DEPARTMENT OF NUCLEAR TECHNOLOGY, FACULTY OF ENGINEERING
CHULALONGKORN UNIVERSITY
PHAYATHAI ROAD, BANGKOK 10330, THAILAND
TEL: (662) 218-6772, (662) 218-6784. FAX: (662) 218-6770
E-mail: fnegbr@eng.chula.ac.th

THAI - CANADIAN

NUCLEAR HUMAN RESOURCES DEVELOPMENT

LINKAGE PROJECT

TRAINING PROGRAM

NUCLEAR PHYSICS & REACTOR THEORY

SUPPLEMENTARY TEXT

Prepared by

John L. Groh

SPONSORED BY
ATOMIC ENERGY OF CANADA LIMITED
CANADIAN INTERNATIONAL DEVELOPMENT AGENCY
ELECTRICITY GENERATING AUTHORITY OF THAILAND
OFFICE OF ATOMIC ENERGY FOR PEACE

SUPPLEMENT TO CHAPTER 8
OF
REACTOR PHYSICS FUNDAMENTALS

This supplement provides mathematical background and detail not in the text. It also reviews the text material from several different points of view. You should be familiar with the text material before studying this supplement.

Reactor Kinetics:
Power Increase for a Step Δk Increase with the Reactor Critical

Very Small Δk

Small Δk

Large Δk

How to Avoid Prompt Criticality

Power Rundown

John L. Groh

Power Increase After a Step Δk Increase

Chapter 8 describes the reactor response when k is suddenly made greater than one. It ignores control system response and reactivity effects caused by change in temperature or by changes in xenon-135 concentration.

The reactor response to a *small* Δk increment (e.g. for normal reactor control) is simple. The simple formula describing this is reviewed in section 1. below. For *larger reactivity changes* a more complicated formula is needed. See section 2. below. Even this formula is no good for very large steps with Δk *close to* β. Reactor behaviour in this case is reviewed in section 3. below.

1. The simple formula $P(t) \propto e^{t/\tau}$ (8.3 on pages 6) can be derived without a differential equation. If the initial thermal neutron population in the critical reactor is N_0, then one generation after the reactivity addition the population is:

$$N_1 = (1+\Delta k)N_0$$

After two generations it is $N_2 = (1+\Delta k)N_1 = (1+\Delta k)^2 N_0$

After three generations it is $N_3 = (1+\Delta k)^3 N_0$

.

.

.

and after n generations it is $N_n = (1+\Delta k)^n N_0$

Equivalently, since neutron power is proportional to the neutron flux

$$P_n = (1+\Delta k)^n P_0$$

This is a perfectly good formula, but to make it look like the formula in the text, use the properties of the natural log to write $(1+\Delta k)^n = e^{n\ln(1+\Delta k)}$

When $\Delta k << 1$, $\ln(1+\Delta k) \approx \Delta k$, so the equation can be written as $P_n = P_0 e^{n\Delta k}$

Finally, the average time for a generation is ℓ, so time $t = n \times \ell$ and

$$P(t) = P_0 e^{(\Delta k/\ell)t}$$

This looks exactly like equation 8.3 if the reactor period is $\tau = \ell/\Delta k$

In this approximation the neutrons are considered as a single group. Their average lifetime $\mathcal{L}$, (calculated on page 10 as $0.993 \times 0 + 0.007 \times 13 = 0.091$ s) can be written using algebra:

$$\mathcal{L} = (1-\beta) \times \ell + \beta/\lambda \approx \beta/\lambda$$

If we use this formula, equation 8.3 becomes.

* $P(t) = P_0\, e^{t/\tau}$ with $\tau = \mathcal{L}/\Delta k = \beta/(\lambda \Delta k)$

The single group approximation is excellent for small Δk values, not more than a tenth of a mk, typical of the reactivity used for normal reactor power manoeuvring.

A feature of an exponential power rise is that the rate of change of the *log* power is constant. Chapter 10 discusses the following useful equation.

$$\ln P(t) = \ln P_0 + \frac{t}{\tau} \quad \text{so} \quad \frac{d \ln P(t)}{dt} = \left(\frac{1}{P}\frac{dP}{dt}\right) = \frac{1}{\tau}$$

The reactor responds to a small reactivity step at low power with a constant fractional increase each second. The physical meaning of the reactor period, τ, is that its inverse measures this constant fractional rate of increase.

Pages 9 & 10 demonstrate a very fast power increases IF all the neutrons are prompt. No such reactor exists, but the reactor behaviour described is much like a prompt critical reactor ($\Delta k \cong \beta$), discussed in section 3. below. Operation in which Δk approaches β cannot be allowed.

