Mathematics - Course 121

OPERATION IN THE WEAROUT REGION

This lesson discusses the following:

(1) the advantage to system reliability of a program of preventative replacement/maintenance of components, and

(2) calculating the reliabilities of missions extending into the wearout region.

I Preventative Replacement versus Wearout Replacement of Components

A program of preventative replacement of system components, ie, replacement before failure and before entering the wearout region of the bathtub curve (see 121.00-8), can improve system reliability very dramatically, as the following example shows.

Example

A system contains 10,000 identical components, all of which are necessary for system survival, ie, the reliability block diagram, see Figure 1, shows all 10,000 components in series.

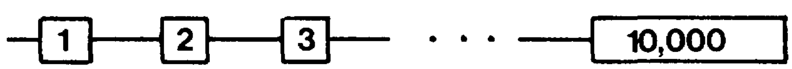

Figure 1 - System Reliability Block Diagram

The system is placed in service at time t = o, and is operated for 10 hours per day. The wearout distribution function is Normal with mean M = 7200 hours and standard deviation σ_1 = 600 hours, see Figure 2. Both early life and useful life failures will be ignored in this example - only wearout failures will be considered.

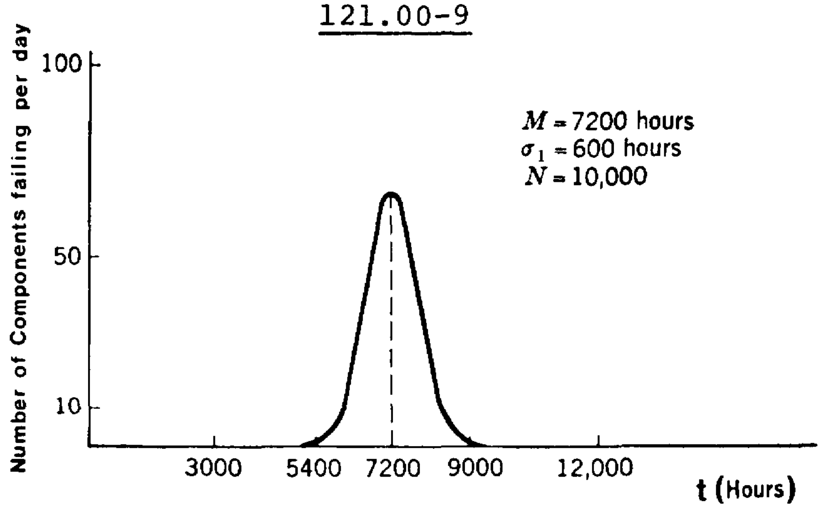

Figure 2 - Wearout Failure Distribution Function

Components are replaced only as they fail. 99.70% of the original components fail within three standard deviations of the mean, ie, between t = 5400 hours and t = 9000 hours. Thus, the second generation of components begins to enter service from about t = 5000 hours on. The failure distribution function (fdf) for second generation failures is centered at t = 2M = 14,400 hours, but the peak height is only about half that for the first generation, and the standard deviation (sd) is about doubled, ie, $\sigma_2 = 2\sigma_1$.

This 'smearing out' of the failure distribution function is due to placing the second generation components in service as the original components fail, ie, gradually over the interval from about t = 5000 hours to t = 9400 hours.

Similarly, the third generation of components, phased in service as second generation components fail, has a fdf centered at t = 3M = 21,600 hours, with a peak height about one-third that of the first generation fdf, and a sd $\sigma_3 = 3\sigma_1$, and so on - see Figure 3.

