Module 5

Neutron Multiplication Factor and Reactivity

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Notes & References

5.1 MODULE OVERVIEW

Neutron Life cycle

This module will explain the ***neutron life cycle***, which is the sequence of events that can happen to fission neutrons between their creation in the fission process and their eventual removal by absorption or leakage. A formula known as ***the six-factor formula*** for the ***neutron multiplication factor*** will be derived from a flow chart illustrating the various processes in the life cycle. This formula allows us to examine the effect of operational changes to the reactor, for example, the movement of control absorbers, build-up of fission products or replacement of fuel. We will discuss the significance of ***neutron leakage*** in determining the overall design of the CANDU reactor. Finally, we will introduce a new quantity, the ***reactivity,*** which is the parameter used on a day-to-day basis to specify changes to the multiplication factor of the system.

5.2 MODULE OBJECTIVES

After studying this module, you should be able to:

i) State the expression for the multiplication factor (k) in terms of the six factors.

ii) Define each of the factors in the six-factor formula.

iii) Sketch a neutron life cycle for the reactor, and explain what happens at each stage.

iv) Calculate the values of the six factors for a life cycle diagram with numerical values on it.

v) Define k_∞ and explain what it means when the core contains a large number of fuel bundles of differing burnup.

vi) Explain why the CANDU reactor is designed in the form of a large cylinder of approximately equal height and diameter.

vii) Define reactivity and the unit used for it (the milli-k).

5.3 THE NEUTRON LIFE CYCLE

In the previous module, we defined the neutron multiplication factor as

Multiplication factor

$$k = \frac{\textit{the number of neutrons in one generation}}{\textit{number of neutrons in the previous generation}} \qquad (5.1)$$

In this section, we will separate out the factors that influence the value of the multiplication factor for a given reactor. This will enable us, in later modules, to examine what happens to the multiplication factor when, for example, the fuel begins to burn up, or temperature changes take place in various components of the reactor.

In a critical reactor, the neutrons that actually maintain the reaction go through a cycle like the one shown below:

Neutron life cycle

Fast neutron born at fission	→	Is slowed down in moderator to become thermalized	→	Thermal neutron is captured in U-235 to cause fission
↑				↓
↑ ← ←		← ← ←		←

On average, 2.43 fast neutrons are produced per fission. In a critical reactor, only one neutron from each fission in U-235 completes this cycle, so that the number of neutrons is constant from one generation to the other (i.e., k = 1). The remaining neutrons are lost by various means. It is important to know how they are lost, because this tells us how we have to design the reactor to reduce this loss, and also how we can regulate reactor power. The best way to understand what is going on is to draw a diagram of the neutron cycle for a typical CANDU, shown in Figure 5.1. For simplicity, we will assume that the only fissile material in the core is U-235, which is the situation at the start of life of a fresh core, before any Pu-239 has had time to build up in the fuel. We start (upper right) with 1,000 fast neutrons produced by thermal fission, and follow their fates as we go round the cycle. The continuous line applies to neutrons that are still contributing to the cycle, while the dotted lines show neutron losses.

The first thing that happens is that we actually gain some neutrons, because fission neutrons are energetic enough for some of them to cause fast fission in the U-238 before they are slowed down below the ***fast fission*** threshold energy (1.2 MeV).

