Module 9

Source Neutron Effects

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9.1 MODULE OVERVIEW

Neutron sources in reactor

Any CANDU reactor has two ***sources*** of neutrons (other than the fission process itself). The first is ***spontaneous fission*** in the fuel (mainly U-238), and the second is ***photoneutrons*** generated when deuterium nuclei are disintegrated by high-energy gamma rays from certain fission products. These neutron sources are important because they maintain an indication of neutron flux on the instrumentation even when the reactor is shut down.

For safety reasons, it is important that we are able to interpret instrumentation readings so we can determine the ***reactivity*** of the shutdown reactor. In this module, we first show that the fission process itself ***amplifies*** the source neutrons by a factor which is ***inversely proportional to the reactivity***. Operationally, we can use this relation to determine the shutdown reactivity by inserting or withdrawing an extra known reactivity, and measuring the corresponding change in indicated power. We also examine the ***stabilization time*** for the power level to settle down after a change in the subcritical reactivity, and show that this increases the closer the reactor is to the critical condition.

9.2 MODULE OBJECTIVES

After studying this module, you should be able to:

i) State the two inherent sources of neutrons in CANDU reactors, and indicate their approximate magnitudes.

ii) State the formula for the **subcritical multiplication** of a neutron source (explaining what the terms mean) and explain how the subcritical reactor acts as a multiplier of the source power.

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iii) Use the subcritical multiplication formula to solve specific problems (such as calculating shutdown system worth).

iv) For a given change in reactivity, state how the magnitude and stabilization time of the resulting power change depends on the initial reactivity of the (subcritical) reactor.

9.3 NEUTRON SOURCES IN THE CANDU REACTOR

The term "source neutrons" refers to neutrons other than prompt or delayed neutrons arising from the induced fission process itself. The CANDU system has two "built-in" sources of neutrons, one arising from the nature of the fuel itself and the other from the use of the heavy water moderator. These sources are:

Spontaneous fission neutrons

i) ***Spontaneous fission neutrons***. As mentioned in Section 2.8.1, a small number of neutrons is generated in the reactor by spontaneous fission of the uranium (the great majority from the U-238 which constitutes most of the fuel). The contribution to reactor power from fissions caused by the spontaneous fission neutrons is tiny (only about 10^{-12}% of full power).

Photoneutrons

ii) ***Photoneutrons*** are produced in a heavy water reactor as a result of the disintegration of deuterium nuclei by high energy (> 2.2 MeV) gamma rays emitted by certain fission products (see Section 2.9). The strength of the photoneutron source depends on the number and energy of the gamma rays present, which in turn depends on how long the reactor has been operating. For prolonged operation at significant power levels (>10% full power), the gamma flux is proportional to the power. The strength of the photoneutron source at power is typically an order of magnitude or more below the delayed neutron fraction. After shutdown, with the disappearance of the delayed neutron source, the photoneutron source becomes important. It decreases as the inventory of fission products which produce the high-energy gamma rays decays away. The contribution to the power from fissions caused by photoneutrons decreases in an essentially exponential fashion after shutdown so that the photoneutron source power one day after shutdown is typically close to $5 \times 10^{-5}\%$. (The actual value depends on the shutdown reactivity and on the operating power in the weeks before shutdown.) The longest-lived relevant fission product decay chain has a half-life of about 15 days; thus, the photoneutron source persists for several weeks after shutdown.

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9.4 SOURCE MULTIPLICATION IN A SUBCRITICAL REACTOR

In the previous section, we quoted values for the contributions which spontaneous fission neutrons and photoneutrons make to the power. The values given were for fissions caused ***directly*** by the source neutrons themselves, that is, we ignored the fact that these fissions will give rise to additional fission neutrons which can, in turn, cause further fissions. The subcritical reactor, in fact, acts as a ***multiplier*** of source neutrons, so that the actual power generated as a result of the neutron source can be much greater than would be produced by the source neutrons alone. The amount of multiplication produced by the subcritical reactor will be a function of its reactivity, as shown below.

Imagine that a source emitting S_0 neutrons ***in each generation*** is inserted into a subcritical reactor whose multiplication factor, k, is slightly less than 1.