For reference, here are text values of the constants.

Diffusion Time	$\ell \approx 0.001$ s	
Average Lifetime	$\mathcal{L} \approx \beta/\lambda \approx 0.1$ s	
Delayed Neutron Decay Constant	$\lambda \approx 0.08\ s^{-1}$	($1/\lambda \approx 12.5$ s)
Delayed Neutron Fraction	$\beta = 0.007$ (fresh fuel), $\beta \approx 0.056$ (equilibrium fuelling)	

$\mathcal{L}$ changes a little with fuel burnup because of the variation in β.

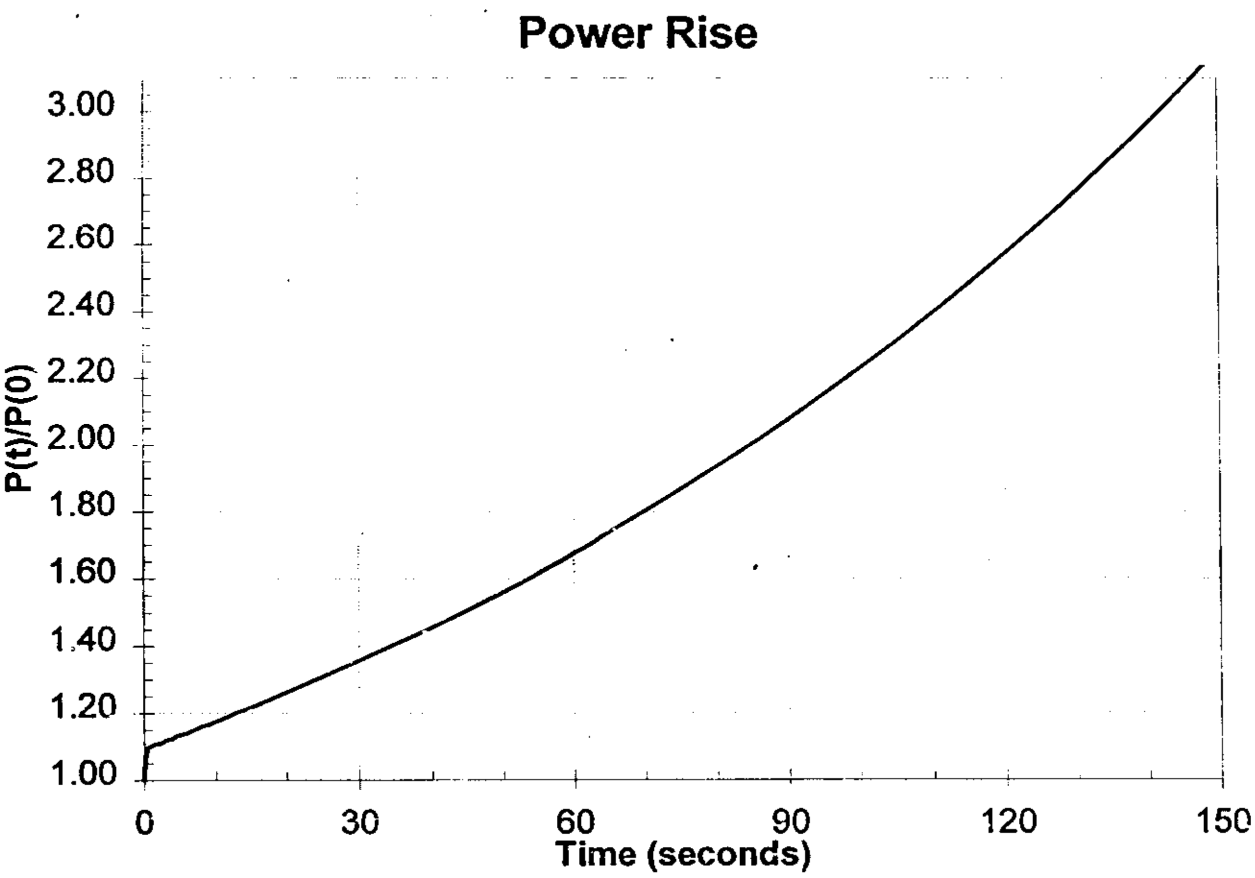
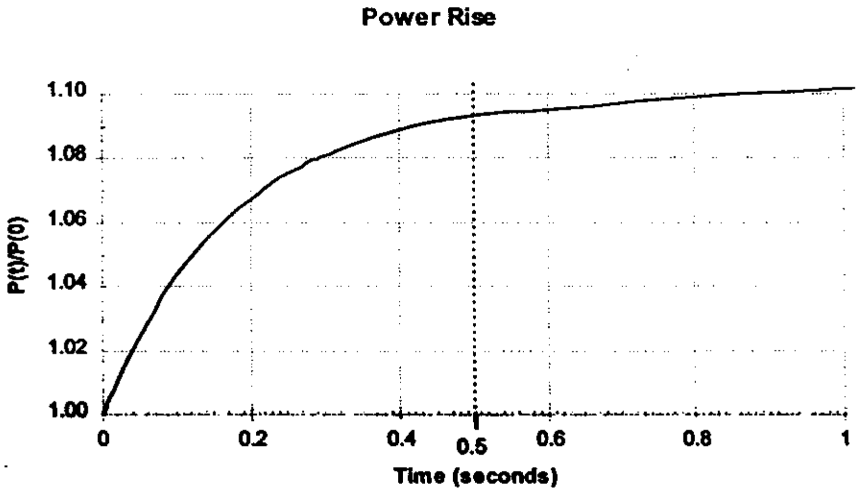