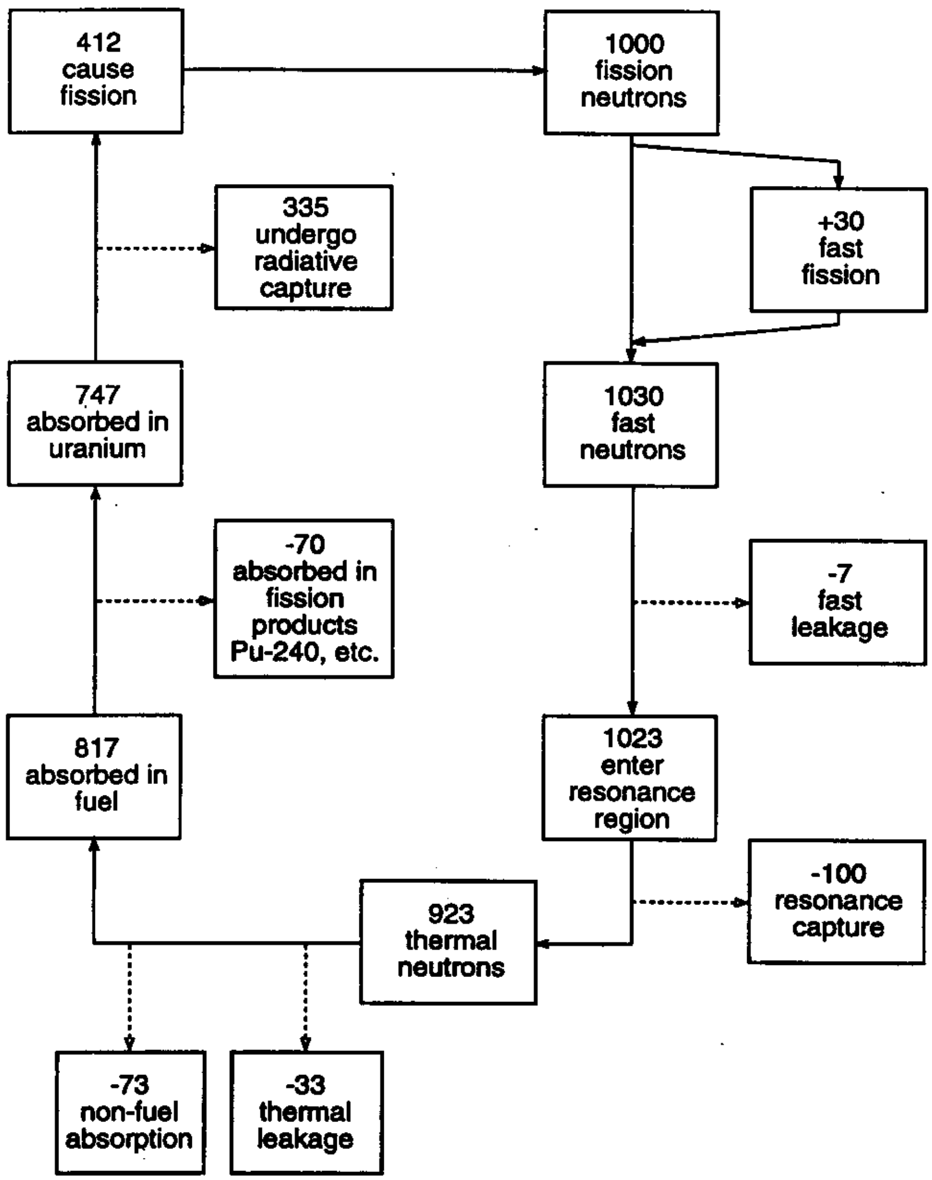

Figure 5.1: Neutron life cycle

The result is an addition of 30 or so neutrons to the cycle at this point. Therefore, 1,030 neutrons will begin their journey down to thermal energies.

On the way down, neutrons are lost from the cycle in two ways. First, and least important, is the loss of fast neutrons which reach the boundary of the reactor and ***leak out.*** The second is capture in the ***U-238 resonances***, which accounts for almost 10% of the losses. Once the neutrons reach thermal energies, some are absorbed in the fuel, but again there are two other processes competing with fuel absorption. A few thermal neutrons will escape from the reactor, and more will be absorbed in ***non-fuel core materials*** , such as the moderator and the pressure and calandria tubes.

