

Nuclear Theory - Course 227  
CHANGES IN REACTOR POWER WITH TIME

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Reactor kinetics is the study of how neutron power changes with time. As a preface to this discussion it must be recognized that neutron density ( $n$ ), neutron flux ( $\phi$ ), and neutron power ( $P$ ) are all related by physical or design constants such that they all behave in a similar manner. Their relationships are:

$$P = EV\Sigma_f\phi$$

$$\phi = nv$$

where:

- $P$  = neutron power
- $E$  = energy released per fission
- $\Sigma_f$  = the macroscopic fission cross section
- $\phi$  = neutron flux
- $V$  = volume of the reactor
- $n$  = neutron density
- $v$  = average neutron velocity

### Prompt Kinetics

First we shall examine the behaviour of a reactor without delayed neutrons. The change of the neutron density in one generation is:

$$\Delta n = kn - n$$

where:

- $kn$  = the neutrons in one generation
- $n$  = the neutrons in the preceding generation
- $k$  = neutron multiplication factor

The time period over which this takes place ( $\Delta t$ ) is one neutron lifetime ( $\ell$ ).

Thus:

$$\frac{\Delta n}{\Delta t} = \frac{kn - n}{\ell}$$

or:

$$\frac{dn}{dt} = \frac{kn - n}{\ell} = \frac{n}{\ell} (k - 1) \times \frac{k}{k}$$

$$\frac{dn}{dt} = \frac{nk}{\ell} \frac{k-1}{k}$$

Recall that:

$$\frac{k-1}{k} = \Delta k$$

Normally  $k$  is very close to one so that:

$$\frac{\ell}{k} \approx \ell$$

We can now rewrite the equation as:

$$\frac{dn}{dt} = \frac{n}{\ell} \Delta k$$

The solution to this equation is:

$$n(t) = n_0 e^{\frac{\Delta k t}{\ell}}$$

where:

$$n_0 = \text{original neutron density}$$

Since both neutron flux and neutron power behave in a similar manner we can write:

$$P(t) = P_0 e^{\frac{\Delta k t}{\ell}} \quad (1)$$

Equation (1) shows that power changes exponentially with time and that the rate of change of power depends on the reactivity ( $\Delta k$ ) and the neutron lifetime ( $\ell$ ).

### Reactor Period

In operating reactors it is convenient to have an indication of how long it takes for power to change by a given amount (e.g. how long it takes for power to double or increase by a certain percentage). The most common measure in Candu reactor is how long it takes power to increase by a factor of  $e^*$ . This time interval is called the reactor period  $T$  (tau).

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\*  $e$  is the base for natural logarithms and is used simply for mathematical convenience.  
 $e = 2.7183$

To illustrate what the reactor period is, consider the reactor power after one reactor period (ie,  $t = T$ ).

$$P(T) = eP_0 \quad (\text{ie, power has increased by a factor of } e)$$

Also:

$$P(T) = P_0 e^{\frac{\Delta k}{\ell} T}$$

Therefore:

$$eP_0 = P_0 e^{\frac{\Delta k}{\ell} T}$$

$$e = e^{\frac{\Delta k}{\ell} T}$$

Clearly:

$$1 = \frac{\Delta k}{\ell} T$$

and:

$$T = \frac{\ell}{\Delta k} \quad (\text{reactor period for a reactor with only prompt neutrons})$$

Thus we can rewrite equation (1) as:

$$P(t) = P_0 e^{\frac{t}{T}} \quad (2)$$

Equation (2) is a valid expression for power as a function of time considering that we have only prompt neutrons.

To gain a feel for what this means, consider such a reactor with  $\Delta k = 0.5 \text{ mk}$ ;

$$T = \frac{0.001 \text{ s}}{0.0005} = 2 \text{ s}$$

This means that, with the reactor only slightly supercritical ( $k = 1.0005$ ), power is increasing by a factor of  $e$  ( $\sim 270\%$ ) every 2 seconds. That is about 176% per second\*.

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\*Do not confuse this with rate  $\log N$  which would be 50%/s for this example.