Figure 1. Power Rise following a +0.5 mk step increase in reactivity ($\Delta k = 0.0005$) in a critical core with $\beta = 0.00582$, $\ell = 0.88$ ms, $\lambda = 0.0758\ s^{-1}$.



2. For analysis of upsets, where larger Δk step increases of 2 or 3 mk could occur, the <u>one group</u> formula is not accurate. A better method keeps track of <u>two separate groups</u> of neutrons: prompt and delayed. The *delayed* neutrons are lumped together in one group with a lifetime ≈ 12.5 s, and a decay constant, $\lambda = (1/12.5)\ s^{-1} = 0.08\ s^{-1}$.

<u>THE SIMPLIFIED TWO NEUTRON GROUP FORMULA</u>

$P(t) = P_0\,[\beta/(\beta-\Delta k)]\,e^{t/\tau}$	[formula 8.6 on page 16]
$\tau = [(\beta-\Delta k)/(\lambda\,\Delta k)]$	[formula 8.5 on page 15]

To get the new formula for P and τ from the old ones, substitute:

$P_0 \rightarrow P_0 \times [\beta/(\beta-\Delta k)]$ into $P(t) = P_0\, e^{t/\tau}$ and

$\lambda \rightarrow \lambda \times [\beta/(\beta-\Delta k)]$ into $\tau = \beta/(\lambda\Delta k)$

The <u>Prompt Jump factor</u>, $[\beta/(\beta-\Delta k)]$ is a new feature, illustrated in the graph opposite. The prompt increase lasts for a few tenths of a second.

For very small Δk, formula 8.6 is identical to 8.3. i.e. When $\Delta k << \beta$, $[\beta/(\beta-\Delta k)] \approx 1$, making $P(t) \approx P_0\, e^{t/\tau}$ and $\tau \approx \beta/(\lambda\Delta k)]$ again. This shows the one group formula is valid when $\Delta k/\beta << 1$.

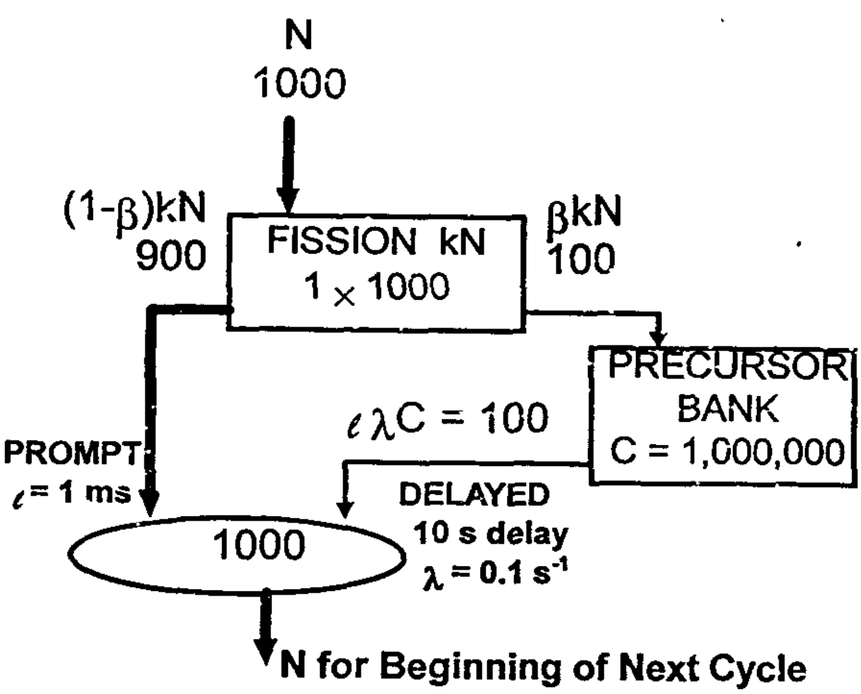
For reference, the accurate formula modelled in figure 8.7 and opposite (with a smoothed prompt jump) can be written:

$$P(t) = P_0 \frac{\beta}{\beta - \Delta k} e^{t/\tau} \times \left[1 - \left(\frac{\Delta k}{\beta}\right) e^{-t/\tau_R}\right] \quad \text{where} \quad \frac{1}{\tau_R} = \frac{\beta - \Delta k}{\ell} + \frac{1}{\tau}$$