All the losses to this point leave just over 80% of the original 1,000 neutrons to be absorbed in the fuel. Some are absorbed in the uranium and some in the fission products and other parasitic absorbers such as Pu-240. Not all of the neutrons absorbed in the uranium give rise to fissions, of course, because some simply undergo radiative capture (in U-235 or U-238). The result, in the case shown, is that only 412 of the original fission neutrons cause fission in the fuel. The average number of fission neutrons per fission in U-235 is 2.43, so that these fissions will produce $412 \times 2.43 = 1{,}001$ fission neutrons (which, allowing for the error introduced by rounding off the numbers used in Figure 5.1, is the same as the number of neutrons we started with). This, of course, is what one expects for a critical reactor where, by definition, the number of neutrons remains constant from one generation to another.

Notes & References

5.4 THE SIX-FACTOR FORMULA

Six-factor formula

Now that we've seen the factors that govern the neutron cycle, let's put the whole thing on a more formal basis. The traditional way of doing this is to introduce a formula known as the **six-factor formula.** We will first define the six factors and then look at how they can help us to describe the neutron cycle. We start by writing the neutron multiplication factor as the product of the six factors (we'll see where this expression comes from in a moment):

$$k = \varepsilon p \eta f \Lambda_f \Lambda_t$$

Now, the definition of each factor:

Fast fission factor

Fast fission factor, denoted by the symbol ε (epsilon). This is the factor by which the fast neutron population increases due to fast fission in U-238 (typically about 1.03 for natural uranium).

$$\varepsilon = \frac{No.of\ neutrons\ from\ thermal\ fission + No.of\ neutrons\ from\ fast\ fission}{No.of\ neutrons\ from\ thermal\ fission} \qquad (5.2)$$

Resonance escape probability, p

Resonance escape probability (p). This is the probability that a neutron will ***not*** undergo resonance capture in U-238 while slowing down. The typical value is 0.90 for natural uranium fuel.

$$p = \frac{Number\ of\ neutrons\ leaving\ resonance\ energy\ range}{Number\ of\ neutrons\ entering\ resonance\ energy\ range} \qquad (5.3)$$

Reproduction factor

Reproduction factor, denoted by the symbol η (eta). This is the number of fission neutrons produced per thermal neutron absorbed in the fuel. Note that in the present course the term "fuel" is taken to include all contents of the fuel bundle with the exception of the cladding (e.g., fissile material, U-238, fission products, Pu-240, etc.).

$$\eta = \nu \frac{\Sigma_f\,(fuel)}{\Sigma_a\,(fuel)} = \nu \frac{\Sigma_f\,(fuel)}{\Sigma_f\,(fuel) + \Sigma_{n,\gamma}\,(fuel)} \tag{5.4}$$

A typical value of η for natural uranium fuel is about 1.2. Note carefully the distinction between η, which is the number of fission neutrons ***per thermal neutron absorbed in the fuel*** and ν, which is the average number of neutrons produced per fission.

Thermal utilization

Thermal utilization (f). This is the fraction of thermal neutrons absorbed by the fuel compared to the total thermal neutrons absorbed in the whole reactor. (It is essential that "fuel" be defined as for η above). A typical value of f is about 0.94.

$$f = \frac{\Sigma_a(fuel)\phi(fuel)}{\Sigma_a(total\ reactor)\phi(total\ reactor)} \tag{5.5}$$

where ϕ (fuel) and ϕ (total reactor) are the average thermal neutron fluxes in the fuel and in the total reactor.

Fast non-leakage probability

Fast non-leakage probability denoted by Λ_f (lambda-f). This is the probability that a fast neutron will not leak out of the reactor. A typical value is about 0.995.

Thermal non-leakage probability

Thermal non-leakage probability, denoted by Λ_f (lambda-t). The probability that a thermal neutron will not leak out of the reactor. A typical value is about 0.98.

The first four factors depend essentially on the materials in the reactor and not on its size. The two non-leakage probabilities depend on the size and shape of the reactor and may also be increased by adding a **reflector**, as we will discuss later. In a later section, we will see how the leakage varies with reactor size and shape.

Let's redraw the cycle diagram using the symbols as defined above instead of the numbers we used before. For convenience, we'll also start at a different point in the cycle and, to make it more general, we won't make the assumption that the system is necessarily a critical one. Instead we'll use the diagram to derive an expression for the neutron multiplication factor for the cycle illustrated (Figure 5.2).