This is an unacceptable rate of power change because it would be mechanically impossible to build a regulating system which could respond to such changes rapidly enough to safely control the reactor. Fortunately the fission process produces delayed neutrons which radically alter the time response from that of prompt neutrons alone.

### Effect of Delayed Neutrons

In Level 3 Nuclear Theory we simply assumed that the delayed neutrons increased the average neutron lifetime. This simple treatment is not only computationally inaccurate but it also fails to predict the physical way in which delayed neutrons affect the reactor. A more complex treatment is required for deeper understanding.

Again we will look at the time rate of change of the neutron density  $\frac{dn}{dt}$ , which can be written as:

$$\frac{dn}{dt} = \underbrace{\frac{kn}{\ell} (1 - \beta)}_{\text{Term 1}} + \underbrace{\lambda C}_{\text{Term 2}} - \underbrace{\frac{n}{\ell}}_{\text{Term 3}}$$

where:

Term 1 represents the production of prompt neutron in the present generation

Term 2 represents the production of delayed neutrons in the present generation

Term 3 represents the total neutrons in the preceding generation

$\lambda$  = delayed neutron precursor decay constant

$C$  = delayed neutron precursor concentration

With some mathematical manipulation:

$$\begin{aligned} \frac{dn}{dt} &= \frac{n}{\ell} (k(1 - \beta) - 1) + \lambda C \\ &= \frac{kn}{\ell} \left( \frac{k - k\beta - 1}{k} \right) + \lambda C \\ &= \frac{kn}{\ell} \left( \frac{k - 1}{k} - \beta \right) + \lambda C \end{aligned}$$

Finally:

$$\frac{dn}{dt} = \frac{kn}{\ell} (\Delta k - \beta) + \lambda C \quad (3)$$

Equation (3) partially describes how 'n' is changing; however,  $\lambda C$  is not a constant. We continually create delayed neutron precursors while other precursors, created earlier, decay to give us delayed neutrons. So we must describe how the precursor concentration changes:

$$\frac{dC}{dt} = \underbrace{\frac{kn}{\ell} \beta}_{\substack{\text{precursor} \\ \text{creation} \\ \text{rate}}} - \underbrace{\lambda C}_{\substack{\text{precursor} \\ \text{decay} \\ \text{rate}}}$$

Recalling that there are six groups of precursors you can see that we would have seven simultaneous differential equations to solve.

For a computationally accurate prediction of power changes, these equations are solved on a computer. For the purpose of understanding; however, we assume an average behaviour of the delayed neutrons which reduces the problem to solving two equations with a solution of the form:

$$P(t) = A_0 e^{a_0 t} + A_1 e^{a_1 t}$$

If we assume:

- a)  $\Delta k < \beta$
- b) a step change in  $\Delta k$  occurs at time zero
- c) neutron density was constant prior to the insertion of reactivity.

The solution (with certain approximations) is:

$$P(t) = P_0 \left( \underbrace{\frac{\beta}{\beta - \Delta k} e^{\frac{\lambda \Delta k}{\beta - \Delta k} t}}_{\text{1st Term}} - \underbrace{\frac{\Delta k}{\beta - \Delta k} e^{-\frac{\beta - \Delta k}{\ell} t}}_{\text{2nd Term}} \right) \quad (4)$$

A mathematical and graphical solution for a typical set of conditions is shown on the next page.

Assume typical values

$$\beta = 0.0065$$

$$\Delta k = 0.001 \text{ (1 mk)}$$

$$\lambda = 0.1 \text{ s}^{-1} \text{ (average for all precursor groups)}$$

$$\lambda = 0.001 \text{ s}$$

$$\frac{P(t)}{P_0} = [1.18 e^{0.0182 t} - 0.18 e^{-5.5 t}]$$

This solution is plotted as figure (1).