When t = 0, the square bracket cancels the jump factor. For typical values, the square bracket becomes ≈ 1 in about ½ s. This equation for P(t) is graphed on the page opposite for a step reactivity insertion of 0.5 mk (Δk = 0.0005) with β = 0.00582, ℓ = 0.88 ms, and λ = 0.0758 s. Notice how little accuracy is lost when the smoothing factor is removed from the "exact" equation, as in equations 8.5 and 8.6 above.

To better understand where this formula comes from, we will now expand on the discussion in section 8.5 of the text.

- Figure 2., represents the initial steady state. Make sure you can reproduce the numbers. Parameters have been changed, as in the text. An unrealistic value for β, the delayed neutron fraction (10% instead of ½%), make it easier to see the effect of the precursor bank.



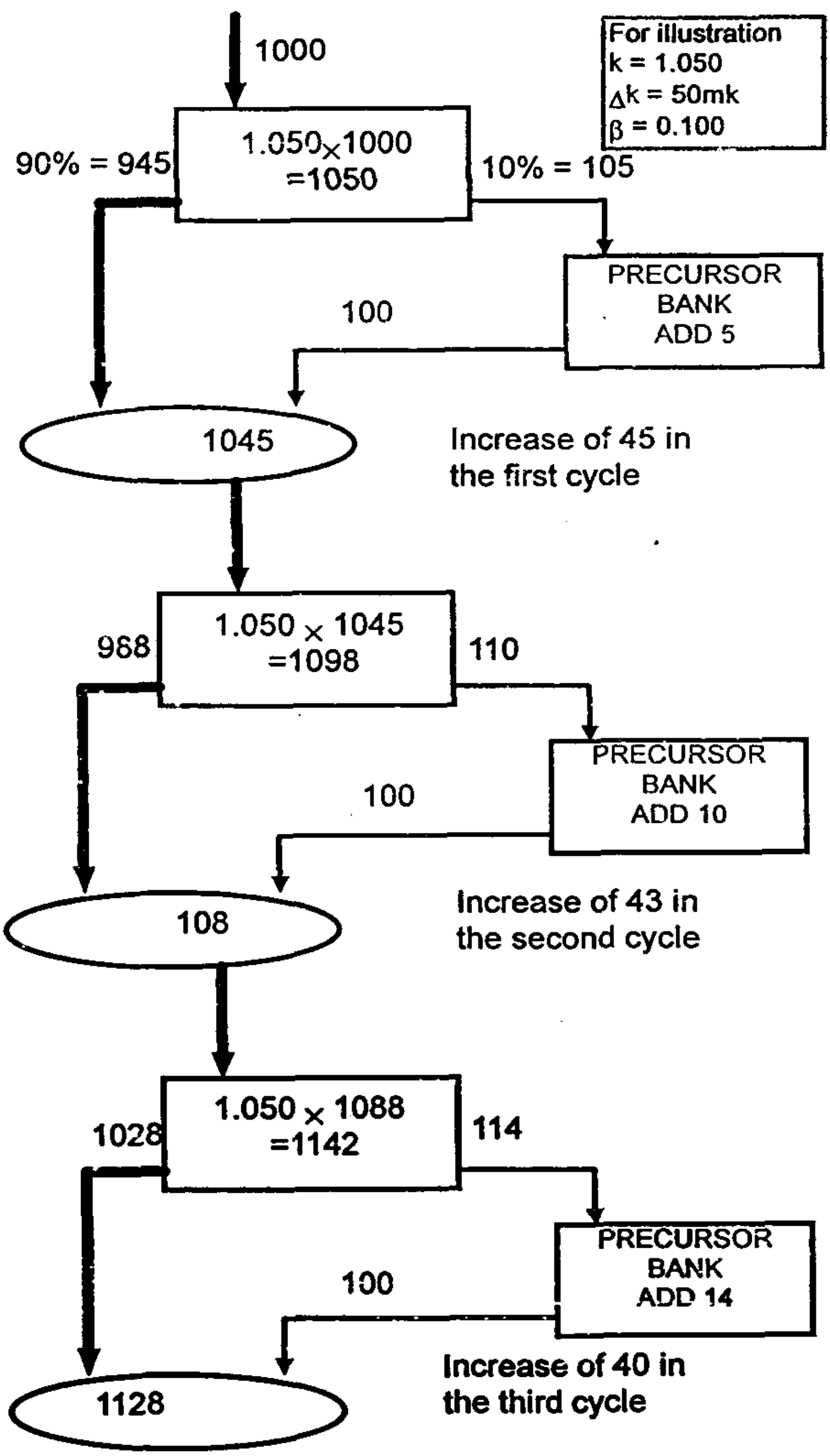
Initially, k =1, N = 1000, with decay constant $\lambda = 0.1\ s^{-1}$, (i.e. delayed neutron lifetime of 10 s), prompt neutron lifetime, $\ell = 0.001$ s and, for illustration $\beta = 0.100$,