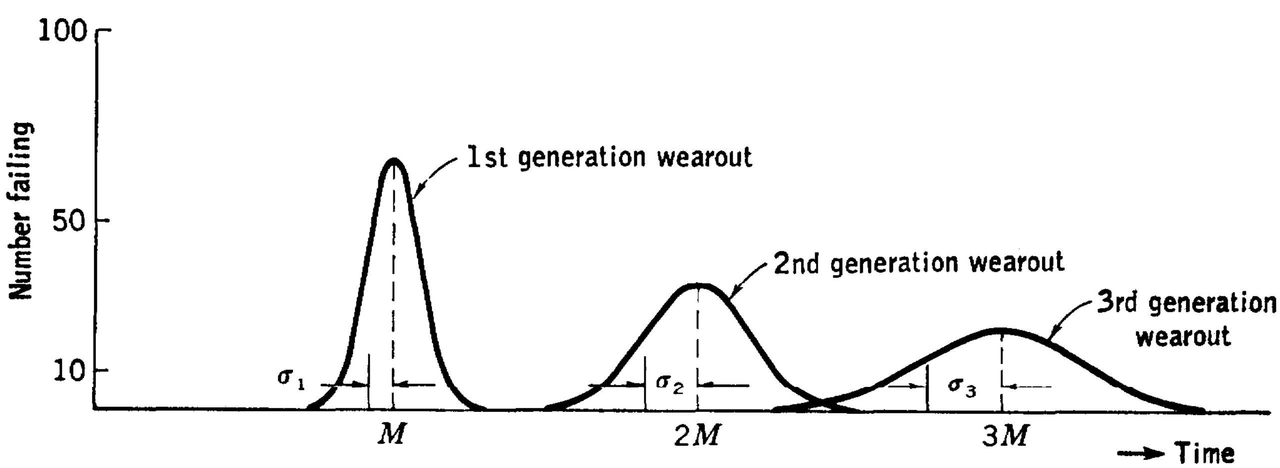

Figure 3 - Wearout Distribution Functions for First, Second and Third Generation Components

Once there is significant overlap of the fdf's for the various generations of components, the system failure rate tends to stabilize to a constant value. The trend to a stable failure rate is evident in Figure 4, which shows not only the system failure rate but also the contributions due to the various generations of components.

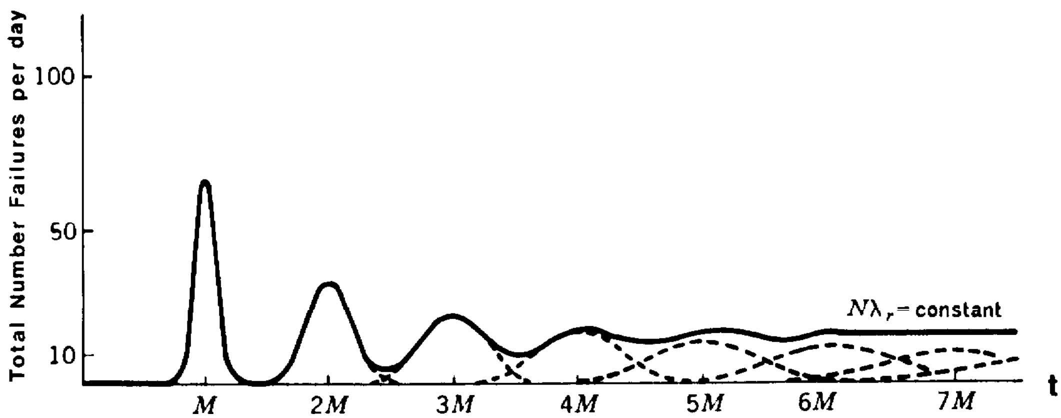

Figure 4 - Stabilization of Failure Frequency

It can be shown that the stabilization time for a system of identical components is

$$T = nM ,$$

where $$n = \frac{M}{3\sigma_1} ,$$

ie, $$T = \frac{M^2}{3\sigma_1} .$$

In the present example, n = 4 and T = 28,800 hours. Figure 4 confirms that the system failure rate is essentially constant after t = 4M. This constant failure rate for a component is the so-called *wearout replacement rate*,

$$\lambda_r = \frac{1}{M} ,$$

and the corresponding system failure rate, due exclusively to wearout failures, is

$$\lambda_s = \frac{N}{M}$$

where N is the number of components.

In this example, $\lambda_r = 0.000139 \text{ hour}^{-1}$ and

$$\lambda_s = 1.39 \text{ h}^{-1} ,$$

which is a very high failure rate. Consequently the system reliability is very low - even for a 1-hour mission the reliability is only

$$R_s(1) = e^{-1.39}$$

$$= 0.37$$

Question: How could system reliability be improved?