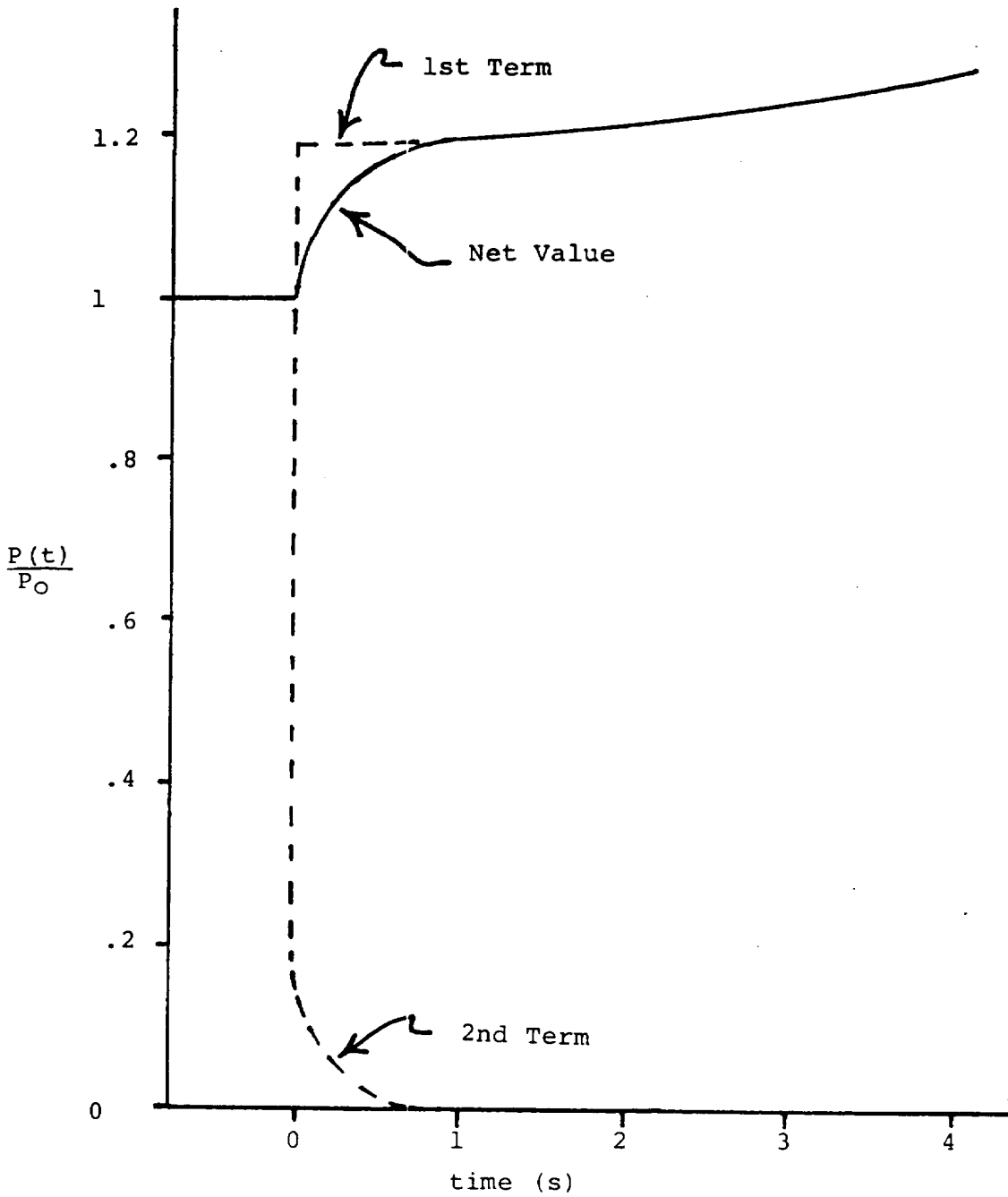


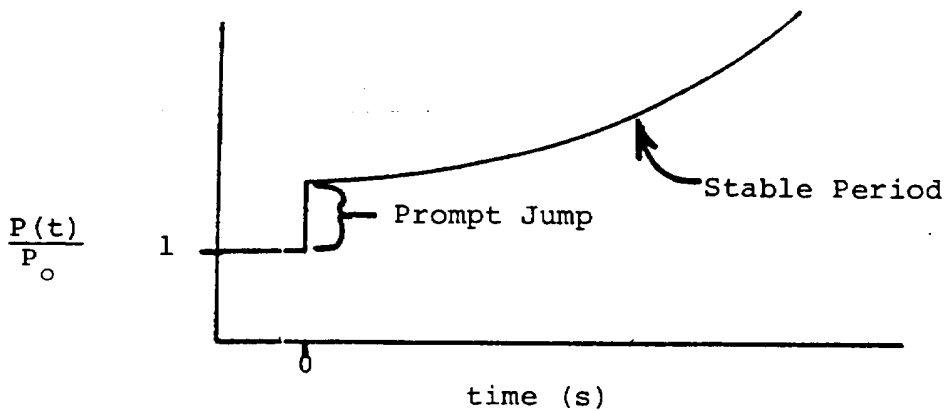
Figure 1

From this example you can see that the second term of equation (4) dies away rapidly and can usually be ignored.

If we do neglect the second term we have:

$$P(t) = \frac{\beta}{\beta - \Delta k} P_0 e^{\frac{\lambda \Delta k}{\beta - \Delta k} t} \quad (5)$$

Graphically this simplified equation is:



This behaviour is called a "prompt jump" followed by a "stable period" where the stable period  $(T) = \frac{\beta - \Delta k}{\lambda \Delta k}$ .