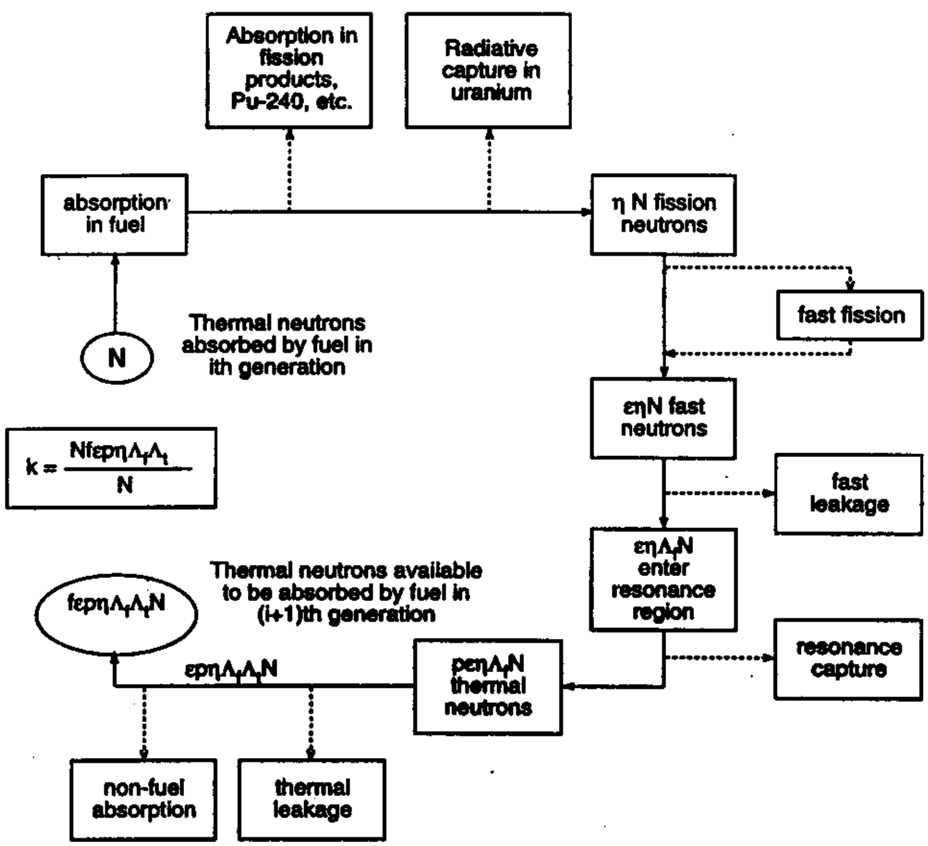

Figure 5.2: Multiplication factor in neutron cycle

Notes & References

The starting point in our diagram this time is the number of thermal neutrons (N) absorbed in the fuel at the beginning of what we'll call the i^{th} generation. From the definition of η given above, the number of fission neutrons resulting from these fuel absorptions will equal ηN. (Remember that the way η is defined takes care of all absorptions in the fuel that do not lead to fissions). On their way down to thermal energies, the fission neutrons will be augmented by fast fission in U-238, so that their number increases to $\varepsilon\eta N$.

We take account of fast leakage by multiplying by the fast non-leakage probability, and of resonance capture in U-238 by multiplying by the resonance escape probability. The number of neutrons that actually reach thermal energies is therefore $\varepsilon p\eta\Lambda_f N$.

We now allow for the thermal neutrons lost by leakage by multiplying by the thermal non-leakage probability; we allow for the loss by absorption in non-fuel materials by multiplying by f, the thermal utilization. We have accounted for all these potential losses, and the expression we have left (the product $N\varepsilon p\eta f\Lambda_f\Lambda_t$), is the number of neutrons absorbed by the fuel at the start of the $(i + 1)^{th}$ generation.

