

## Fluid Mechanics - Course 223

## FORCE - MOMENTUM

In mechanics, Newton's second law states that if a body experiences an acceleration then a force must have been applied.

Thus  $F = m.a$  where  $F$  is the force  
 $m$  is the mass  
 $a$  is the acceleration.

Acceleration is the rate of change of velocity, ie,  $dv/dt$ .  
 Mass  $\times$  velocity is momentum - thus the force due to a change in velocity is equal to the rate of change of momentum.

$$\text{Thus } F = \frac{m(V_2 - V_1)}{t}$$

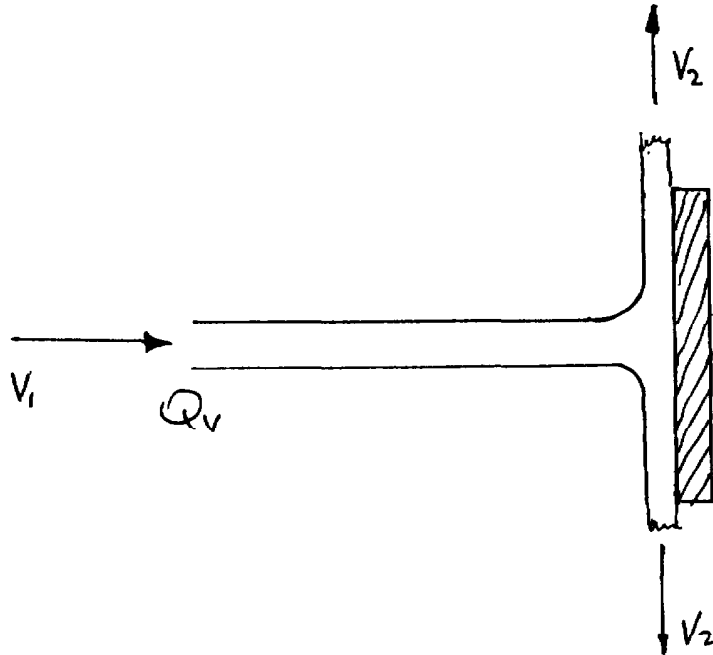
This relationship may be applied to fluid flow. If a change in velocity occurs, then there is a resultant force. If we consider the flow situation

$$F = \frac{m}{t}(V_2 - V_1) \quad V_2 - V_1 = \text{Change in acceleration}$$

$$\frac{m}{t} = \text{mass flow}$$

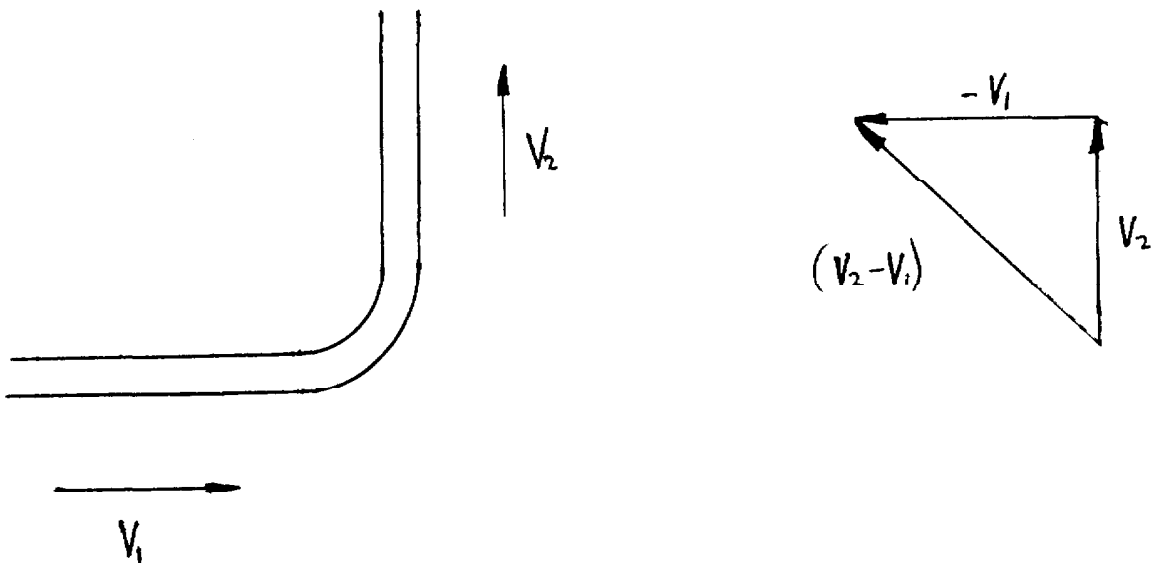
$$\therefore F = \rho AV_1(V_2 - V_1)$$

Thus the force due to the change in velocity of the fluid =  $F = \rho Q_V(V_2 - V_1)$ . When the change in velocity is considered, it should be remembered that the direction must be taken into account as well as the magnitude.



Consider a jet striking a flat plate. The final velocity of the fluid is at  $90^\circ$  to the original velocity and the jet is split in radial directions, thus  $V_2$  is zero. Thus the change in velocity =  $V_2 - V_1 = 0 - V_1$ . Thus the force exerted by the plate, to deflect the jet, has to act in the opposite direction to the flow.