Answer: By replacing components preventatively, before wearout. For example, if all 10,000 components in this system were replaced every 3600 hours, the system would become almost failure free. The component unreliability for a 3600 hour mission is the area under the normalized distribution function of Figure 2, to the left of t = 3600 h.

Using the techniques of lesson 121.00-7 and an extended version of the Normal Distribution Table, component unreliability,

$$R_c(3600) = F(-6)$$

$$= 1 \times 10^{-9}$$

Thus system unreliability,

$$Q_s(3600) \doteq 10{,}000\ Q_c(3600)$$

$$= 1 \times 10^{-5}$$

ie, system reliability for each 3600 hour mission between component replacements is

$$R_s(3600) = 0.99999$$

Points Illustrated by this Example

1. That there is a second mode of operation which results in a constant failure rate, namely, *wearout replacement*. (The first mode is, of course, useful life operation, when failures are purely random in time). Thus a constant failure rate, in the absence of further information, cannot be regarded as proof of useful life operation, since failures might be due almost entirely to wearout. The only way to verify useful life operation is to conduct tests and careful statistical analyses to discover the useful life failure rates of components.

2. That if large numbers of components are placed effectively in series, and operated on the wearout replacement scheme, then system reliability can be very poor even if the individual components are highly reliable.

3. That the reliability of a system can be very high even though it contains a large number of components effectively in series, providing components are replaced systematically prior to wearout, and providing the replacements are burned in.

II Missions Extending into the Wearout Region

This section deals with finding the reliability of missions extending into the wearout region.

With reference to Figure 5, let the total failure rate $\lambda(t)$ be written as

$$\lambda(t) = \lambda_u + \lambda_w(t) \quad ,$$

where λ_u is the constant useful life failure rate, and

$\lambda_w(t)$ is the variable wearout contribution to the failure rate.

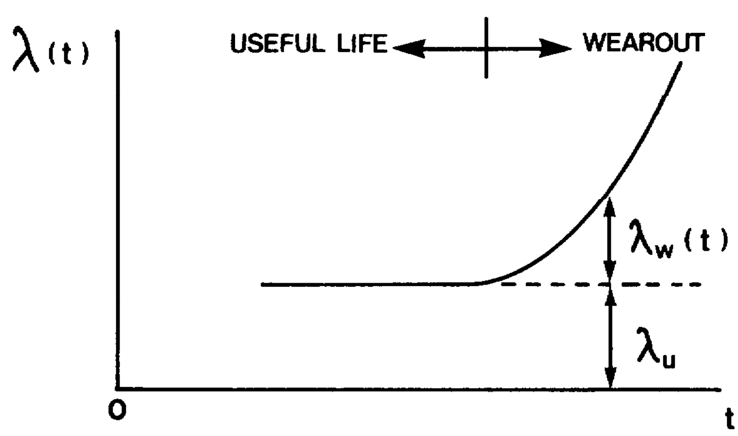

Figure 5 - Failure Rate Curve Showing Useful Life and Wearout Contributions

Let $R(T+t|T)$ represent the reliability of a mission beginning at time T and lasting t units of time, ie, $R(T+t|T)$ is the probability of system survival until time T+t conditional that it survives to time T. Then by PR6,

$$R(T+t|T) = \frac{R(T+t)}{R(T)} .$$

Using the General Reliability Function derived in 121.00-8 gives

$$R(T+t|T) = \exp\left[-\int_T^{T+t} \lambda(t)dt\right]$$

$$= \exp\left[-\int_T^{T+t} (\lambda_u dt + \lambda_w(t))dt\right]$$

$$= \exp\left[-\int_T^{T+t} \lambda_u dt\right] \exp\left[\int_T^{T+t} \lambda_w(t)dt\right]$$

$$= R_u(T+t|T)\ R_w(T+t|T)$$

Thus the mission reliability can be expressed as a product of useful life and wearout factors. The useful life factor,

$$R_u(t) = e^{-\lambda_u t} ,$$

and the wearout factor,

$$R_w(T+t|T) = \frac{R_w(T+t)}{R_w(T)}$$

$$= \frac{\int_{T+t}^{\infty} f_w(t)dt}{\int_T^{\infty} f_w(t)\ dt} ,$$

where $f_w(t)$ is the wearout failure distribution function. If the wearout distribution function is assumed to be Normal, then $R_w(T+t|T)$ is easily evaluated using the Normal Distribution Table of 121.00-7.