We can now write the expression for the neutron multiplication factor, k, which is simply the number of neutrons in the $(i + 1)^{th}$ generation divided by the number in the i^{th} generation, that is

Six-factor formula

$$k = \frac{N\varepsilon p\eta f\Lambda_f\Lambda_t}{N} = \varepsilon p\eta f\Lambda_f\Lambda_t$$

You'll see that this is the formula (5.1) quoted at the beginning of this section. If the reactor happens to be critical, of course, the product of the six factors is equal to 1. In fact, ***what we do in operating a reactor at steady power is to adjust one or more of the six factors to achieve a k-value of 1.***

Of the six factors, the only ones which can be varied conveniently to make short-term changes in reactivity are the thermal utilization, f, and the leakage terms. Thermal utilization can be changed by varying the amount of absorption in the core, for example, by adjusting the liquid control zones or removing boron from the moderator. In some of the earlier CANDUs, the leakage terms were varied by changing the level of the moderator (and thereby the effective size of the core). The factors ε and p are fixed by the initial design of the core, but η can be varied by removing exhausted fuel bundles and replacing them with fresh fuel, as is done in on-load refuelling.

Before we leave this topic, let's go back to the statement made earlier that the first four factors in the six-factor formula depend on the composition of the reactor and the last two on its size. For an infinitely large reactor, of course, the non-leakage probabilities would both be equal to one, since no neutrons could escape from an infinite system. In that case, k would simply equal the product of the first four terms, and we could write

Definition of k_∞

$$k_\infty = \varepsilon p \eta f \qquad (5.6)$$

where the terminology k_∞ is used to indicate that the k-value here is that for an infinitely large reactor of the composition specified by the appropriate values of ε, p,η and f. Equation (5.6) is known as the **four-factor formula.**

For a reactor of finite size, the relation between k_∞, as defined by the above equation, and the multiplication factor k, is

$$k = k_\infty \Lambda_f \Lambda_t$$

where, as before, the quantities Λ_f and Λ_t are the fast and thermal non-leakage probabilities.

Once a reactor has operated for some time, there will be a wide variation in the composition of the fuel bundles, since each will have a different degree of burnup of U-235 and buildup of Pu-239 and fission product poisons. We may still find it useful, as we will in Module 7, to analyze the changes taking place in an individual bundle as irradiation proceeds. We do this by considering the k_∞ value of that particular bundle (by which we mean the k-value of an infinite reactor whose composition is the same as that of the bundle). The equilibrium core will consist of a large number of bundles with k_∞ values determined by their individual degrees of burnup. The lower k_∞ values of the more highly irradiated bundles will be compensated by the higher k_∞ values of the less irradiated ones.

5.5 NEUTRON LEAKAGE

As mentioned above, fast and thermal non-leakage probabilities depend on the size and shape of the reactor. Let's consider how leakage might be affected by these factors. It's obvious that the ***size*** of the core is going to be a major factor. A single fuel bundle in a large vat of heavy water will be far from critical because too many fission neutrons escape from the fuel never to return (that is, the non-leakage probabilities Λ_f and Λ_t are too low). Let us add more and more properly spaced fuel bundles until the reactor does become critical. This is the minimum size of this assembly of fuel and moderator which will yield a self-sustaining chain reaction, and is called the ***critical size***.

Critical size

The ***shape*** of the reactor also has an important influence on leakage. For example, imagine that we could remove all the fuel bundles from a CANDU reactor and arrange them end-to-end in a single line, surrounded by heavy water. Would this system still be critical? It should be fairly obvious that it would not, because the leakage is far too great; neutrons escaping into the moderator would have a much lower chance of ever encountering fuel again than they did in the previous configuration.

The reason why an extreme geometry of this kind will not work is that the ***production rate*** of neutrons is proportional to the ***volume*** of the core, whereas the ***leakage rate*** is proportional to its ***surface area*** (because that is where the leakage occurs). To reduce the leakage factor, therefore, we want to minimize the ratio of the core surface to its volume. The geometrical shape with the lowest surface to volume ratio is a sphere, but this can be approximated fairly well by using the more practical arrangement of a cylinder whose height is approximately equal to its diameter.