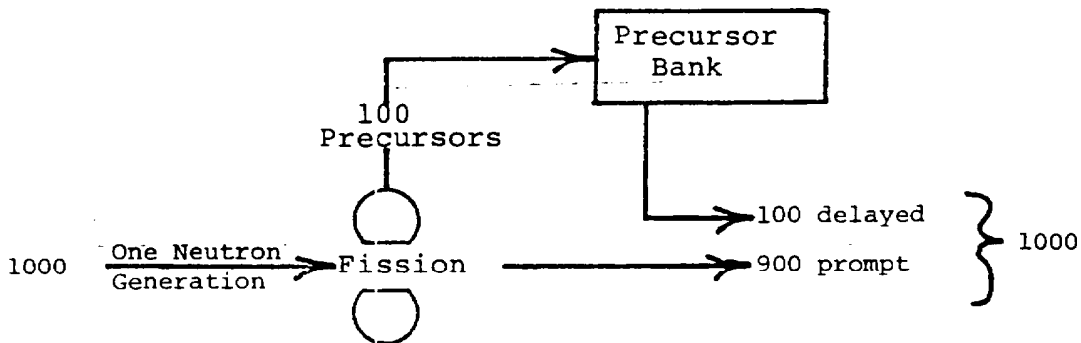
Physical Effect of Delayed Neutrons

To understand what is physically happening we will look at a simple numerical example using a greatly exaggerated value for  $\beta$ . This is done only for numerical simplicity and in no way alters the qualitative results.

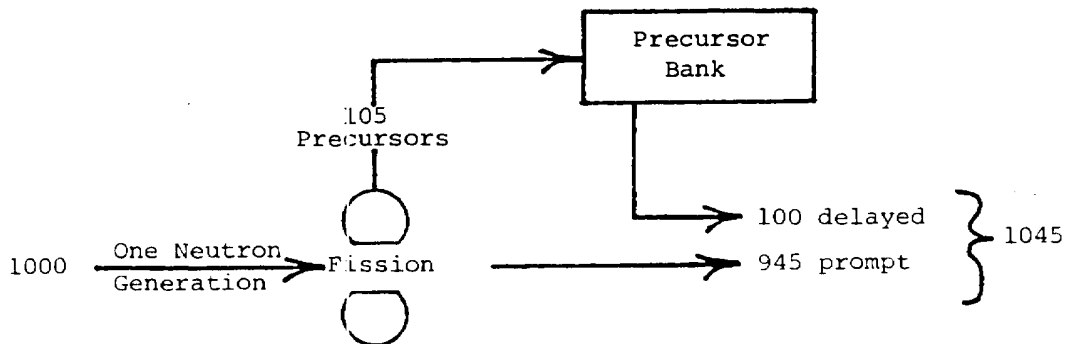
Assume:

$$\begin{aligned} \beta &= .1 \\ \Delta k &= .05 \\ n_0 &= 1000 \end{aligned}$$

Prior to the reactivity insertion:



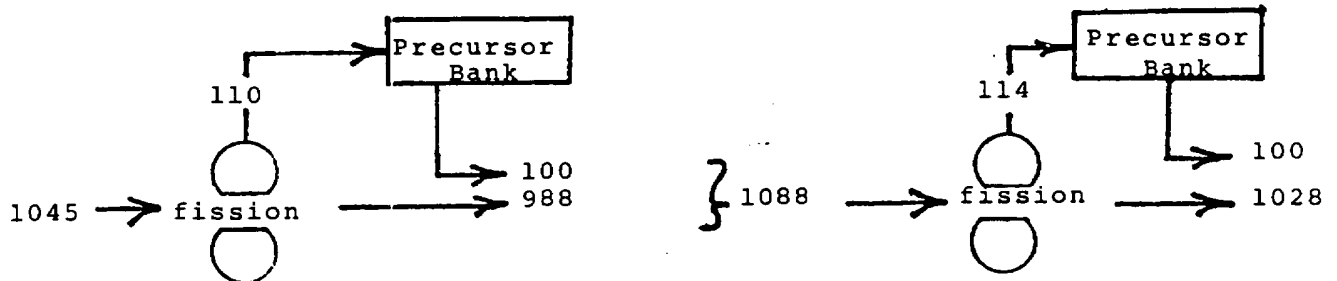
The chain reaction is being maintained at a level of 1000 neutrons per generation. Now insert 50 mk of reactivity such that  $k = 1.05$ :



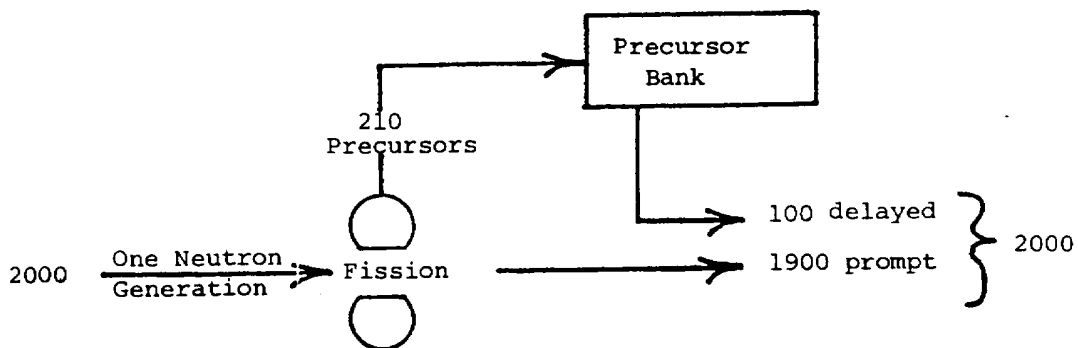


Even though we create 105 precursors we get only 100 delayed neutrons from the precursor bank since it contains precursors produced earlier.

This chain proceeds as follows:



If we assume the output of the precursor bank does not change for a second, we have time for one thousand prompt generations in which time the series will converge to:



Thus in a very short time period we get a jump in the prompt neutron level but power can not increase beyond a certain point until more precursors start to decay. Therefore, after the prompt jump, the rate of power increase is determined by the decay rate of the delayed neutron precursors. Calculating the magnitude of the prompt jump for this problem using equation (5).

$$P = \frac{\beta}{\beta - \Delta k} P_0 = \frac{.1}{.1 - .05} 1000 = 2000$$

### Approximate Numerical Effect of Delayed Neutrons

Going back to the problem we solved without delayed neutrons we shall now see what the effect of delayed neutrons really is.