The precursor bank concentration, C, is inferred from the decays per second, λC, multiplied by the time for one cycle, ℓ, to give 100 decays per cycle.

Figure 2. Neutron Balance for a Critical Reactor

- In figure 3 notice the rise in prompt neutron population in the first 3 cycles after adding reactivity. The increase is slightly less in successive generations. This is because no extra neutrons have arrived from the precursor bank (delay tank).

Precursors accumulate, but to increase the delayed neutron contribution from 100 to 101, the precursor tank must increase by 10,000 neutrons.



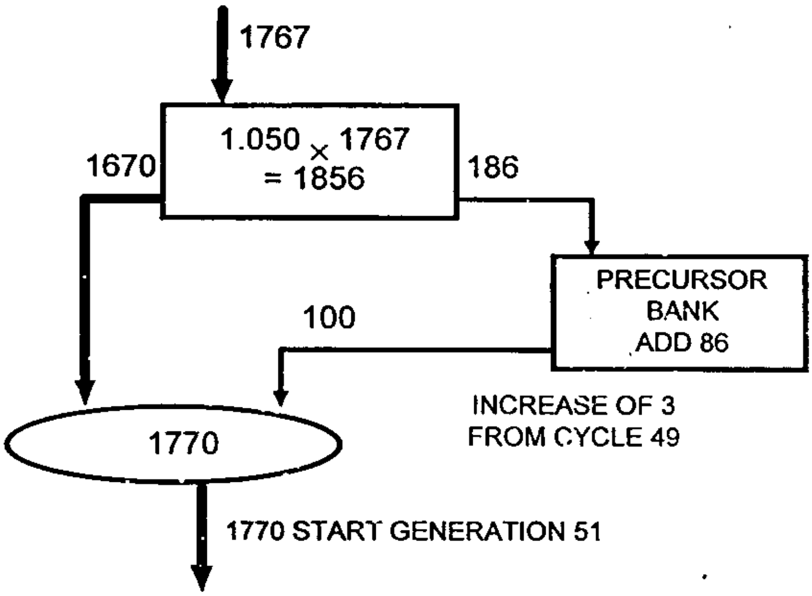
These figures reproduce exactly the values in figure 8.4 of the text.

Initially there is a rapid (but decreasing) increase in the prompt neutron population.

At the same time, the accumulation in the precursor bank starts out small, but increases from one generation to the next.

Continuing with this arithmetic for 50 cycles brings us to figure 5.

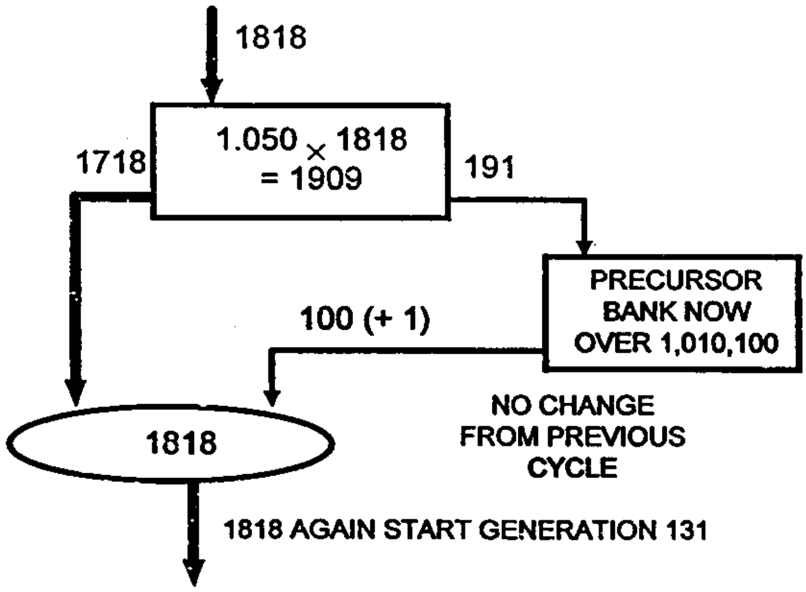
Figure 4. The First Three Cycles after a Step Reactivity Addition



After 50 cycles the prompt population increases by only 3 from one cycle to the next, but precursors are accumulating at about 86 each cycle.