With a full-size CANDU, the height and diameter are both about 6 meters. At this size, the leakage is very small (about 2% for thermal neutrons and 0.5% for fast). This core size is much larger than the minimum needed for a critical mass; for a CANDU 600 with fresh fuel and a full moderator, a "spherical" arrangement of only 100 bundles would be sufficient to produce a critical system. In a cylindrical arrangement of ***filled*** channels around the core central axis, criticality would be achieved with only 16 channels (containing $16 \times 12 = 192$ bundles) loaded. The multiplication factor for a completely fuelled core would clearly be considerably greater than one; this excess is initially held down by added absorbers such as adjuster rods and moderator poison. The moderator poison is gradually removed to compensate for factors such as xenon production and core burnup.

5.6 REACTIVITY

When $k = 1$ and the effects of source neutrons are negligible, the neutron flux and the power level will remain constant. It is important to note that a reactor may be critical at any power level. Telling people that a reactor is critical tells them nothing about the reactor's power output. (By analogy: if I tell you that a car is not accelerating, do you know how fast it is going?).

Under normal operational conditions, a reactor is operating close to criticality (that is, k is nearly equal to 1). It is more convenient under these circumstances to talk in terms of the amount by which k differs from 1 than it is to keep quoting the value of k itself. When we do a precise analysis of the equations that describe the behavior of the reactor power when k is changed, it turns out that the solutions involve a quantity, called reactor *reactivity*, which is defined by the relation.

Reactivity

$$Reactivity = \frac{k-1}{k} \qquad (5.8)$$

If, as is normally the case, we are dealing with values of k which are close to 1, the above expression is nearly equal to k-1 and we can use the following approximation:

$$Reactivity = k - 1 = \Delta k \qquad (5.9)$$

In the rest of this course, we will simply take reactivity to be given by this approximate expression.

Milli-k unit

Because the reactivity changes involved in normal reactor control are always quite small, they are measured in a smaller unit, the ***milli-k*** (abbreviated ***mk***). An example will illustrate the use of this unit:

Suppose we have $k = 1.002$

Then, $\Delta k = 1.002 - 1$

$= 0.002$

In this case, Δk is equal to 2 parts in a thousand, which we call 2 mk, i.e.,

$$\Delta k = 2\ mk$$

A typical CANDU liquid zone control system, as employed for adjusting the reactor power in normal day-to-day operation, has a reactivity range (or "worth") of about ±3 mk.

ASSIGNMENT

1. Put your notes away. Write the six-factor formula, define each of the terms, and sketch the neutron life cycle with the terms used correctly.

