy-plane* into four *quadrants*, as in Figure 1.

A uniform scale of length units is marked on each axis, starting from 0. The position of a point in the plane is specified by its distance from the y-axis (the *x co-ordinate* or *abscissa*) and its distance from the x-axis (the *y co-ordinate* or *ordinate).* For example the point P(2, 3), with x co-ordinate 2 and y co-ordinate 3, is located in the plane at the intersection of perpendiculars erected to the x-axis, 2 units from 0, and to the y-axis, 3 units from 0 (see Figure 2).

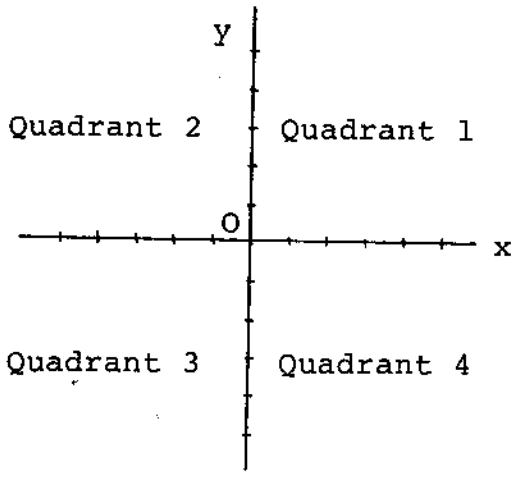


Figure 1

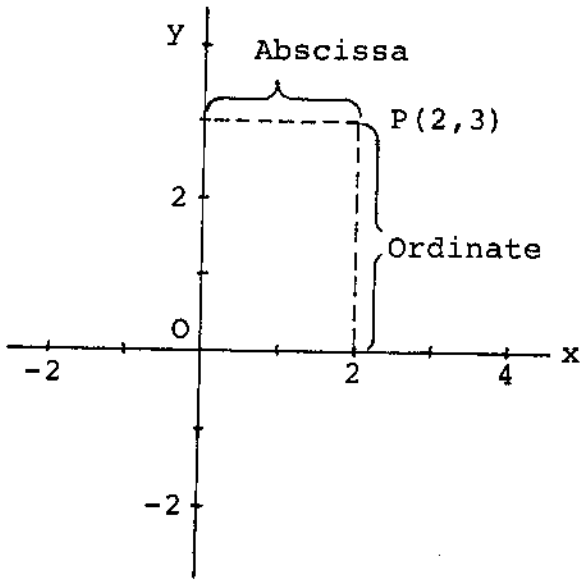


Figure 2

The tedious process of constructing perpendiculars is usually eliminated by the use of *squared paper*.

III Data Graphs

Steps to Plotting a Data Graph

Step 1:

Select a piece of graph paper of suitable dimensions and size of grid squares to display data.

Step 2:

Select the *independent variable* (the one deliberately varied in an experiment) to be displayed horizontally and the *dependent variable* (the one which responds to changes in the independent variable) to be displayed vertically. For example, temperature would normally be plotted vertically on a temperature - time graph. Note, however, that choice of variable to be displayed vertically is often a matter of personal judgement or convenience - eg, graph of voltage vs current, where either variable could be independent.

Step 3:

Choose display ranges and scales to spread data over about two-thirds or more of available space along either axis. Draw axis and mark on scales.

Step 4:

Label axes with respective quantities and units thereof.

Step 5:

Plot data. Make data points visible by some method such as circling them.

Step 6:

Indicate the pattern or trend of the data by

(a) joining successive data points by straight line segments to produce a *LINE GRAPH*, if the data does not obey a simple relationship, or

(b) drawing a smooth averaging *CURVE* through the data ("*curve*" here includes the case of the straight line), if the data does obey a simple law.

Step 7:

Place a suitable title on the graph.

Example 1: Hospital Patient's Temperature Chart

The following table indicates a patient's temperature readings taken at 6-hour intervals May 1 to 3 inclusive. Plot a temperature-time graph for the patient.