Consider a reactor with U-235 as the fuel then:

$$\begin{aligned}\beta &= 0.0065 \\ \lambda &\approx 0.1 \text{ s}^{-1}\end{aligned}$$

The reactor period with  $\Delta k = 0.5 \text{ mk}$  is:

$$\begin{aligned}T &= \frac{\beta - \Delta k}{\lambda \Delta k} = \frac{0.0065 - 0.0005}{(0.1)(0.0005)} \\ &= 120 \text{ s}\end{aligned}$$

and the overall power function is:

$$\begin{aligned}P(t) &= P_0 \frac{\beta}{\beta - \Delta k} e^{\frac{t}{T}} \\ \frac{P(t)}{P_0} &= 1.08 e^{\frac{t}{120}}\end{aligned}$$

After the initial prompt jump to 108% of  $P_0$  power increases with a period of 120 s.

Figure 2 shows the power rise for a step insertion of reactivity. As you can see the average lifetime approximation (from Level 3) fails to predict the rapid initial rise in power caused by the multiplication of prompt neutrons. This rapid rise in power is an important consideration in the design of all reactivity mechanisms. In order to limit any rapid increase in power all reactivity mechanisms are designed to limit the rate of reactivity addition.

### Prompt Criticality

It may have occurred to you to ask why we restricted  $\Delta k$  to being less than  $\beta$ . For one thing the equations we derived are no longer valid but more importantly the increase in power is no longer dependent on delayed neutrons if  $\Delta k > \beta$ . Return to our numerical example with a value of  $\Delta k = .15$  with  $\beta = .1$ .

$$\begin{array}{l} 100 \text{ delayed} \\ 900 \text{ prompt} \end{array} \quad 1000 \times 1.15 \times (1 - .1) \rightarrow 1040 \text{ prompt}$$