2. For k = 0.95, calculate the value of the reactivity obtained from the correct (that is, non-approximated) formula.

3. Calculate each of the six factors, k and k_∞ for the neutron life cycle shown below.

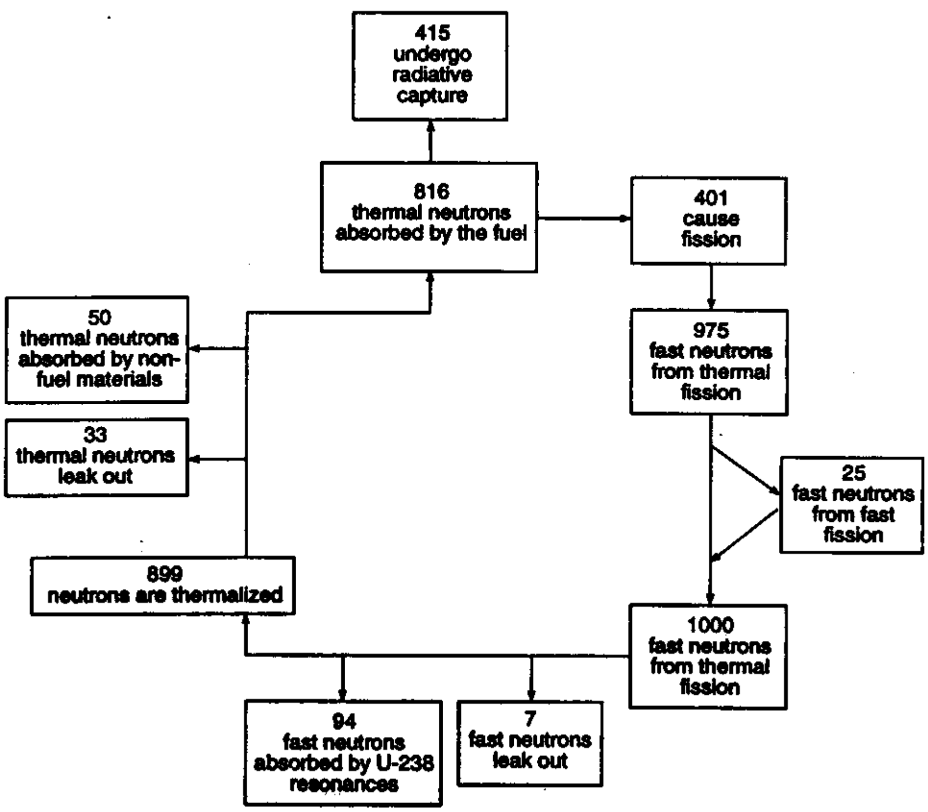

Diagram 5.1

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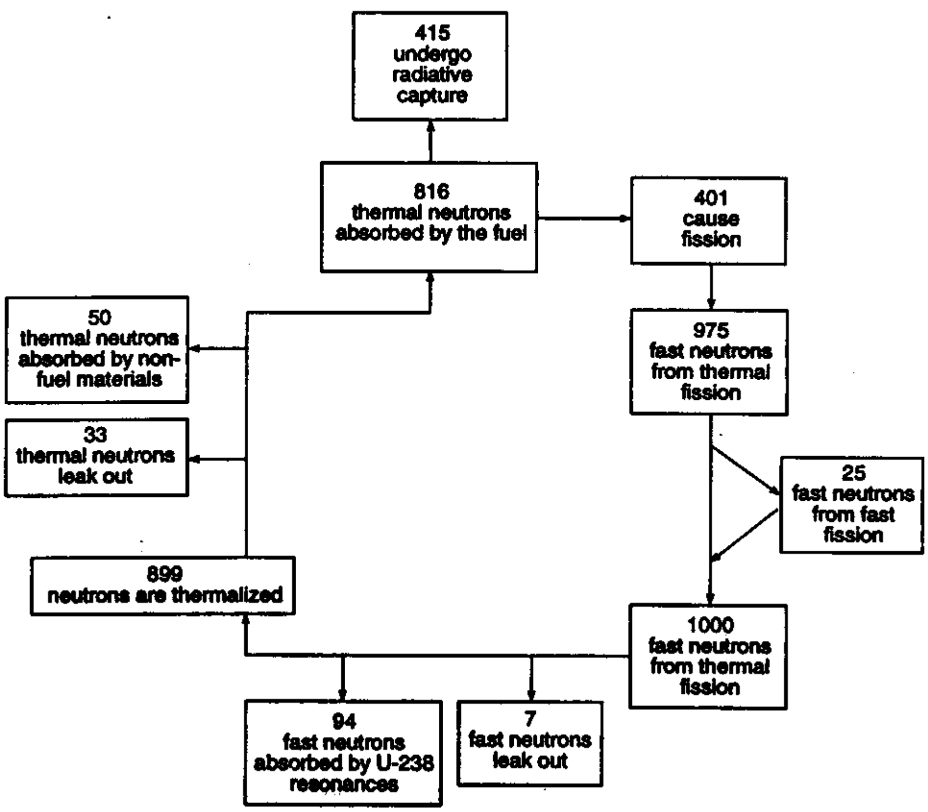

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