Day	May 1				May 2				May 3			
Time (hr)	0000	0600	1200	1800	0000	0600	1200	1800	0000	0600	1200	1800
Temp (°C)	37.6	37.3	37.1	36.9	36.9	36.9	37.1	38.9	38.1	37.2	36.9	36.9

Note that all the above temperature readings lie between 36.9° and 38.9°C. The above data have been plotted in Figure 3, using an unsuitable temperature display range of 0° to 40°C, and again in Figure 4 using a temperature display range of 36.8° to 39.0°C. Figure 4 is obviously much easier to read and interpret than Figure 3. This contrast between Figures 3 and 4 illustrates the importance of choosing a suitable scale and display range (step 3 above).

Note that a *line graph* has been produced (step 6 above), since there is no simple law relating a patient's temperature with time.

Figure 3: HOSPITAL PATIENT'S TEMPERATURE CHART

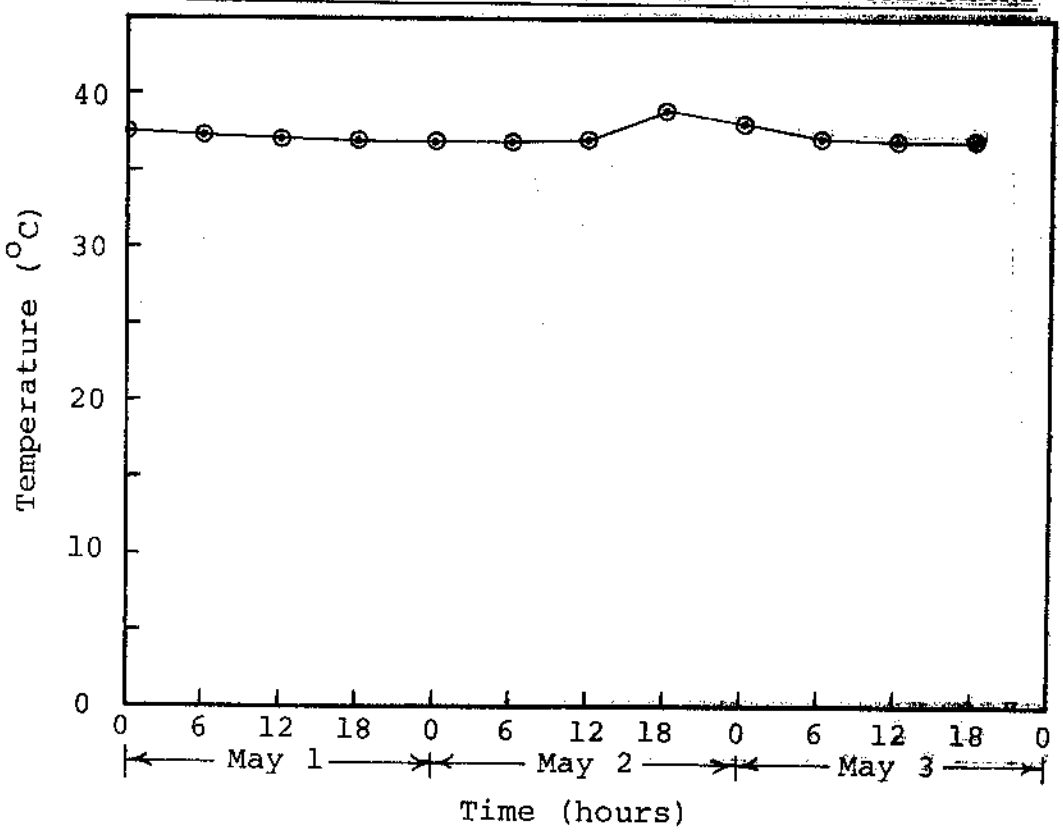
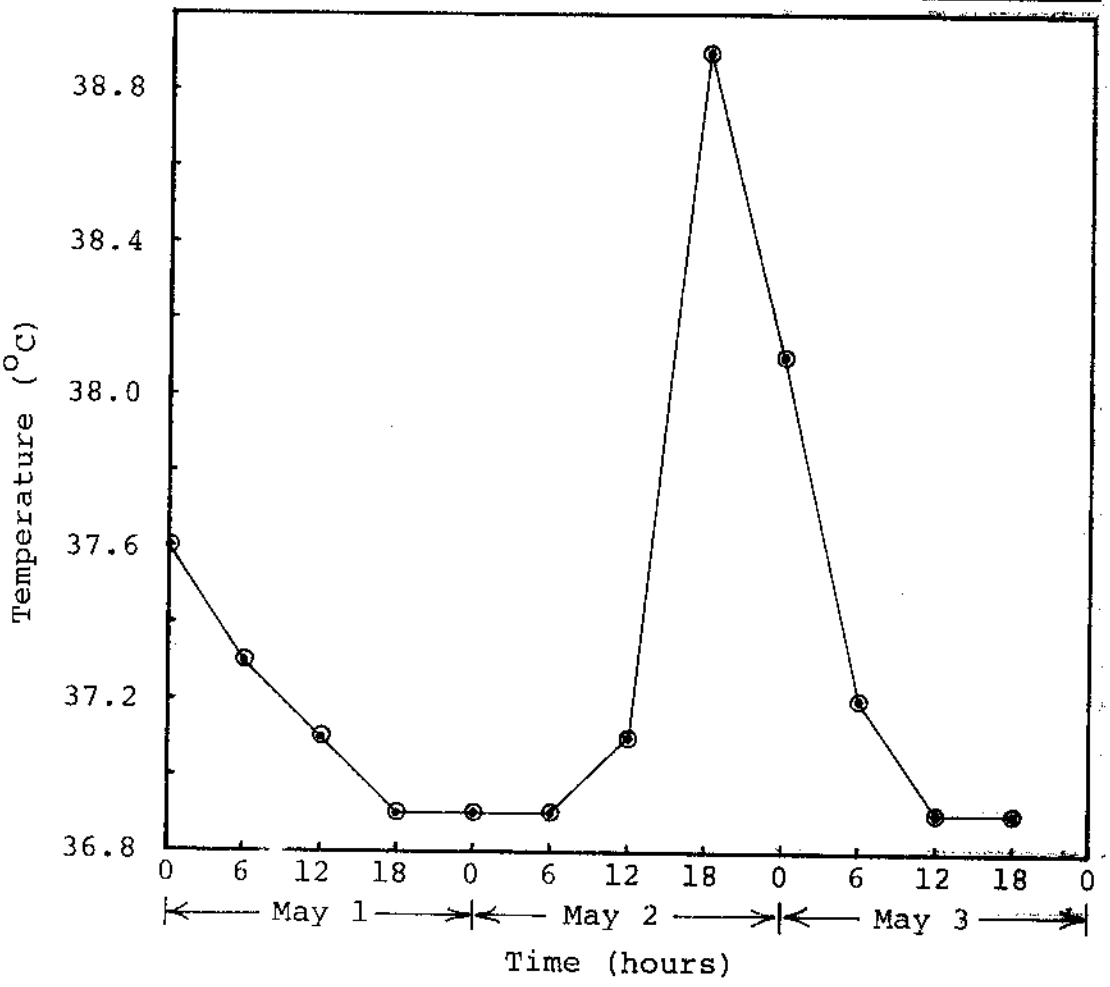