The prompt rise is approaching its end, and the precursor bank is increasing by nearly the same large amount each cycle.

Figure 5. 50 cycles after the initial step reactivity increase



Finally, in figure 6., the precursor bank is big enough to produce an extra delayed neutron.

Just as the prompt rise runs out of steam, extra precursors begin to arrive to continue the rise, which now increases from cycle to cycle.

Figure 6. 130 Cycles After a Step Reactivity Increase

Notice that the short time duration of the prompt jump is governed essentially by the prompt neutron lifetime. It is all over in a few hundred cycles, a fraction of a second. The continued rise after the prompt jump ends depends on the rate of decay of the precursor bank, and is essentially governed by the lifetimes of the delayed neutrons. Of course the size and duration of the prompt jump and the size of the stable reactor period, τ, also depend on the reactivity inserted, Δk, and on the delayed neutron fraction, β, both of which are artificially large in this numerical example.

3. Analysts use numerical methods to solve the power rise problem accurately, keeping track of the individual yields and lifetimes of each group of delayed neutrons. (See table 2.3 in Chapter. 2).

The method just used can be extended using a spreadsheet with separate precursor banks, or other methods may be used. Simulations may keep track separately of as many as 33 groups of delayed neutrons, 6 each from fission products from U-235, U-238 fast fissions, Pu-239 and Pu-241 together with 9 groups of photo-neutrons.

The result is plotted, inaccurately, as Figure 8.8 in the text. An accurate plot allows the stable reactor period to be found for any Δk insertion. The graph duplicates the formula values of τ for $\Delta k << \beta$. (The formula, with smoothing term, also works for values of $\Delta k >> \beta$!)

The graph can be used to find the period for larger values of Δk, including values with $\Delta k \approx \beta$, the <u>prompt critical</u> reactor. The condition for prompt criticality is that the reactor be critical on prompt neutrons alone, ignoring the effect of the delayed neutrons. From figure 2 this condition is:

$$k(1-\beta)N = N \Rightarrow (1 + \Delta k)(1 - \beta) = 1 \Rightarrow 1 - \beta + \Delta k - \beta\Delta k = 1 \Rightarrow \beta = \Delta k/k$$

For CANDU reactors the period at prompt criticality is $\tau \approx 1$ s. There is no sudden change in reactor behaviour at prompt criticality. Effective control is lost well before prompt criticality is reached. Figure 8.8 shows a trip should occur if the period drops as low as 10 s.

CANDU Features That Reduce The Chance Of Prompt Criticality

The features most often quoted in CANDU literature are:

- reactivity devices with slow rates of reactivity addition and limited positive reactivity worth.
- two independent, diverse, fast acting shutdown systems (SDSs) act to limit the power increase and to hold the reactor shut down if other devices fail to limit reactivity addition.

Reactivity additions are inhibited by software, by hardwired logic & by mechanical design. If the regulating system or rod drive interlocks fail, the equipment limits the rate of reactivity addition.

- For example, the small tubing used in the LZC system limits reactivity addition to about + 0.15 mk/s even if the outlet tubing is sheared off.
- Small motors used for rod drives limit the maximum rate of rod withdrawal.

Design constraints are enforced by commissioning, testing, change control, maintenance, control of the operating envelope, defined impairment levels, prompt CRO actions on alarms or impairments, SDS availability.

Additional design features are less often quoted:

- The CANDU positive void coefficient (its worst $+\Delta k$ accident) is "small", and the maximum rate of voiding allows sufficient time for the SDSs to act. *Note that a large negative void coefficient is also unsafe. The core could get into an abnormal voided state followed by void collapse caused, e.g. by actions taken to cool the fuel.*
- The prompt neutron lifetime, $\ell \approx 1$ m s, is longer than the light water reactor prompt neutron lifetime, $\ell \approx 0.3$ ms. This limits the rate of rise. [The longer lifetime results because D_2O moderates less quickly than H_2O and because we over moderate, achieving a well thermalized spectrum.]
- The reactivity insertion for prompt criticality is higher than for reactors with very high fuel burnup, i.e. with high plutonium content. [High plutonium content decreases the Δk required to get a high rate of rise.]