The number of neutrons present at the end of the first generation will be S_0, representing the neutrons emitted during that time. Throughout the next generation, the original S_0 neutrons will be multiplied by the fission process to kS_0, and an additional S_0 will be fed in from the source. The total number of neutrons at the end of the second generation will therefore be

$$S_0 + kS_0$$

Throughout the third generation, these neutrons will be multiplied to $k(S_0 + kS_0)$, and another S_0 will be added from the source, amounting to a total at the end of the third generation of

$$S_0 + kS_0 + k^2S_0$$

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If we extend this argument indefinitely, we will end up, after a large number of generations, with a neutron population (S_∞) given by

$$S_\infty = S_0 + kS_0 + k^2S_0 + k^3S_0 + \dots$$
$$= S_0(1 + k + k^2 + k^3 + \dots)$$

With k less than 1, the sum in the brackets is equal to

$$\frac{1}{1-k}$$

so that we can say that

$$S_\infty = \frac{S_o}{1-k} \quad (9.1)$$

Since k-1 = Δk, this can also be written as

$$S_\infty = \frac{S_o}{-\Delta k} \quad (9.2)$$

Since the fission rate in the subcritical reactor is proportional to the neutron population, we can write equations 9.1 and 9.2 in terms of power rather than neutron population. Thus, the power level in the reactor is

Subcritical multiplication

$$P_\infty = \frac{P_o}{1-k} \quad (9.3)$$

where P_0 is the power that would be generated by the source neutrons themselves in the absence of any multiplication by the fission process in the fuel (that is, if ν were equal to zero).

It is important to realize that even in a reactor that is well below critical (e.g., – 40 mk, typical of a reactor trip), the equilibrium source level (S_∞) is 25 times greater than the actual photoneutron source:

$$S_\infty = \frac{S_o}{1-0.96} = \frac{S_o}{0.04} = 25S_o$$

The factor 1/(1 - k) is called the ***subcritical multiplication factor***. In the example above, the subcritical multiplication factor is 25. Thus, the indicated source level (S_∞) is 25 times greater than the actual source level (S_o). This means that the great majority of neutrons in the system are produced by fission rather than directly by the source. The amount of subcritical multiplication depends only on the value of k. For example, if we used only half of the shutoff rods in the previous example, such that we had -20 mk

$$S_\infty = \frac{S_o}{1-0.98} = 50S_o$$

the subcritical multiplication factor would be 50.

In a subcritical reactor without any neutron source, the neutron population would eventually dwindle to zero. When the source is present, however, we must remember that (at least for k > 0.5) most of the neutrons in the reactor at any time are not neutrons from the source, but neutrons which originate in fissions caused when the source neutrons are multiplied by subcritical multiplication in the fuel.

Example of use of subcritical multiplication

Example of subcritical multiplication

Suppose a reactor is shut down with a constant indicated power of 2×10^{-5}%. The operator inserts +1 mk by withdrawing an adjuster and power stabilizes at 3×10^{-5}%. Find the original value of k.

For the first case, ***before the reactivity addition:***

$$P_{\infty} = \frac{P_o}{1-k_i} \quad (k_i = initial\ multiplication\ factor)$$

$$2 \times 10^{-5}\% = \frac{P_o}{1-k_i}$$

After the reactivity addition

$$3 \times 10^{-5}\% = \frac{P_o}{1-(k_i+0.001)}$$

Since P_0 is the same in both cases, the equations may be solved for k_i.

$$P_0 = (1 - k_i) \times 2 \times 10^{-5}\%$$

$$P_0 = [1 - (k_i + 0.001)] \times 3 \times 10^{-5}\%$$

Hence

$$2(1 - k_i) = 3(0.999 - k_i)$$

$$2 - 2k_i = 2.997 - 3k_i$$

$$k_i = 0.997$$

You can always find the value of k in a shutdown reactor by changing reactivity, noting the power before and after the change and doing a simple calculation.

Notes & References

9.5 STABILIZATION TIME AFTER REACTIVITY CHANGES IN A SUBCRITICAL REACTOR

In a subcritical reactor, power will increase to a new equilibrium value each time positive reactivity is added. The magnitude of the increase and the time it takes for the power to stabilize at the new value will depend on the initial and final values of k. The closer k is to one, the larger the power increase for a given Δk, and the longer the time required to stabilize. This is shown in Figure 9.1, where we start at a power level of $1 \times 10^{-5}\%$ at a k value of 0.90 and add reactivity in +10 mk steps ($\Delta k = 0.010$), allowing P to stabilize after each reactivity addition. (The values chosen are purely for illustration, as making 10 mk *steps* would be quite impossible in practice.)

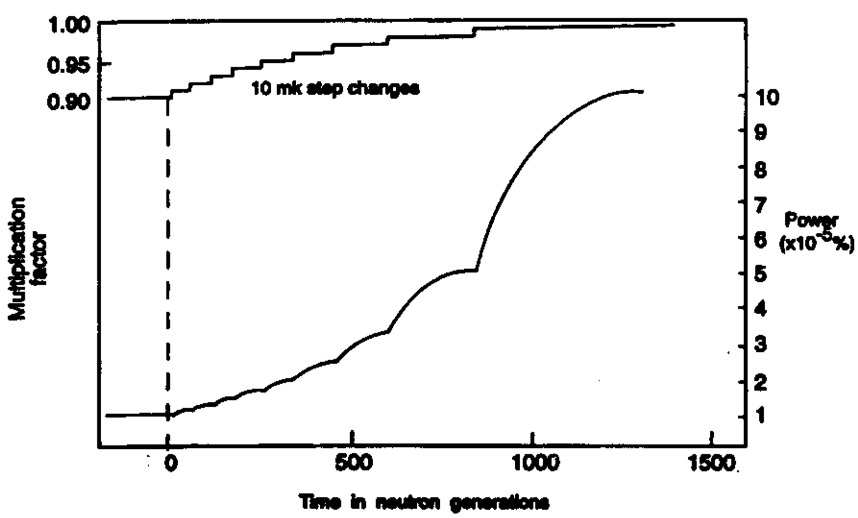

Figure 9.1: Stabilization time after change in multiplication factor

The fact that the change in power for the same reactivity step (Δk) increases as k approaches 1 can be seen in two specific cases:

1. **k changes from 0.90 to 0.91:**

Power at k = 0.90 is $P_{0.90} = \frac{P_o}{1-0.90} = 10P_o$

Power at k = 0.91 is $P_{0.91} = \frac{P_o}{1-0.91} = 11.1P_o$

The change in power = 1.1 P_o (or $P_{0.91}/P_{0.90} = 1.11$; power increases by 11%)

2. **k changes from 0.98 to 0.99:**

Power at k = 0.98 is $P_{0.98} = \frac{P_o}{1-0.98} = 50P_o$

Power at k = 0.99 is $P_{0.99} = \frac{P_o}{1-0.99} = 100P_o$

The change in power = 50 P_o (or $P_{0.99}/P_{0.98} = 2$; power doubles)

It is therefore clear that the change in power for a given Δk increases as k tends towards 1. The fact that the time required for the power to stabilize at the new value becomes larger as k tends to 1 is more subtle. To simplify things, we'll ignore the effect of delayed neutrons. As shown earlier, the final neutron population arising from a source that provides S_o neutrons per neutron generation is

$$S_\infty = S_0(1 + k + k^2 + \ldots..)$$

Formally, this is an infinite series, but, because k is less than 1, each successive term is smaller than the previous one. For practical purposes, we can regard the series as having terminated after n generations if the value of k^n has become negligible in comparison with the first term (that is, 1). Suppose we assume that the series can be considered to have terminated after k^n has become as low as 0.001. Then the number of generations we would have to wait for the power to stabilize would be n, where n is the solution of the equation:

$$k^n = 0.001$$

or

$$n \log k = \log(0.001) = -3$$

(Note: for convenience, use log to base 10, not natural log)

Let's illustrate by taking three selected values of k:

k = 0.80

n log (0.80) = –3, giving n = 31

k = 0.90

n log (0.90) = 3, giving n = 66

k = 0.95

n log (0.95) = –3, giving n = 135

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stabilization time

This clearly shows that the stabilization time increases as we get closer to the critical condition. In practice, especially as we get close to critical, the waiting times for the power to stabilize after each reactivity step become much larger than the calculations above suggest because of the time required by the delayed neutrons (and the photoneutrons) to come into equilibrium at the new reactivity. The effect of delayed neutrons can be roughly taken into account by using an "average" neutron lifetime which is a weighted mean of those of the prompt and delayed neutrons. This gives us a value of about 0.09 seconds (see Section 8.4.2) instead of the 0.001 seconds for the prompt neutrons alone. Using this value, we would estimate that the stabilization time required after the reactor was brought to a k-value of 0.98 from a considerably lower value, would be in the order of 30 seconds (the time required to come within a few percentage points of the equilibrium value). This agrees approximately with predictions from more sophisticated calculations.

9.6 EFFECT OF SOURCES IN THE CRITICAL REACTOR

When the reactor is **critical**, equation (9.1) doesn't apply, since its derivation is based on the assumption that the series $(1 + k + k^2 + k^3 + k^4 + ...)$ has a finite sum. When k = 1, the fission reaction alone is able to maintain the neutron numbers constant from one generation to another, but the source will be adding in an extra S_0 new ones every generation, so that the neutron population will simply increase indefinitely at a constant rate of S_0 neutrons per generation. With the reactor at any significant power level, this rate of increase is negligible in comparison with the neutron population already present, and will be obscured by the automatic regulation of the reactor. In a **supercritical** reactor, any effects of the sources can be ignored.

Notes & References

ASSIGNMENT

1. Describe the two inherent ***sources*** of neutrons in CANDU reactors.

2. Explain why the neutron population in a subcritical CANDU reactor is significantly higher than the source level.

3. Define the ***subcritical multiplication factor***, and write the argument which will enable us to derive an expression for it.

4. The power level in a shutdown reactor with a reactivity of – 40 mk is 8 kW. Calculate the power level after an additional –10 mk of reactivity is inserted.

5. A reactor has a k-value of 0.999. A source of strength S_0 neutrons per generation is placed in it. If the mean neutron lifetime is 0.1 s, calculate the time required for the neutron population to attain a value within 0.1% of the equilibrium value.