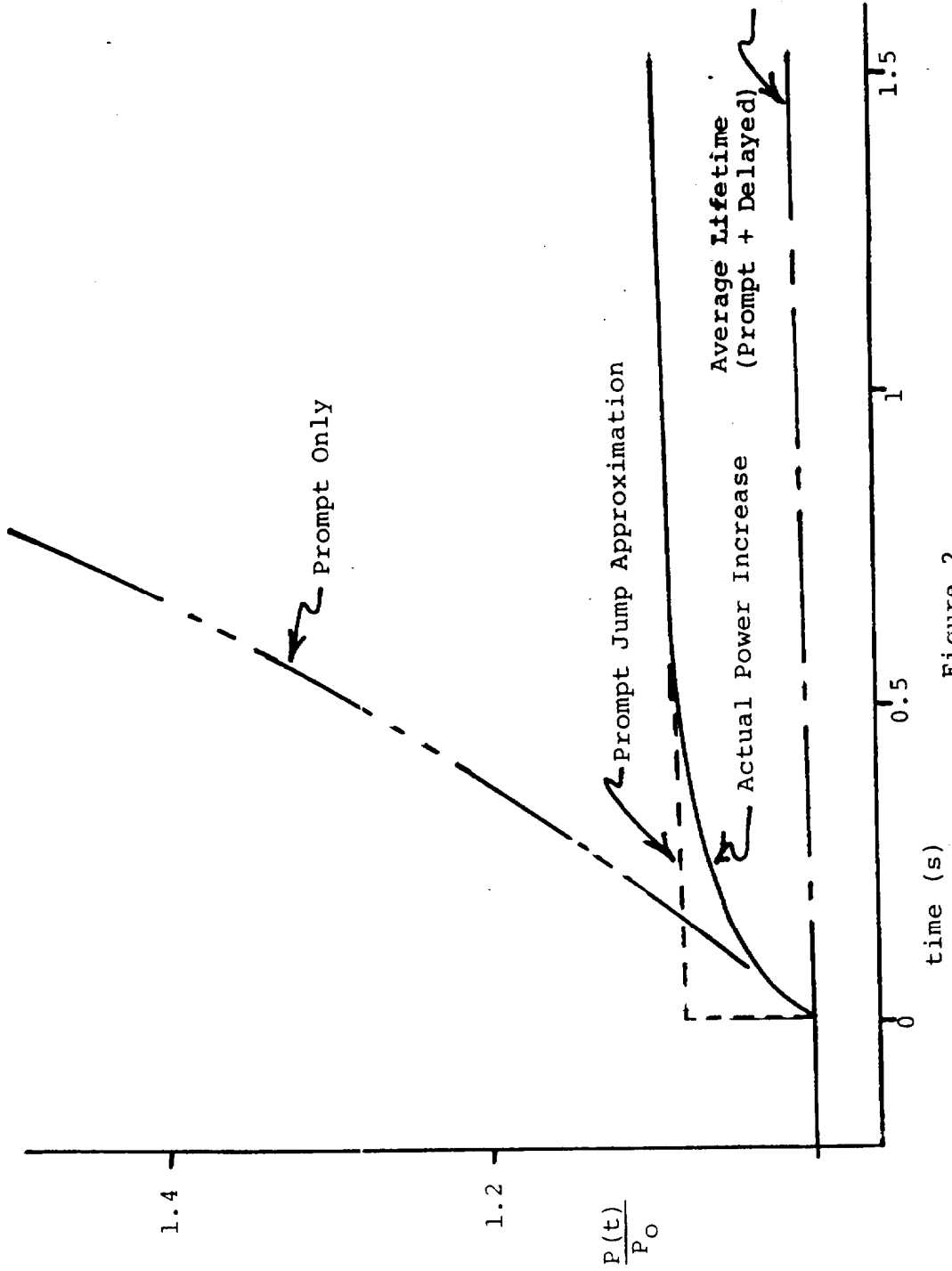


Figure 2

Power vs Time  
 (for a step insertion of  
 0.5 mk at  $t = 0$ )

As you see power is increasing without having to "wait" for the delayed neutrons. This shortens the reactor response time.

When  $\Delta k = \beta$  the reactor is critical on prompt neutrons alone, hence the name "prompt critical".

Figure 3 shows reactor period versus  $\Delta k$  for a reactor with a prompt lifetime of .001 seconds (i.e. the Candu reactors).