The magnitude of the force =  $\rho Q_v V_1$ . Consider a fluid flowing in a pipe and turning through  $90^\circ$ .



The change in velocity =  $V_2 - V_1$ . ( $V_2 - V_1$ ) is the direction in which the force has to be applied to prevent the pipework moving.

Example

A 12" SCH 40 line carries water at  $0.4 \text{ m}^3/\text{s}$ . Calculate the force exerted on a  $90^\circ$  bend.

$$V_1 = \frac{Q}{A} = \frac{0.4}{722.1 \times 10^{-4}} = \underline{5.54 \text{ m/s}}$$

The change in velocity =  $V_2 - V_1$ .

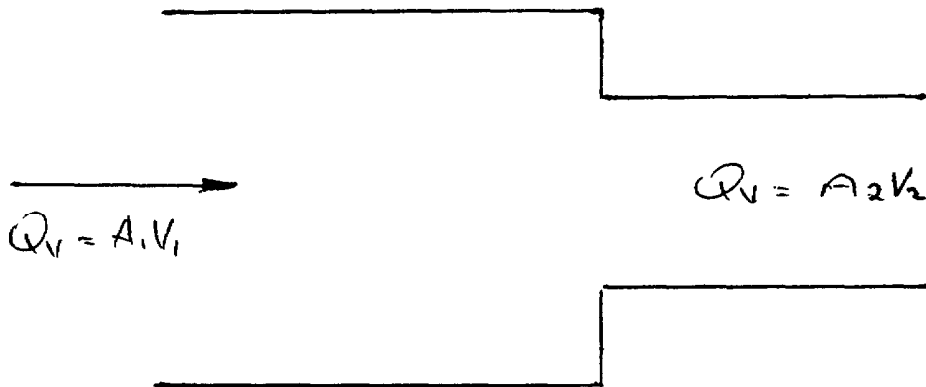
$$\begin{aligned} &= \sqrt{5.54^2 + 5.54^2} \\ &= \underline{7.83 \text{ m/s}} \end{aligned}$$

$$\text{Force} = \ell \times Q_V \times (V_2 - V_1)$$

$$= 1000 \times 0.4 \times 7.83$$

$$= 3130\text{N at } 45^\circ \text{ away from the centre of the bend.}$$

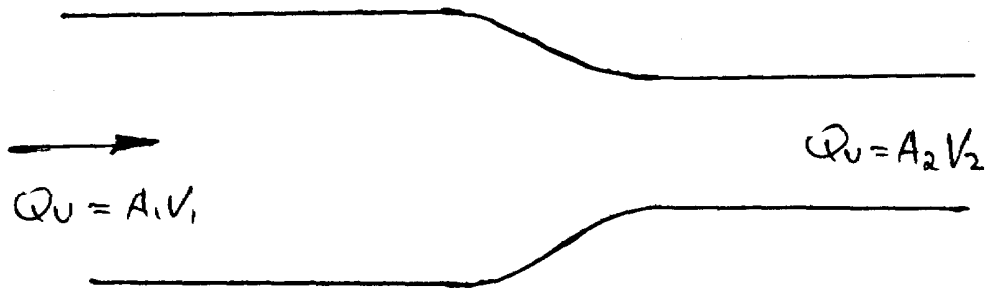
Consider a change in section of pipe.



Force due to change in area.

$$F = \ell \times Q_V \times (V_2 - V_1) \text{ in the direction of } V_2.$$

Consider the reaction force from a jet or nozzle.



$$\text{Force on nozzle} = \ell \times Q_V \times (V_2 - V_1)$$

Example

A 3" fire hose has a 1" nozzle and discharges 500 gpm. Calculate the force required to hold the nozzle steady.

$$F = \ell \times Q_V \times (V_2 - V_1)$$

$$500 \text{ gpm} = \frac{500 \times 10}{62.4 \times 60} = \underline{1.34} \text{ cfs}$$

$$V_2 = \frac{Q_V}{A_2} = \frac{1.34 \times 4 \times 144}{\pi \times 1^2} = \underline{245.7} \text{ fps}$$

$$V_1 = \frac{V_2}{9} = \underline{27.3} \text{ fps}$$

$$\begin{aligned} \text{Force} &= \ell \times Q_V \times (V_2 - V_1) \\ &= 1.94 \times 1.34 \times (245.7 - 27.3) \\ &= \underline{567.8} \text{ lbs} \end{aligned}$$

ASSIGNMENT

1. A 6" line discharges 0.2 m<sup>3</sup>/s, of oil,  $d = 0.8$ , at a plate, which is at 90° to the jet. What is the force required to deflect the jet.
2. A 14" SCH 40 line carries water at 0.35 m<sup>3</sup>/s. Calculate the force on the pipe due to a 60° bend in the line.

3. Oil flows at 1800 gpm to the oil coolers, via a 6" line. The line reduces from 6" to 4". Calculate the thrust on the line due to the section change.  $d = 0.75$ .
4. A 6" line discharges water into a tank. The thrust on the line is 600 lbs. Calculate the flowrate.

J. Irwin-Childs