POWER RUNDOWN

The formulas for power increase (fission rate increase) also work when neutron absorber is inserted. Operationally, CANDU reactivity devices, designed to insert positive reactivity slowly, can insert large negative reactivity quickly, giving a large, fast prompt drop. This is discussed in more detail in Chapter 10.

* Neutron Power Rundown: (see Chapter10, figure 10.2).

A prompt drop, $\beta/(\beta - \Delta k)$, with Δk a negative number, is followed by a more gradual power decrease, $\propto e^{-\lambda t}$, as the D.N. precursor bank decays. Since there are several groups of delayed neutrons, there is a sequence of such decays that drop out one at a time as the precursors with shorter half lives disappear. This is followed by the slow gradual rundown as the photo-neutron sources decay.

Photo-neutrons, neutrons generated in heavy water by energetic gamma rays, are amplified by the core configuration. This produces measurable flux until the source fission products decay. Subcritical multiplication of neutron sources is discussed in detail in Chapter 9.

* Thermal Power Rundown: (See figure 10.1).

Notice that thermal power is not proportional to fission rate, especially at low power. This is mainly because of the decay heat, which decreases slowly. Pump heat and ambient losses contribute to non-linearity between thermal and neutron power.

At full power (or <u>immediately</u> after shutdown), the fission product decay heat is 6% to 7% of F.P. This decays to about 3% in 3 minutes, and to below 1% in 8 hours. 3% is a significant number. Many CANDU backup systems (e.g. auxiliary boiler feedwater pump) are sized to handle decay heat after shutdown. Typically they are sized for about 3% full power.

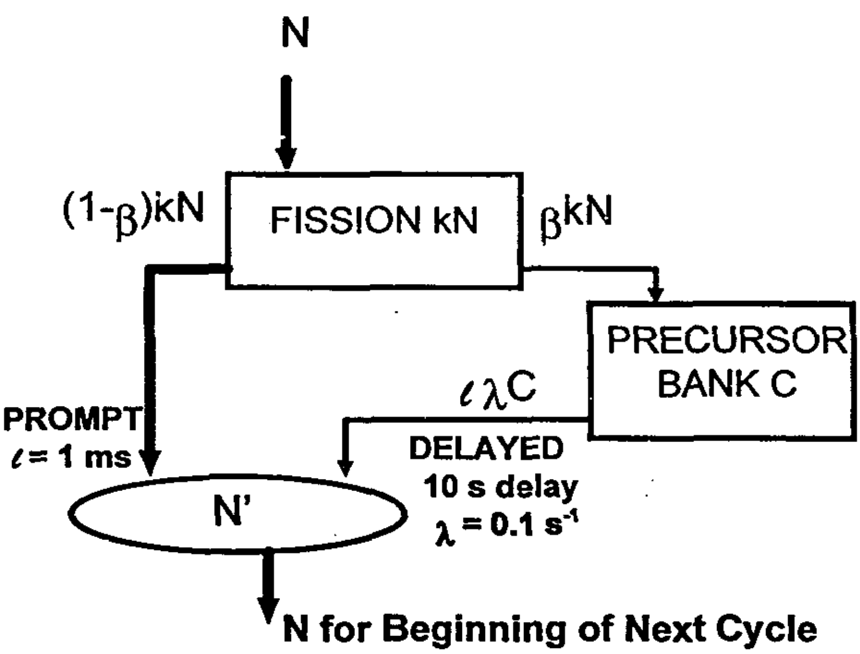
Differential Equations for the Time Response

The differential equations for the thermal neutron density, N(t), and the delayed neutron precursor concentration, C(t), derived in many textbooks, are (for a single delayed neutron group):

$$RateLog(N) = \frac{d\,ln(N)}{dt} = \frac{1}{N}\frac{dN}{dt} = \frac{(\rho - \beta)}{\ell} + \frac{\lambda C}{N} \quad \text{and}$$

$$\frac{dC}{dt} = \frac{\beta k N}{\ell} - \lambda C$$

These equations are consistent with the numerical model, redrawn in figure 7 using algebraic symbols.



$$\frac{dN}{dt} = \frac{(N' - N)}{\ell}$$

$$= \frac{(1-\beta)kN}{\ell} - \frac{N}{\ell} + \lambda C$$

rearranging

$$\frac{1}{N}\frac{dN}{dt} \approx k\frac{\left(k - \frac{1}{k}\right) - \beta}{\ell} + \frac{\lambda C}{N}$$

This is the equation above with (k-1)/k = ρ and setting the overall multiplier k = 1 (small Δk)