The foregoing discussion shows that reliability is a conditional probability. The definition of reliability given in 121.00-2 was called a "working definition", and is adequate for the purposes of this course. A more rigorous definition which takes into consideration component age and the conditional probability character of reliability is given here for reference:

DEFINITION: *The reliability of a component is its conditional probability of performing its function within specified performance limits at a given age, for the period of time intended, and under the operating stress conditions encountered.*

Example

A system has a useful life failure rate of 10 failures per million hours. The wearout failure distribution function is Normal with mean 1000 hours and Standard deviation 100 hours. Calculate the reliability for a mission beginning 300 hours into the system's life and lasting 750 hours.

Solution

The wearout failure distribution function is sketched in Figure 6.

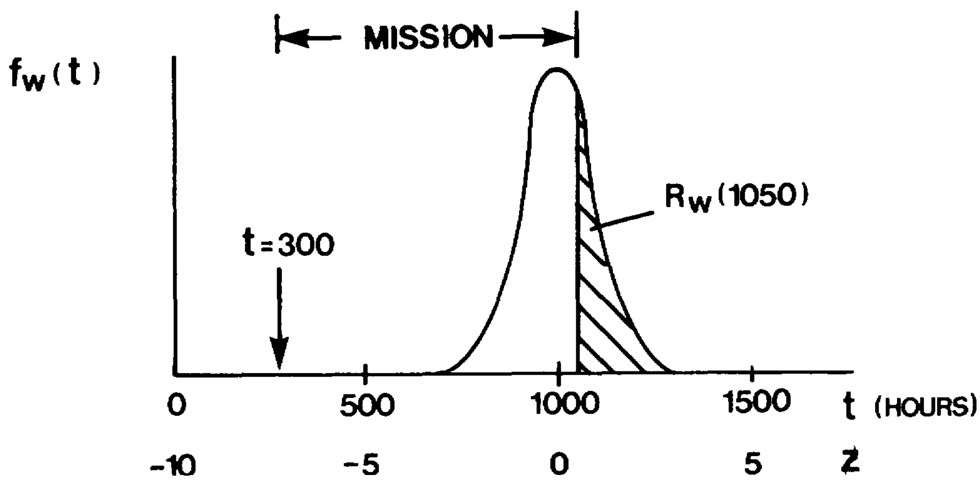

Figure 6 - Wearout Failure Distribution Function

$$R(1050|300) = R_u(1050|300)\ R_w(1050|300)$$

where $R_u(1050|300) = e^{-10^{-5} \times 750}$

$$= 0.9925$$

$$\text{and } R_W(1050|300) = \frac{R_W(1050)}{R_W(300)}$$

$$= \frac{1-F\left(\frac{1050-1000}{100}\right)}{1-F\left(\frac{300-1000}{100}\right)}$$

$$= \frac{1-0.6915}{1-0.0000} \quad \text{(from Normal Distribution Table)}$$

$$= 0.3085$$

$$\therefore R(1050|300) = \underline{\underline{0.3062}}$$

ie, mission reliability is 0.3062. Note that mission reliability is dominated by the wearout factor.

ASSIGNMENT

1. The useful life failure rate of a transformer is 10^{-6} failures per hour. The wearout density function is Normal with mean 50,000 hours and standard deviation 5,000 hours. Calculate the reliability of the transformer for a 1,000 hour mission, assuming the transformer has already seen 45,000 hours' service.

2. List and briefly describe two modes of system operation which result in constant system failure rates.

3. Explain how a system employing a large number of reasonably reliable components, all of which are necessary to system survival, can be operated such that the system reliability is also reasonably high.

4. Explain why following a program of preventative replacement/ maintenance of components improves the system reliability dramatically over that obtained by following a program of wearout replacement of components.

L. Haacke