Figure 4: HOSPITAL PATIENT'S TEMPERATURE CHART



Example 2: Average Weight vs Age of Teenage Boys

The following table gives the weights of boys of various ages. Draw a graph to illustrate this variation. From the graph, find the average weight of (a) a 12½ year old boy (b) a 16 year old boy.

Age (years)	10	11	12	13	14	15
Weight (kg)	34.7	36.3	38.6	41.7	45.8	51.7

A step-by-step solution is given for this example:

Step 1:

Graph paper with one millimeter squares is suitable for this application.

Step 2:

Weight will be plotted vertically and age horizontally. (Weight is responding to age, not age to weight.)

Step 3:

y-axis: weights from 34 to 60 kg, at scale 2 kg = 1cm

x axis: ages 10 to 16 years at scale 1 year = 2 cm.

Steps 4 and 5:

See Figure 5 for axis labels, data plot.

Step 6:

Since there is an obvious relationship between average weight and age of boys 10 to 15 years old, a smooth curve is drawn through the data.

Step 7:

See Figure 5 for title.

The curve drawn in Figure 5 represents the relationship between average weight and age of boys of all ages from 10 to 15 years, inclusive. Thus the average weight of a 12½ year old boy, from Figure 5, is 40.1 kg. This is an example of *INTERPOLATION* - estimating values of variables between given data points.

By assuming that the trend of the relationship continues to age 16 years, one can estimate the average weight of a 16 year old boy. As seen from Figure 5, this weight is 60.0 kg. This is an example of *EXTRAPOLATION* - estimating values of variables outside of the range of the given data.

Note that interpolation gives more reliable estimates than extrapolation because the former is guided by given data on both sides of the estimate, whereas the latter is guided by data on one side only of the estimate, and the assumption that the trend of the data continues as far as the estimated value.

Example 3: Load - Effort Relationship for a Machine

The following table contains experimentally determined values of the effort required to move various loads, using a certain machine. Draw a graph to show the load-effort relationship.

Load (kg)	30	40	60	70	80
Effort (kg)	2.13	2.6	3.8	4.3	5.1

The required graph is shown in Figure 6. Note that the curve best fitting the data in this example is a straight line. The curve itself represents an estimate of the true relationship between load and effort. The various data points lie slightly above or below the curve simply because of the uncertainty inherent in the experimental measurements.

When the curve best fitting data is deemed to be a straight line, as in this example, the relationship between the variables is said to be *linear* (noun "line"; adjective "linear").