Figure 7. A Typical Neutron Generation

Similarly, the accumulation in the precursor bank during the *cycle* is βkN - ℓλC so (dC/dt), accumulation per *second*, is given by $\frac{dC}{dt} = \frac{\beta kN}{\ell} - \lambda C$

Looking at the two terms that contribute to the rate log in

$$RateLog(N) = \frac{d\,ln(N)}{dt} = \frac{1}{N}\frac{dN}{dt} = \frac{(\rho-\beta)}{\ell} + \frac{\lambda C}{N}$$

you may be able to convince yourself that this equation models the dynamic behaviour. Before the reactivity insertion $\rho = 0$ and the rate of change of neutron density (left hand side of the equation) is also zero so we have an initial precursor bank concentration $C_0 = \beta N_0/(\ell\lambda)$

With a step increase in positive reactivity, ρ, the rate goes up quickly as the first term on the right hand side increases immediately. As N increases the $\lambda C_0/N$ term drops and the rate of increase slows as $(\lambda C_0)/(N/\ell)$ approaches $(\beta-\rho)$. Notice that $N \rightarrow N_0\,[\beta/(\beta-\rho)]$ is exactly the change in N required to reduce the rate of increase to zero if the delayed neutron concentration, C, stays at its initial value, C_0. i.e the prompt jump $N_0\,[\beta/(\beta-\rho)]$, increases the population of neutrons, N, just enough to make the right hand side of the equation zero again (temporarily).

As C gradually increases from the initial value, C_0, N and C increase together so there is a constant rate log, corresponding to an exponential power increase. The value of N increases in lock step with C.

Notice that if, somehow, the delayed neutrons could be turned off, [set C = 0 in the equation], with a reactivity of zero the power would be decreasing from one generation to the next. The reactor is subcritical without the delayed neutrons; it depends on the delayed neutrons to "top up" the neutrons in each neutron cycle to keep the reactor critical. On power maneuvers, the dynamic response of the reactor is controlled, apart from the prompt jump, by the slow rate of change of the delayed neutron precursor concentrations.

The initial prompt jump represents a step increase in fission rate over a relatively small number of neutron cycles. The prompt jump is halted when this higher fission rate cannot be "topped up" by the delayed neutrons which have not yet begun to increase.

With a higher production rate of delayed neutron precursors, the concentrations of the delayed neutrons gradually increase, each according to its half life. Subsequent increased decay from the "delayed neutron bank" (or "reservoir") then drives the power up.

If the initial step reactivity, ρ, increase is too big (equal to or greater than the delayed neutron fraction, β) it is impossible for the delayed neutron term, $\lambda C_0/N$, to decrease enough to bring the sum of two terms back to zero, halting the rise. The prompt jump continues without limit and the delayed neutrons do not get a chance to take control. The reactor is prompt critical, i.e. critical on prompt neutrons alone.

The reactor must be operated with $\rho \ll \beta$ to keep the delayed neutrons in control

In summary, the initial rate of rise (prompt jump) is governed by the prompt neutron lifetime, independent of the delayed neutron lifetimes. The subsequent stable rate of rise depends on the rate of buildup of delayed neutron precursors, which is governed by the delayed neutron precursor decay constants.

Time Response in Three Dimensions.

Modelling the flux shape changes that occur during a reactor transient is outside the scope of this course, but it may be useful to introduce some of the jargon here, to make it easier to read the advanced textbooks.

Time dependant reactor behaviour is often introduced in textbooks using the time dependant diffusion equation. This is a differential equation that models the time dependence of the thermal neutron flux distribution. For homogeneous reactors with simple geometric shapes the easiest solution is to separate this equation into time and space equations. The space equation is solved for the flux shape, shown in Chapter 6, and solution of the time equation produces the time dependence discussed in Chapter 8.

This is the *point reactor model*. The flux at each point in the reactor increases at exactly the same rate, leaving the shape unchanged. This model works quite well for fast transients, where shape changes are slower than the power rise. Accurate simulations of reactor behaviour use more sophisticated models.

For example, the separation into space and time dependant parts may be modified to allow slow time dependant shape changes. This results in a modified point reactor model. Parameters in the time equation are averaged over the whole core and these averages change as the shape changes.

Other models solve the static flux shape diffusion equation for the fundamental flux shape and its *harmonic modes*. (The mathematical form of the equations for a cylindrical homogeneous reactor are the same as the solutions for the vibrations of a cylindrical membrane!). The actual flux shape is a superposition of the fundamental and its harmonics, and time dependence of the flux shape is determined from time dependant equations for the amplitudes of the shape components.