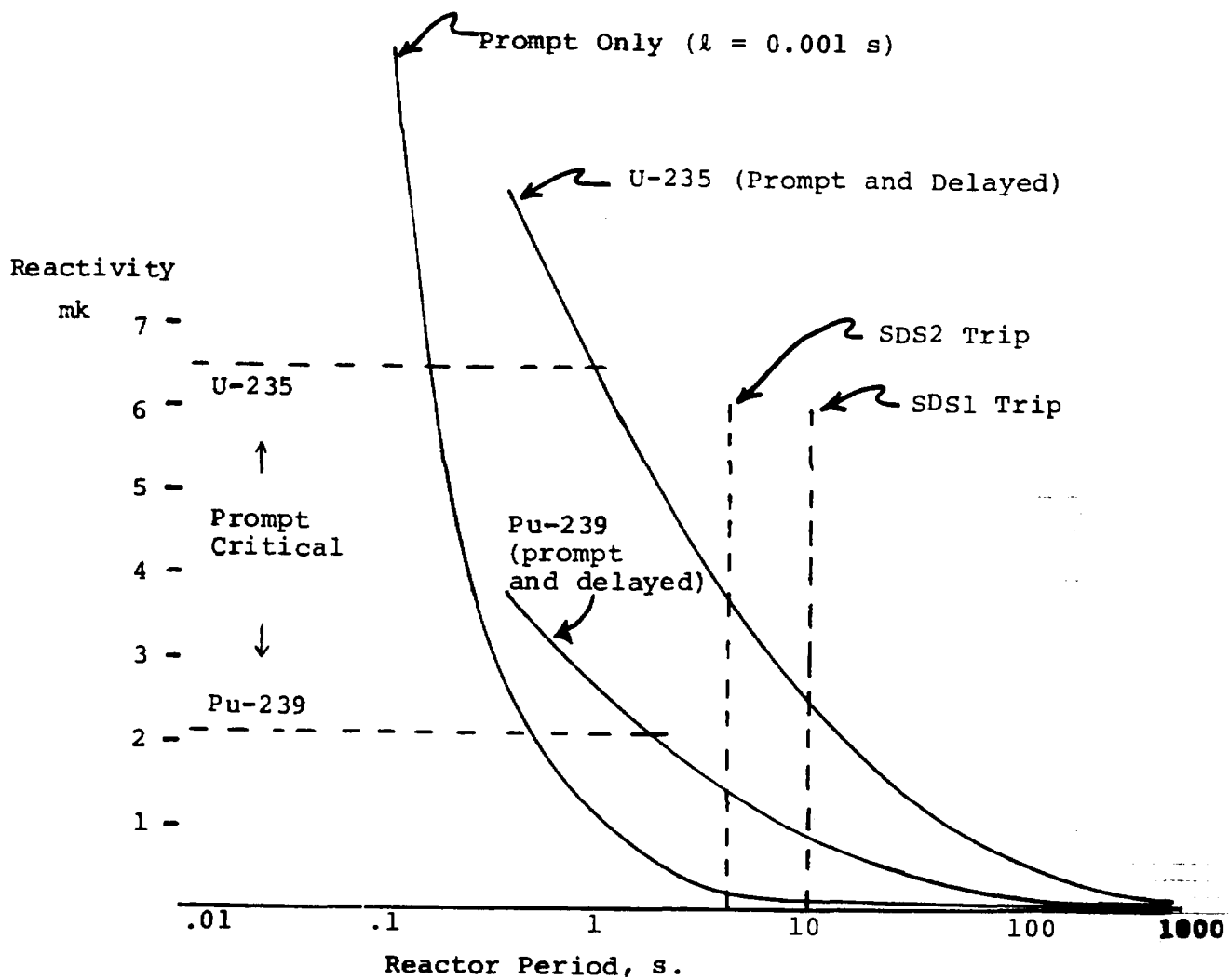


Figure 3

Reactivity v Period

As you can see nothing radically happens when the reactor approaches prompt criticality, the chain reaction is simply becoming less dependent on the delayed neutrons, hence power is changing more rapidly. In this regard we use the avoidance of prompt criticality as a design limit. (SDS1 trips the reactor at a period of 10 s and SDS2 trips the reactor at a period of 4 s. Both of these are below prompt criticality,  $T \approx 1$  s).

Also included on figure (3) is the plot of period versus reactivity for a reactor with only Pu-239 ( $\beta = 0.0021$ ) as a fuel. You will note that SDS1 and SDS2 trip set points provide adequate protection even in this situation. As we approach equilibrium fuel in our reactors we get closer to this situation. At equilibrium fuel prompt criticality occurs at  $\Delta k \approx 0.0035$ . The practical consequence of this is that the reactor regulating and protection system design must be based on the worst case which is equilibrium fuel.

#### Large Negative Reactivities (Reactor Trips)

The equations developed for the prompt jump are equally valid for any insertion of negative reactivity except that you have a prompt drop followed by a stable negative period. Recalling that:

$$T = \frac{\beta - \Delta k}{\lambda \Delta k}$$

if:

$$|\Delta k| \gg |\beta|$$

$$T \approx -\frac{1}{\lambda}$$

Thus the stable reactor period will be determined by the decay constant of the delayed neutron precursors. In fact it will be determined by the longest lived group of precursors, thus, the shortest reactor period possible after the prompt drop will be -80 s. In our reactors we have a very significant production of neutrons from the photoneutron reaction with deuterium thus the actual period will be somewhat longer.

#### ASSIGNMENT

1. Define reactor period (T).
2. Write the expression for reactor period considering both prompt and delayed neutrons.
3. Explain physically the way in which delayed neutrons effect the time response of neutron power.

4. Calculate reactor power 10 seconds after a step insertion of +2 mk of reactivity for fresh fuel ( $\beta = 0.0065$ ) and for equilibrium fuel ( $\beta = 0.0035$ ).  $P_0 = 50\%$  and  $\lambda = 0.1 \text{ s}^{-1}$ .
5. What do we measure on our reactors that is related to reactor period? What is the relationship?

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