Figure 5: AVERAGE WEIGHT vs. AGE FOR TEENAGE BOYS

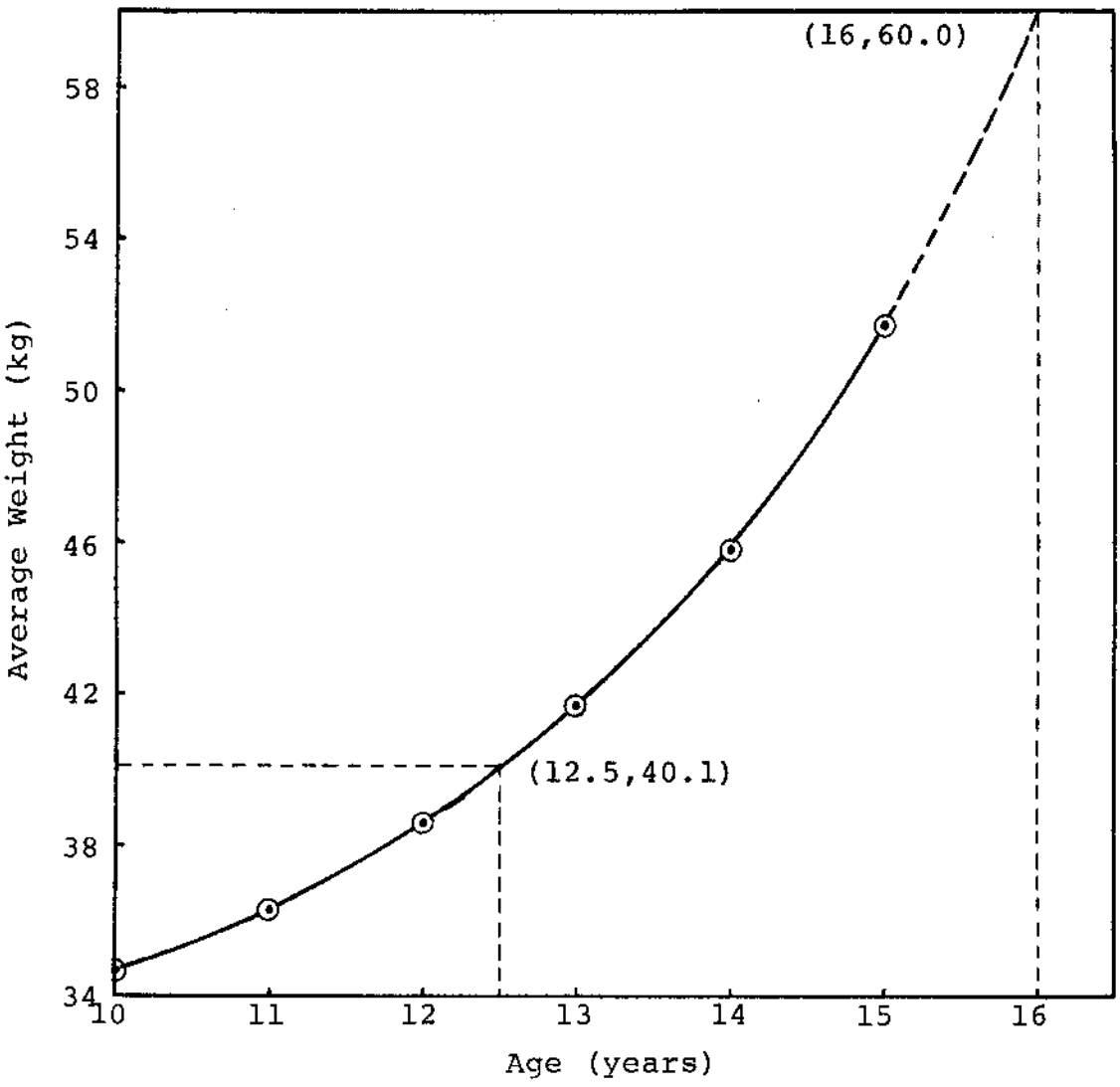
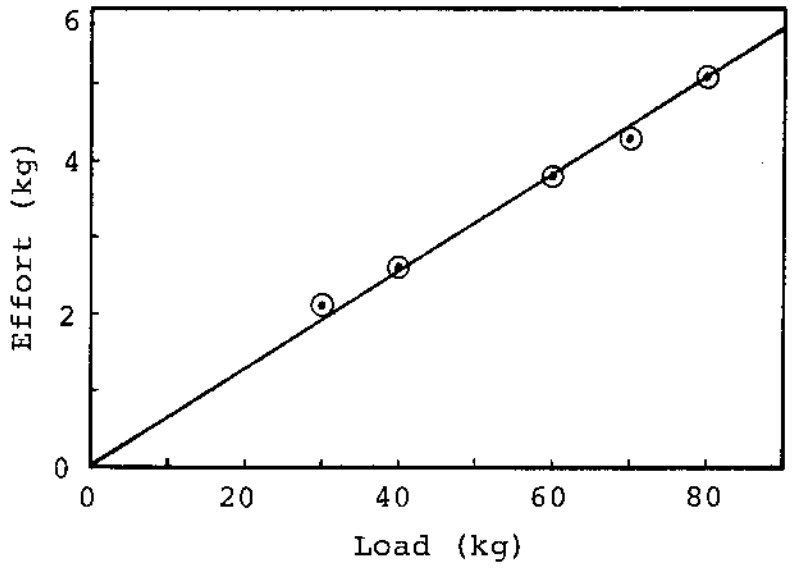


Figure 6: LOAD-EFFORT RELATIONSHIP FOR A MACHINE



ASSIGNMENT

1. Plot the following points:

   (a) P(3,4)
   (b) Q(-2,4)
   (c) R(-5,-4)
   (d) S(4,-2)
   (e) T(15,10)
   (f) U(-3,8)

2. The following table shows the temperature at two-hourly intervals for one day. Plot a graph to illustrate this variation in temperature.

Time	2am	4	6	8	10	Noon	2pm	4	6	8	10	12
Temp (°C)	9	8	9	12	14	18	23	26	22	20	16	14

3. The following table gives the current, I, in a circuit, for various values of the resistance, R, when the voltage remains constant.

R(ohms)	2	4	8	12	16	20	40	60
I(amperes)	60	30	15	10	7.5	6	3	2

Plot a graph showing how the current varies with the resistance and estimate

(a) The current when R = 10 ohms and
(b) The resistance required to give a current of 50 amperes.

4. The pressure, P, at different depths, h, in a liquid is found to be as follows:

h(cm)	0	10	20	30	35
P(kPa)	103	261	419	577	656

Plot the graph and from the graph estimate:

(a) The pressure at a depth of 50 cm.
(b) The depth at which the pressure is 300 kPa.

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